CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the May/June 2014 series

4037 ADDITIONAL MATHEMATICS

4037/12 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

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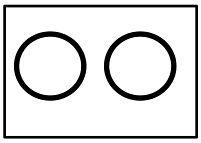
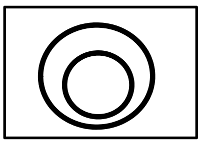
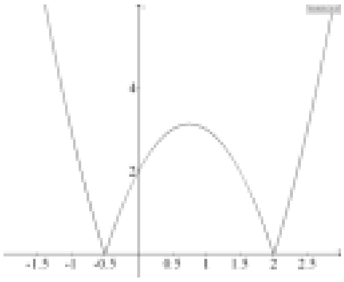
1	$\frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)\cos A}$	M1	**M1** for obtaining a single [illegible] correctly
	$\frac{\cos^2 A + 1 + 2\sin A + \sin^2 A}{(1+\sin A)\cos A}$	M1	**M1** for expansion of $(1+\sin A)$ and use of identity
	$= \frac{2(1+\sin A)}{(1+\sin A)\cos A}$	DM1	**DM1** for factorisation and cancelling of $(1+\sin A)$ factor
	$= \frac{2}{\cos A} = 2\sec A$	A1	**A1** for use of $\frac{1}{\cos A} = \sec A$ and final answer
	Alternative:		
	$\frac{\cos A(1-\sin A)}{(1+\sin A)(1-\sin A)} + \frac{1+\sin A}{\cos A}$	M1	**M1** for multiplying first term by $\frac{1-\sin A}{1-\sin A}$
	$= \frac{\cos A(1-\sin A)}{1-\sin^2 A} + \frac{1+\sin A}{\cos A}$		
	$= \frac{\cos A(1-\sin A)}{\cos^2 A} + \frac{1+\sin A}{\cos A}$	M1	**M1** for expansion of $(1-\sin A)(1+\sin A)$ and use of identity
	$= \frac{1-\sin A}{\cos A} + \frac{1+\sin A}{\cos A}$	M1	**M1** for simplification of the 2 terms
	$= \frac{2}{\cos A} = 2\sec A$	A1	**A1** for use of $\frac{1}{\cos A} = \sec A$ and final answer
2 (a) (i)		B1	
(i)		B1	
(b) (i)	6	B1	
(ii)	5	B1	
(iii)	9	B1	

3 (i)		**B1** **B1** **B1**	**B1** for shape **B1** for $y = 2$ (must have a graph) **B1** for $x = -0.5$ and 2 (must have a graph)
(ii)	Maximum point occurs when $y = \frac{25}{8}$	**M1**	**M1** for obtaining the value of y at the maximum point, by either completing the square, differentiation, use of discriminant or symmetry.
	so $k > \frac{25}{8}$	**A1**	Must have the correct sign for **A1** Ignore any upper limits
4	$\int_0^a \sin 3x \, dx = \frac{1}{3} \, dx = \frac{1}{3}$	**B1,B1**	**B1** for $k\cos 3x$ **only**, **B1** for $-\frac{2}{3}\cos 3x$ **only**
	$\left[-\frac{2}{3}\cos 3x\right]_0^a = \frac{1}{3}$	**M1**	**M1** for correct substitution of the correct limits into their result
	$\left(-\frac{2}{3}\cos 3a\right) - \left(-\frac{2}{3}\right) = \frac{1}{3}$	**A1**	**A1** for correct equation
	$\cos 3a = 0.5$	**M1**	**M1** for correct method of solution of equation of the form $\cos ma = k$
	$3a = \frac{\pi}{3}$, $a = \frac{\pi}{9}$	**A1**	**A1** allow 0.349, must be a radian answer
5 (i)	$2^{5x} \times 2^{2y} = 2^{-3}$ leads to $5x + 2y = -3$	**B1, B1** **DB1**	**B1** for 2^{2y}, **B1** for 2^{-3}, **B1** for dealing with indices correctly to obtain given answer
(ii)	$7^x \times 49^{2y} = 1$ can be written as $x + 4y = 0$	**B1** **B1**	**B1** for either 7^{4y} or 7^0 seen **B1** for $x + 4y = 0$
	Solving $5x + 2y = -3$ and $x + 4y = 0$ leads to	**M1**	**M1** for solution of their simultaneous equations, must both be linear
	$x = -\frac{2}{3}$, $y = \frac{1}{6}$	**A1**	**A1** for both, allow equivalent fractions only

6 (a)	**YX** and **ZY**	**B1,B1**	**B1** for each, must be [illegible] order,
(b)	$\mathbf{B} = \mathbf{A}^{-1}\begin{pmatrix} 3 & 9 \\ -6 & -3 \end{pmatrix}$,	**M1**	**M1** for pre-multiplication by $\mathbf{A}^{-1}$
	$= -\frac{1}{3}\begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} 3 & 9 \\ -6 & -3 \end{pmatrix}$	**B1,B1**	**B1** for $-\frac{1}{3}$, **B1** for $\begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix}$
	$= -\frac{1}{3}\begin{pmatrix} -9 & 3 \\ -18 & 21 \end{pmatrix}$ or $\begin{pmatrix} 3 & -1 \\ 6 & -7 \end{pmatrix}$	**DM1** **A1**	**DM1** for attempt at matrix multiplication **A1** allow in either form
	Alternative method:		
	$\begin{pmatrix} 5 & -2 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 3 & 9 \\ -6 & -3 \end{pmatrix}$	**M1**	**M1** for a complete method to obtain 4 equations
	Leads to $5a - 2c = 3$, $5b - 2d = 9$ $-4a + c = -6$, $-4b + d = -3$	**A2,1,0**	–1 for each incorrect equation
	Solutions give matrix	**M1**	**M1** for solution to find 4 unknowns
	$-\frac{1}{3}\begin{pmatrix} -9 & 3 \\ -18 & 21 \end{pmatrix}$ or $\begin{pmatrix} 3 & -1 \\ 6 & -7 \end{pmatrix}$	**A1**	**A1** for a correct, final matrix

7	(i)	$\sin\frac{\theta}{2}=\frac{6}{8}$, $\frac{\theta}{2}=0.8481$ or better or $12^2=8^2+8^2-128\cos\theta$	**M1**	**M1** for a complete method for either θ or $\frac{\theta}{2}$
		$\theta=1.6961$ or better	**A1**	Answer given.
		or using areas $\frac{1}{2}\times 12\times 2\sqrt{7}=\frac{1}{2}8^2\sin\theta$ oe $\sin\theta=0.9922$, $\theta=1.4455$ or 1.6961	**M1** **A1**	**M1** for using the area of the triangle in 2 different forms **A1** for choosing the correct angle.
	(ii)	Arc length = $(2\pi-1.696)\times 8$	**M1**	**M1** for correct attempt at a minor or major arc length
		$(36.697$ or $36.7)$	**A1**	**A1** for correct major arc length, allow unsimplified
		Perimeter = $12+(2\pi-1.696)\times 8$ $=48.7$	**A1**	**A1** for 48.7 or better
	(iii)	Area $=\frac{8^2}{2}(2\pi-1.696)+\frac{8^2}{2}\sin 1.696$	**M1,M1**	**M1** for correct attempt to find area of major sector
		$=178.5,\ 178.6,$ awrt 179	**A1**	**M1** for correct attempt to find area of triangle, using any method
		Alternative: Area = $\pi 8^2-\left(\frac{1}{2}8^2(1.696)-\frac{8^2}{2}\sin 1.696\right)$		**M1** for attempt at area of circle – area of minor sector **M1** for area of triangle

8 (a) (i)	720	**B1**	
(ii)	240	**B1**	
(iii)	Starts with either a 2 or a 4: 48 ways	**B1**	allow unevaluated
	Does not start with either a 2 or a 4: 96 ways (i.e. starts with 1 or 5)	**B1**	allow unevaluated
	Total = 144	**B1**	must be evaluated
	Alternative 1:		
	Ends with a 2, starts with a 1,4 or 5 : 72 ways Ends with a 4, starts with a 1,2 or 5 : 72 ways Total =144	**B1** **B1** **B1**	
	Alternative 2:		
	$240-\left(2\times 2\times {}^{4}P_{3}\right)$ or $\left(4\times {}^{4}P_{3}\times 2\right)-\left(2\,{}^{4}P_{3}\right)$ $=144$	**B2** **B1**	**B2** for correct expression seen, allow *P* notation
	Alternative 3:		
	${}^{3}P_{1}\times {}^{4}P_{3}\times {}^{2}P_{1}$ or $3\times 4\times 2$ $=144$	**B2** **B1**	Allow *P* notation here, for **B2**
(b)	With twins : ${}^{16}C_{4}$ $(=1820)$	**B1**	
	Without twins: ${}^{16}C_{6}$ $(=8008)$	**B1**	
	Total: 9828	**B1**	
	Alternative:		
	${}^{18}C_{6}-\left(2\times {}^{16}C_{5}\right)$ $=9828$	**B1,B1** **B1**	**B1** for ${}^{18}C_{6}-....$, **B1** for $2\times {}^{16}C_{5}$

9	**(i)**	$h = \dfrac{4000}{\pi r^2}$ or $\pi r^2 h = 4000$	**B1**	
		$A = 2\pi rh + 2\pi r^2$		
		$A = 2\pi r \dfrac{4000}{\pi r^2} + 2\pi r^2$	**M1**	**M1** for substitution of h or πrh into *their* equation for A
			A1	**A1** Answer given
	(ii)	$\dfrac{\mathrm{d}A}{\mathrm{d}r} = -\dfrac{8000}{r^2} + 4\pi r$	**B1, B1**	**B1** for each term correct
		When $\dfrac{\mathrm{d}A}{\mathrm{d}r} = 0$, $r^3 = \dfrac{8000}{4\pi}$	**M1**	**M1** for equating to zero and attempt to find r^3
		leading to $A = 1395,\ 1390$	**M1**	**M1** for substitution of their r to obtain A.
			A1	**A1** for 1390 or awrt 1395
		$\dfrac{\mathrm{d}^2 A}{\mathrm{d}r^2} = \dfrac{16000}{r^3} + 4\pi$, which, is positive so a minimum.	**√B1**	**√B1** for a complete correct method and conclusion.

10 (i)	Velocity $= 26 \times \frac{1}{13}(5\mathbf{i} + 12\mathbf{j})$	**M1**	**M1** for $\frac{1}{13}(5\mathbf{i} + 12\mathbf{j})$
	$= 10\mathbf{i} + 24\mathbf{j}$	**A1**	
	Alternative 1:		
	$\lvert 10\mathbf{i} + 24\mathbf{j}\rvert = \sqrt{10^2 + 24^2}$ $= 26$	**M1**	**M1** for working from given answer to obtain the given speed
	Showing that one vector is a multiple of the other, hence same direction	**A1**	**A1** for a completely correct method
	Alternative 2:		
	$\sqrt{5^2 + 12^2} = 13$, $13k = 26$, so $k = 2$ Velocity $= 2(5\mathbf{i} + 12\mathbf{j})$,	**M1**	**M1** for attempt to obtain the 'multiple' and apply to the direction vector
	Velocity $= 10\mathbf{i} + 24\mathbf{j}$	**A1**	**A1** for a completely correct method
	Alternative 3:		
	Use of trig: $\tan\alpha = \frac{12}{5}$, $\alpha = 67.4°$		
	Velocity $26\cos 67.4°\mathbf{i} + 26\sin 67.4\mathbf{j}$	**M1**	**M1** for reaching this stage
	Velocity $= 10\mathbf{i} + 24\mathbf{j}$	**A1**	**A1** for a completely correct method
(ii)	Position vector $= 4(10\mathbf{i} + 24\mathbf{j})$ or $40\mathbf{i} + 96\mathbf{j}$	**B1**	Allow either form for **B1**
(iii)	$(40\mathbf{i} + 96\mathbf{j}) + (10\mathbf{i} + 24\mathbf{j})t$ oe	**M1**	**M1** for *their* $(\mathbf{ii}) + (10\mathbf{i} + 24\mathbf{j})t$ or $(10\mathbf{i} + 24\mathbf{j}) \times (t + 4)$
		A1	**A1** correct answer only
(iv)	$(120\mathbf{i} + 81\mathbf{j}) + (-22\mathbf{i} + 30\mathbf{j})t$ oe	**B1**	
(v)	$40 + 10t = 120 - 22t$ or $96 + 24t = 81 + 30t$	**M1**	**M1** for equating like vectors
	$t = 2.5$ or 18:30	**A1**	**A1** Allow for $t = 2.5$
	Position vector $= 65\mathbf{i} + 156\mathbf{j}$	**DM1**	**DM1** for use of t to obtain position vector
		A1	**A1** cao

11	(a)	$\tan x(\tan x + 5) = 0$ $\tan x = 0, \quad x = 0°, 180°$ $\tan x = -5, \quad x = 101.3°$	 **B1,B1** **B1**	 **B1** for each , must be from c[illegible] work
	(b)	$2(1 - \sin^2 y) - \sin y - 1 = 0$ $2\sin^2 y + \sin y - 1 = 0$ $(2\sin y - 1)(\sin y + 1) = 0$ $\sin y = \frac{1}{2}, y = 30°, 150°$ $\sin y = -1, y = 270°$	**M1** **A1,A1** **A1**	**M1** for use of correct identity and attempt to solve resulting 3 term quadratic equation.
	(c)	$\cos\left(2z - \frac{\pi}{6}\right) = \frac{1}{2}$ $\left(2z - \frac{\pi}{6}\right) = \frac{\pi}{3}$ $z = \frac{\pi}{4}$ or 0.785 or better	**M1** **A1**	**M1** for dealing with sec correctly and obtaining $\frac{\pi}{3}$ or 1.05
		$\left(2z - \frac{\pi}{6}\right) = \frac{5\pi}{3}$ $z = \frac{11\pi}{12}$ or 2.88 or better	**M1** **A1**	**M1** for obtaining a second equation $\left(2z - \frac{\pi}{6}\right) = 2\pi - their\ \frac{\pi}{3}$ oe