CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the October/November 2013 series

4037 ADDITIONAL MATHEMATICS

4037/13 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

Note: B2 or A2 means that the candidate can earn 2 or 0.
B2, 1, 0 means that the candidate can earn anything from 0 to 2.

1 **(i)** ${}^6C_2\,(2^4)\,(px)^2$ or $\binom{6}{2}2^4(px)^2$	B1	Seen or implied, unsimplified
$240p^2 = 60$	M1	M1 for their coefficient of $x^2 = 60$ and attempt to solve
$p = \dfrac{1}{2}$	A1 [3]	
(ii) coefficients of the terms needed	M1	M1 for realising that 2 terms are involved
$(-1)\,{}^6C_1\,(2)^5\,p + (3 \times 60)$	B1	B1 for $(-1)\,{}^6C_1\,(2)^5\,p$ or $-192p$, using their p.
$= 84$	A1 [3]	
2 $\lg \dfrac{y^2}{5y+60} = \lg 10$	B1 B1	B1 for $2\lg y = \lg y^2$ B1 for $1 = \lg 10$ or equivalent, allow when seen
Or $\lg y^2 = \lg 10\,(5y + 60)$	M1	M1 for use of $\log A - \log B = \log A/B$ or $\log A + \log B = \log AB$
$y^2 - 50y - 600 = 0$ leading to $y = -10, 60$	DM1	DM1 for forming a 3 term quadratic equation and an attempt to solve
y must be positive so $y = 60$	A1 [5]	A1 for $y = 60$ only

3 $\tan^2\theta - \sin^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$		**Marks are awarded only if they can lead to a complete proof for the methods other than those shown below**
$= \dfrac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$	M1	M1 for dealing with tan and a fraction
$= \dfrac{\sin^2\theta\left(1-\cos^2\theta\right)}{\cos^2\theta}$	M1	M1 for factorising
$= \dfrac{\sin^4\theta}{\cos^2\theta}$	M1	M1 for use of identity $\cos^2\theta + \sin^2\theta = 1$
$= \sin^4\theta\sec^2\theta$	A1 [4]	A1 for all correct
Alt solution 1		
Using $\tan^2\theta = \sin^2\theta\sec^2\theta$		
LHS $= \sin^2\theta\sec^2\theta - \sin^2\theta$	M1	M1 use of $\tan^2 x = \sin^2 x\sec^2 x$
$= \sin^2\theta(\sec^2\theta - 1)$	M1	M1 for factorising
$= \sin^2\theta\tan^2\theta$	M1	M1 for use of identity
$= \sin^4\theta\sec^2\theta$	A1	A1 for all correct
Alt solution 2		
RHS $= \sin^4\theta\sec^2\theta$		
$= \dfrac{\sin^2\theta\sin^2\theta}{\cos^2\theta}$	M1	M1 for splitting $\sin^4\theta$ and use of identity
$= \dfrac{\sin^2\theta\left(1-\cos^2\theta\right)}{\cos^2\theta}$	M1	M1 for multiplication
$= \dfrac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$	M1	M1 for writing as two terms and cancelling
$= \dfrac{\sin^2\theta}{\cos^2\theta} - \dfrac{\sin^2\theta\cos^2\theta}{\cos^2\theta}$	A1	A1 for all correct
$= \tan^2\theta - \sin^2\theta$		

4 **(i)** $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x+3)^2\, 2\mathrm{e}^{2x} - \mathrm{e}^{2x}\, 2(x+3)}{(x+3)^4}$	M1	M1 for attempt at quotient rule
	A2, 1, 0	–1 for each error
$= \frac{2e^{2x}(x+2)}{(x+3)^3}$, $A = 2$	A1	Must be convinced of correct simplification e.g. sight of $(x + 3 – 1)$ or $(x + 2)(x + 3)$
	[4]	
Alt solution		
$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{2x}\left(-2(x+3)^{-3}\right) + 2e^{2x}(x+3)^{-2}$	M1	M1 for attempt at product rule
	A2,1,0	–1 for each error
$= \frac{2\mathrm{e}^{2x}(x+2)}{(x+3)^3}$, $A = 2$	A1	Must be convinced of correct simplification e.g. sight of $(x + 3 – 1)$ or $(x + 2)(x + 3)$
(ii) $x = –2$, $y = \mathrm{e}^{–4}$	B1, B1 [2]	Accept $1/\mathrm{e}^4$
5 **(i)** $\mathrm{f}^2(x) = \mathrm{f}(2x^3)$		
$= 2(2x^3)^3$ or $2\left(2\left(\frac{1}{2}\right)^3\right)^3$	M1	M1 for $= 2(2x^3)^3$ or $2\left(2\left(\frac{1}{2}\right)^3\right)^3$
$= 2^{-5}$	A1	For 2^{-5} only
	[2]	
Alt method		
$\mathrm{f}\left(\frac{1}{2}\right) = \frac{1}{4}$ $\quad$ $\mathrm{f}\left(\frac{1}{4}\right) = 2^{-5}$	M1	M1 for f of their f $\left(\frac{1}{2}\right)$
	A1	For 2^{-5} only
(ii) $\mathrm{f}'(x) = \mathrm{g}'(x)$	B1	B1 for $6x^2$
$6x^2 = 4 – 10x$	B1	B1 for $4 – 10x$
Leading to $(3x – 1)(x + 2) = 0$	M1	M1 for solution of quadratic equation obtained from differentiation of both
$x = \frac{1}{3}, -2$	A1 [4]	A1 for both

6 Area under the curve:		
$\int_0^{\sqrt{2}} 4 - x^2 \; dx = \left[4x - \frac{x^3}{3} \right]_0^{\sqrt{2}}$	M1 A1	M1 for attempt to integrate
$= \left(4\sqrt{2} - \frac{2\sqrt{2}}{3} \right) - (0)$	DM1	DM1 for application of limits
$= \frac{10\sqrt{2}}{3}$		
Area of trapezium =		
$\frac{1}{2}(4+2)(\sqrt{2}) = 3\sqrt{2}$	B1	B1 for area of trapezium, allow unsimplified
Shaded area = $\frac{10\sqrt{2}}{3} - 3\sqrt{2}$	M1	M1 for subtraction of the two areas
Shaded area = $\frac{\sqrt{2}}{3}$	A1 [6]	Must be in this form
Or: Equation of chord:		
$y = 4 - \sqrt{2}x$	B1	B1 for the equation of the chord unsimplified
Shaded area = $\int_0^{\sqrt{2}} 4 - x^2 - 4 + \sqrt{2}x \, dx$	M1 M1	M1 for subtraction M1 for attempt to integrate
$\left[\frac{\sqrt{2}}{2}x^2 - \frac{x^3}{3} \right]_0^{\sqrt{2}} = \frac{\sqrt{2}}{3}$	√A1 DM1 A1 [6]	√A1for $\left[-m\frac{x^2}{2} - \frac{x^3}{3} \right]$ or equivalent, where *m* is the gradient of their chord DM1 for application of limits

7 **(i)**	$2t^2 - 2(t^2 - t + 1)$	B1	Correct determinant seen unsimplified
	Leading to, $t = \frac{3}{2}$	M1 A1 [3]	M1 for simplification and solution A1 for solution of det **A**=1only, not 1/det **A**=1
(ii)	$\mathbf{A} = \begin{pmatrix} 6 & 2 \\ 7 & 3 \end{pmatrix}$, $\mathbf{A}^{-1} = \frac{1}{4}\begin{pmatrix} 3 & -2 \\ -7 & 6 \end{pmatrix}$	B1, B1	B1 for $\frac{1}{4}$, B1 for matrix
	$\begin{pmatrix} 6 & 2 \\ 7 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 11 \end{pmatrix}$	B1	B1 for dealing correctly with the factor of 2
	$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 3 & -2 \\ -7 & 6 \end{pmatrix}\begin{pmatrix} 10 \\ 11 \end{pmatrix}$	M1	M1 for pre-multiplying their $\begin{pmatrix} 10 \\ 11 \end{pmatrix}$ by their $\mathbf{A}^{-1}$ to obtain a column matrix
	$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$, leading to $x = 2$, $y = -1$	A1 [5]	Allow $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ for A1
8 **(i)**	$\frac{1}{2}(4^2)\sin\theta = 7.5$	M1	M1 for attempt to find the area of the triangle and equate to 7.5
	$\sin\theta = \frac{15}{16}$, $\theta = 1.215\ldots$	A1 [2]	A1 for solution to obtain the given answer Solution must include 1.2153…. or 1.2154
(ii)	$\sin\frac{\theta}{2} = \frac{\frac{1}{2}CD}{4}$, $(CD = 4.567)$	M1	M1 for attempt to find CD
	Arc length = 6(1.215)	B1	B1 for arc length
	Perimeter = 2 + 2 + 6(1.215) + their CD	M1	M1 for sum of 4 appropriate lengths
	= awrt 15.9	A1 [4]	
(iii)	Area = $\frac{1}{2}6^2(1.215) - 7.5$	B1 M1	B1 for sector area M1 for subtraction of the 2 areas
	= 14.4 (awrt)	A1 [3]	

9 **(a)** **(i)** $6(1-\cos^2 x) = 5 + \cos x$ $6\cos^2 x + \cos x - 1 = 0$ $(3\cos x - 1)(2\cos x + 1) = 0$ $x = 70.5°$ $x = 120°$	M1 M1 A1, A1 [4]	M1 for use of $\sin^2 x = (1-\cos^2 x)$ correctly M1 for solution of a 3 term quadratic in cos and attempt at solution of a trig equation A1 for each correct solution
(ii) $\cos x = \sin y$ $\sin y = \frac{1}{3}$ only so $y = 19.5°, 160.5°$	 DM1 √A1, √A1 [3]	 DM1 for relating cos x and sin y or other correct method of solution
(b) $\cot z(4\cot z - 3) = 0$ $\cot z = 0$, $z = \frac{\pi}{2}$ $\cot z = \frac{3}{4}$, $\tan z = \frac{4}{3}$ so $z = 0.927$	M1 B1 M1 A1 [4]	M1 for attempt to use a factor B1 for $\frac{\pi}{2}$ (1.57) M1 dealing with cot and attempt at solution
10 **(i)** lg s	B1 [1]	Allow in table or on graph if no contradiction <u>No marks for graph unless lg$t$ against lg$s$ (or ln$t$ against ln$s$)</u>
(ii) lg$s$: 0.3, 0.6, 0.78, 0.9 lg$t$: 1.4, 0.8, 0.44, 0.19	M1 DM1 A1 [3]	M1 for 3 or more points correct DM1 for a line through 3 or 4 correct points A1 all points correct with a straight line extending at least from first point to last point
(iii) <u>No marks in this part unless lg$t$ v lg$s$ graph is used</u> Gradient : $n = -2$ (allow $-2.1 \to -1.9$) Intercept : log k, or other method $k = 100$ (allow $90 \to 120$)	 M1A1 M1, A1 [4]	 M1 calculates gradient A1 for $n = -2$ M1 for use of intercept and dealing with logarithm correctly (can use another point)
Alt method Using simultaneous equations, points used must lie on the plotted line.	M2 A1, A1	Must attempt to solve 2 valid equations. $k = 100$ and $n = -2$
(iv) When $t = 4$, lg $t = 0.6$ so lg $s = 0.69$ $s = 4.9$ (allow $4.8 \to 5.2$)	M1 A1 [2]	M1 for valid method using either the correct graph or using lg$t = n$lg$s$ + lg$k$ or $t = ks^n$ using their n and their k

11 **(i)** $\left[e^{2x}+\frac{5}{4}e^{-2x}\right]_0^k$	B1, B1	B1 for each term integrated correctly, allow unsimplified
$\left(e^{2k}+\frac{5}{4}e^{-2k}\right)-\left(1+\frac{5}{4}\right)=3$	M1	M1 for application of limits to an integral of the form $Ae^{2x} \pm Be^{-2x}$
$e^{2k}+\frac{5}{4}\ e^{-2k}\ -\frac{12}{4}=0$	M1	M1 for equating to $\frac{3}{4}$ and attempt to rearrange to obtain a 3 term equation. Must be using an integral of the form $Ae^{2x} \pm Be^{-2x}$
$4e^{4k}-12e^{2k}+5=0$	A1 [5]	Answer given, so must be convinced
(ii) $4y^2-12y+5=0$	M1	M1 for solution of quadratic equation
leading to $e^{2k}=\frac{5}{2},\quad e^{2k}=\frac{1}{2}$	M1	M1 for solving equations involving exponentials
$k=0.458,\quad -0.347$	A1, A1 [4]	A1 for each