CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the October/November 2012 series

4037 ADDITIONAL MATHEMATICS

4037/23 Paper 2, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2012 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW –1,2 This is deducted from A or B marks when essential working is omitted.

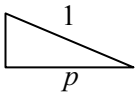
PA –1 This is deducted from A or B marks in the case of premature approximation.

S –1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX –1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

1	1.2 $5x + 7 = -13$ or $25x^2 + 70x + 49 = 169$ $5(5x - 6)(x + 4) = 0$ -4	B1 M1 A1 [3]	correct positive value correct method to find second value correct final answer
2	**(i)** $\frac{1}{6\times 7 - 8\times 4}\begin{pmatrix} 6 & -8 \\ -4 & 7 \end{pmatrix}$ **(ii)** $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{10}\begin{pmatrix} 6 & -8 \\ -4 & 7 \end{pmatrix}\begin{pmatrix} 39 \\ 23 \end{pmatrix}$ $= \begin{pmatrix} 5 \\ 0.5 \end{pmatrix}$	B1B1 [2] M1 A1 [2]	B1 for each part of the inverse pre-multiply $\begin{pmatrix} 39 \\ 23 \end{pmatrix}$ by their inverse correct answers, correctly associated
3	$(3\sqrt{3} - 1)^2 = 27 - 6\sqrt{3} + 1$ or $(3\sqrt{3} - 1)(2\sqrt{3} + 3) = 18 + 7\sqrt{3} - 3$ $\times \frac{2\sqrt{3}+3}{2\sqrt{3}+3}$ or $28 - 6\sqrt{3} = \frac{a\sqrt{3}+b}{3}(2-3)$ $\frac{38\sqrt{3}+48}{3}$ or $a = 38,\ b = 48$	M1 A1 M1 A1 [4]	multiplication, including $a\sqrt{3} \times b\sqrt{3} = 3ab$ a correct expansion valid method to obtain a value for a or b correct answers

4		
$\overrightarrow{XZ} = \begin{pmatrix} 16 \\ 20 \end{pmatrix}$	B1	correct vector for $\overrightarrow{XZ}$
$\overrightarrow{OY} = \begin{pmatrix} 4 \\ -27 \end{pmatrix} + \frac{3}{4}\begin{pmatrix} 16 \\ 20 \end{pmatrix}$ or $\begin{pmatrix} 20 \\ -7 \end{pmatrix} + \frac{1}{4}\begin{pmatrix} -16 \\ -20 \end{pmatrix}$	M1	valid method for $\overrightarrow{OY}$
$= \begin{pmatrix} 16 \\ -12 \end{pmatrix}$	A1	correct vector for OY
$\left\lvert\overrightarrow{OY}\right\rvert = \sqrt{16^2 + (-12)^2}$ oe	M1	uses Pythagoras to find length of $\overrightarrow{OY}$
unit vector in direction of $\overrightarrow{OY} = \begin{pmatrix} 0.8 \\ -0.6 \end{pmatrix}$ oe	A1 [5]	correct vector expression
OR $\overrightarrow{OY} - \overrightarrow{OX} = 3\overrightarrow{OZ} = 3\overrightarrow{OY}$	B1	correct vector equation
$4\overrightarrow{OY} = \begin{pmatrix} 4 \\ -27 \end{pmatrix} + 3\begin{pmatrix} 20 \\ -7 \end{pmatrix} = \begin{pmatrix} 64 \\ -48 \end{pmatrix}$	M1	collect $\overrightarrow{OY}$ s and substitute for $\overrightarrow{OX}$ and $\overrightarrow{OZ}$
$\overrightarrow{OY} = \begin{pmatrix} 16 \\ -12 \end{pmatrix}$ etc.	A1	correct vector for OY
OR $\overrightarrow{OY} = \frac{\overrightarrow{OX} + 3\overrightarrow{OZ}}{4}$	B1	correct use of intercept theorem
$= \frac{\begin{pmatrix} 4 \\ -27 \end{pmatrix} + 3\begin{pmatrix} 20 \\ -7 \end{pmatrix}}{4}$	M1	substitute for $\overrightarrow{OX}$ and $\overrightarrow{OZ}$ and divide
$= \begin{pmatrix} 16 \\ -12 \end{pmatrix}^4$ etc.	A1	correct vector for OY

5	$mx + 2 = mx^2 + 7x + 11$ $mx^2 + 7x - mx + 9 = 0$ $(7 - m)^2 - 4 \times m \times 9 \sim 0$ $m^2 - 50m + 49 \sim 0$ $(m - 1)(m - 49)$, $m = 1, 49$ $1 < m < 49$	M1* A1 M1dep* A1 M1 A1 [6]	eliminates either y or x correct equation compares discriminant with 0 correct quadratic solves 3-term quadratic for m correct answer
6	**(a)**		
	$\sec^2 x = \frac{1}{p^2}$	B1	correct expression for $\sec^2 x$ in terms of p
	$\tan^2 x = \sec^2 x - 1 = \frac{1}{p^2} - 1$	M1 A1 [3]	substitution in correct formula (ps only) correct answer, oe
	OR $\sin^2 x = 1 - p^2$	B1	correct expression for $\sin^2 x$ in terms of p
	$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1 - p^2}{p^2}$	M1 A1	substitution in correct formula (ps only) correct answer, oe
	OR $\sqrt{1 - p^2}$  p	B1	'opposite' is $\sqrt{1 - p^2}$
	$\tan x = \frac{\sqrt{1 - p^2}}{p}$	M1	$\tan x$ = their opposite ÷ their adjacent
	$\tan^2 x = \frac{1 - p^2}{p^2}$	A1	correct answer, oe
	(b) $\cot^2\theta + 2(\cot\theta \tan\theta) + \tan^2\theta$	B1	correct squaring of bracket
	$\cot^2\theta = \text{cosec}^2\theta - 1$ or $\tan^2\theta = \sec^2\theta - 1$ completion "**AG**"	B1 B1 [3]	use of a correct relevant formula correct completion

7	**(a)**		
	$\int\left(x^{\frac{3}{2}}+3x^{\frac{1}{2}}\right)\mathrm{d}x$	B1	correct expression in terms of indices
		M1	increase fractional power by 1
	$\frac{2}{5}x^{\frac{5}{2}}+2x^{\frac{3}{2}}(+c)$	A1	correct answer, ISW
		[3]	
	(b)		
	$\frac{k}{2x+5}$ oe	M1	integral of correct form, k a constant
	$\frac{-10}{2x+5}$ oe	A1	correct integral, ignore '+ c'
	$\frac{k}{2\times10+5}-\frac{k}{5}$	M1	their integral with x =10 subtract their integral with $x = 0$
	1.6	A1√	correct answer, ft their $k\left(=\frac{-4}{25}k\right)$
		[4]	
8	gradient $\frac{9-3}{1-(-2)}(=2)$	B1	correct gradient
	(AD) $y-5=2(x-4)$ or $y=2x-3$	B1√	correct equation for AD, ft their m_{AD}
	(CD) $y-9=-\frac{1}{2}(x-1)$ or $x+2y=19$	M1	uses $m_1m_2 = -1$ and $x = 1$ and $y = 9$ in equation of line
		A1	correct equation for CD
	solves equation for AD with equation for CD	M1	solving equations for a value of x or y
	D is (5, 7)	A1	$x = 5$, $y = 7$
	$\text{area}=\frac{1}{2}\begin{vmatrix}4&-2&1&5&4\\5&3&9&7&5\end{vmatrix}=\frac{1}{2}\lvert26-66\rvert$	M1	a correct method to calculate the area of the trapezium
	or $=\frac{1}{2}\left(\sqrt{5}+\sqrt{45}\right)\sqrt{20}$		
	$=20$	A1	correct answer
		[8]	
	OR (X on BC, AX//DC)		
	$\text{gradient}=\frac{9-3}{1-(-2)}(=2)$	B1	correct gradient
	(BC) $y-9=2(x-1)$ or $y=2x+7$	B1	correct equation for BC
	(AX) $y-5=-\frac{1}{2}(x-4)$ or $2y=-x+14$	M1	uses $m_1m_2 = -1$ and $x = 4$ and $y = 5$ in equation of line
		A1	correct equation for AX
	solves equation BC with equation AX	M1	solving equations for a value of x or y
	X(0, 7)	A1	$x = 0$, $y = 7$
	areaΔ + area rectangle	M1	
	$=\frac{1}{2}\sqrt{20}\times\sqrt{20}+\sqrt{20}\times\sqrt{5}$		a correct method to calculate the area
	$=20$	A1	correct answer

9 **(i)** x^3

	B1 [1]	correct answer		

(ii)

x^3	1	8	27	64
x^2y	9.41	5.16	–6.21	–28.32

M1	graph of x^2y against x^3, linear axes	
A2,1,0 [3]	4, 1–3, 0 points plotted correctly	

(iii) $a = 9.5$ to 10.5

B1	correct answer
M1	finding numerical value for the gradient
A1 [3]	correct answer

gradient = $\frac{y_2 - y_1}{x_2 - x_1}$

$b = -0.6 \pm 0.01$

(iv) $y = \frac{a}{13.69} + 3.7b$ or $13.69y = a + 50.653b$

M1	appropriate substitutions or read graph at 50.653 and divide value by 13.69
A1 [2]	correct answer

$= -1.48 \pm 0.04$

10 **(i)** $x^2 + 80^2$ seen

B1	
B1 [2]	

$\text{time} = \frac{\text{distance}}{\text{speed}}$, oe

(ii)

$\left(\frac{dT}{dx} =\right) \frac{-1}{10} + \frac{x}{6\sqrt{x^2 + 6400}}$

M1*	attempt to differentiate given expression
A1A1	A1 each correct unsimplified term

$\frac{x}{6\sqrt{x^2 + 6400}} = \frac{1}{10}$ oe

M1dep*	attempt to solve $\frac{dT}{dx} = 0$, to include squaring both sides

$x = 60$

A1	correct answer for x

$T = 30\frac{2}{3}(30.7)$

A1 [6]	correct answer for T

11 (a)		
$2^{x-2} = 100^2, \frac{x-2}{2} = \log_2 100$	B1	correct expression
or $2^{\left(\frac{x}{2}-1\right)} = 100$		
$x = 2 + \frac{4}{0.301...}$	M1	valid attempt to obtain value for x
$= 15.3$	A1 [3]	correct answer
(b)		
$\log_y 512 = 3$ or $3 = \log_y y^3$	B1	correct relevant use of rule for logarithms
or $\log_y k = \frac{\log k}{\log y}$ (twice)		
$y^3 = 512$ or $2 = \frac{y^3}{256}$	M1	attempt to solve
$y = 8$	A1 [3]	correct answer
(c)		
$\frac{6^{5z-2}}{6^{2z}} = \frac{6^{3(z-1)}}{6^{2(3-z)}}$	M1	attempt to express at least two elements in terms of 6^z or log6
or $\log 6^{(5z-2)} - \log 6^{2z} = \log 6^{3(z-1)} - \log 6^{2(3-z)}$	A1	correct expression
$5z - 2 - 2z = 3z - 3 - (6 - 2z)$ oe	M1	uses rule of indices or logarithms correctly, accept index/log format
$z = 3.5$	A1 [4]	correct answer
12E (i) $(2x + 8)^2 - 9$ or $a = 2, b = 8, c = -9$	B1B1B1 [3]	B1 for each correct value
(ii) $f^{-1}(x) = \frac{\sqrt{(x+9)} - 8}{2}$ oe	M1	inverse of form $\frac{\sqrt{(x \pm c)} \pm b}{a}$
	A2,1,0√ [3]	3, 1 – 2, 0 correct values, ft their a, b and c
(iii) $\left(\frac{2}{x} + 8\right)^2 - 9 = 135$ or $\frac{4}{x^2} + \frac{32}{x} + 55 = 135$	M1	apply fg (not gf) or replace x by $\frac{1}{x}$
$\frac{2}{x} + 8 = 12 (\text{or } -12)$ or $80x^2 - 32x - 4 = 0$	A1	correct equation
	M1	valid method for solving their equation
$x = 0.5$ oe, only	A1 [4]	correct answer

12O (i)	3.5	B1 [1]	correct answer
(ii)	$y^2 + 7 = 2x$	M1	attempt at inverse, involving squaring
	$h^{-1}(x) = \frac{x^2 + 7}{2}$	A1 [2]	correct inverse
(iii)	$\frac{3x - 4}{x - 2} = x,\ x^2 - 5x + 4 = 0$	M1	equate $k(x)$ with x and obtain quadratic equation
	$(x - 4)(x - 1)$	M1	solve three term quadratic
	$x = 4$ only	A1 [3]	correct answer
(iv)			
	$\frac{3\left(\frac{3x-4}{x-2}\right) - 4}{\left(\frac{3x-4}{x-2}\right) - 2}$	M1	substitute to obtain expression for k^2
		A1	correct unsimplified expression
	$\frac{3(3x-4) - 4(x-2)}{3x - 4 - 2(x-2)}$	M1	multiply numerator and denominator by $(x - 2)$, oe
	$5 - \frac{4}{x}$	A1 [4]	correct answer