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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

2504157352

ADDITIONAL MATHEMATICS **4037/13**

Paper 1 **October/November 2011**

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use	
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Total	

This document consists of **16** printed pages.

DC (NF/CGW) 35871/4

UNIVERSITY *of* CAMBRIDGE
International Examinations

Mathematical Formulae

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1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1} b + \binom{n}{2}a^{n-2} b^2 + \ldots + \binom{n}{r}a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

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1 Given that $\dfrac{\left(6x^{\frac{3}{2}}y^{\frac{4}{5}}\right)^4}{2x^{\frac{1}{2}}y^{-1}} = ax^p y^q$, find the values of the constants a, p and q. [3]

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2 Express $\sqrt{\dfrac{1 - \cos^2\theta}{4\sec^2\theta - 4}}$ in the form $k\cos\theta$, where k is a constant to be found. [4]

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3 **(i)** Given that $\mathbf{A} = \begin{pmatrix} 4 & 3 \\ -8 & -2 \end{pmatrix}$, find $\mathbf{A}^{-1}$. [2]

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(ii) Hence find the matrix $\mathbf{M}$ such that $\begin{pmatrix} 4 & 3 \\ -8 & -2 \end{pmatrix} \mathbf{M} = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$. [3]

4 **(a)** Sets A and B are such that $n(A) = 11$, $n(B) = 13$ and $n(A \cup B) = 18$.
Find $n(A \cap B)$. [2]

(b) Sets $\mathscr{E}$, X and Y are such that

$$\mathscr{E} = \{\theta : 0 \leqslant \theta \leqslant 2\pi\},\ X = \{\theta : \sin\theta = -0.5\},\ Y = \left\{\theta : \sec^2\theta = \frac{4}{3}\right\}.$$

(i) Find, in terms of π, the elements of the set X. [1]

(ii) Find, in terms of π, the elements of the set Y. [2]

(iii) Use set notation to describe the relationship between the sets X and Y. [1]

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5 It is given that $\lg p^3 q = 10a$ and $\lg\left(\frac{p}{q^2}\right) = a$.

(i) Find, in terms of a, expressions for $\lg p$ and $\lg q$. [5]

(ii) Find the value of $\log_p q$. [1]

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6 A curve has equation $y = 6\cos\frac{x}{2} + 4\sin\frac{x}{2}$, for $0 < x < 2\pi$ radians.

(i) Find the x-coordinate of the stationary point on the curve. [5]

(ii) Determine the nature of this stationary point. [2]

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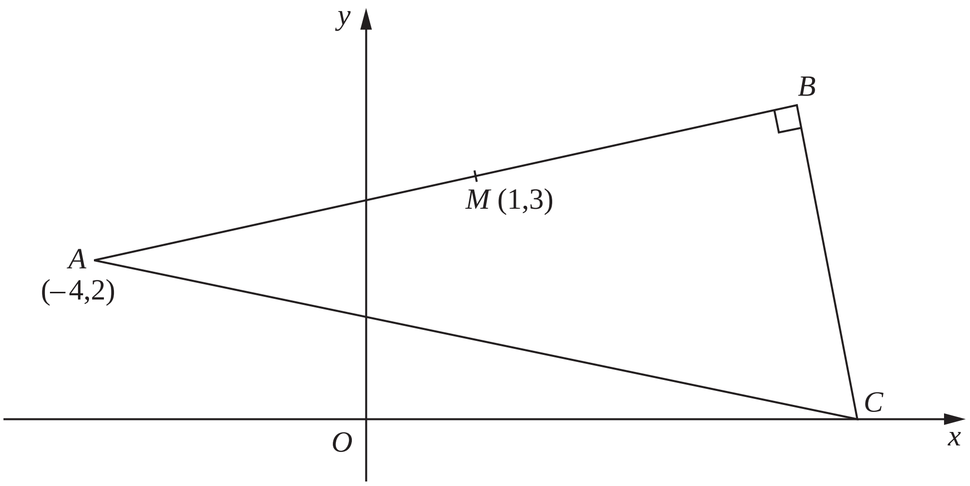

The figure shows a right-angled triangle *ABC*, where the point *A* has coordinates $(-4,2)$, the angle *B* is 90° and the point *C* lies on the *x*-axis. The point *M*(1,3) is the midpoint of *AB*. Find the area of the triangle *ABC*. [7]

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8 Vectors **a** and **b** are such that $\mathbf{a} = \begin{pmatrix} 3 + m \\ 5 - 2n \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 4 - 2n \\ 10 + 3m \end{pmatrix}$.

(i) Given that $3\mathbf{a} + \mathbf{b} = \begin{pmatrix} 1 + n \\ -5 \end{pmatrix}$, find the value of m and of n. [4]

(ii) Show that the magnitude of **b** is $k\sqrt{5}$, where k is an integer to be found. [2]

(iii) Find the unit vector in the direction of **b**. [1]

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9 The function f is defined, for $0° \leqslant x \leqslant 360°$, by $f(x) = 2\sin 3x - 1$.

(i) State the amplitude and period of f. [2]

(ii) State the maximum value of f and the corresponding values of x. [3]

(iii) Sketch the graph of f. [2]

10 **(a)** Differentiate $\tan(3x + 2)$ with respect to x. [2]

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(b) Differentiate $\left(\sqrt{x} + 1\right)^{\frac{2}{3}}$ with respect to x. [3]

(c) Differentiate $\dfrac{\ln(x^3 - 1)}{2x + 3}$ with respect to x. [3]

11 A particle moves in a straight line so that, t s after leaving a fixed point O, its velocity $v\,\text{ms}^{-1}$ is given by $v = 3e^{2t} + 4t$.

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(i) Find the initial velocity of the particle. [1]

(ii) Find the initial acceleration of the particle. [3]

(iii) Find the distance travelled by the particle in the third second. [4]

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12 Answer only **one** of the following two alternatives.

EITHER

A function f is such that $f(x) = \ln(5x - 10)$, for $x > 2$.

(i) State the range of f. [1]

(ii) Find $f^{-1}(x)$. [3]

(iii) State the range of f^{-1}. [1]

(iv) Solve $f(x) = 0$. [2]

A function g is such that $g(x) = 2x - \ln 2$, for $x \in \mathbb{R}$.

(v) Solve $gf(x) = f(x^2)$. [5]

OR

A function f is such that $f(x) = 4e^{-x} + 2$, for $x \in \mathbb{R}$.

(i) State the range of f. [1]

(ii) Solve $f(x) = 26$. [2]

(iii) Find $f^{-1}(x)$. [3]

(iv) State the domain of f^{-1}. [1]

A function g is such that $g(x) = 2e^{x} - 4$, for $x \in \mathbb{R}$.

(v) Using the substitution $t = e^{x}$ or otherwise, solve $g(x) = f(x)$. [5]

Start your answer to Question 12 here.

Indicate which question you are answering.

EITHER	
OR	

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Continue your answer to Question 12 here.

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Continue your answer here if necessary.

For Examiner's Use