UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER				

ADDITIONAL MATHEMATICS **4037/11**

Paper 1 **October/November 2011**

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use	
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This document consists of **14** printed pages and **2** blank pages.

UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \ .$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}\, b + \binom{n}{2}a^{n-2}\, b^2 + \ldots + \binom{n}{r}a^{n-r}\, b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}\, bc \sin A$$

1 **(a)** Sets A and B are such that $n(A) = 15$ and $n(B) = 7$. Find the greatest and least possible values of

   (i) $n(A \cap B)$, [2]

   (ii) $n(A \cup B)$. [2]

(b) On a Venn diagram draw 3 sets P, Q and R such that
$P \cap Q = \varnothing$ and $P \cup R = P$. [2]

2 The function f is such that $f(x) = 4x^3 - 8x^2 + ax + b$, where a and b are constants. It is given that $2x - 1$ is a factor of $f(x)$ and that when $f(x)$ is divided by $x + 2$ the remainder is 20. Find the remainder when $f(x)$ is divided by $x - 1$. [6]

3 Variables t and N are such that when $\lg N$ is plotted against $\lg t$, a straight line graph passing through the points $(0.45, 1.2)$ and $(1, 3.4)$ is obtained.
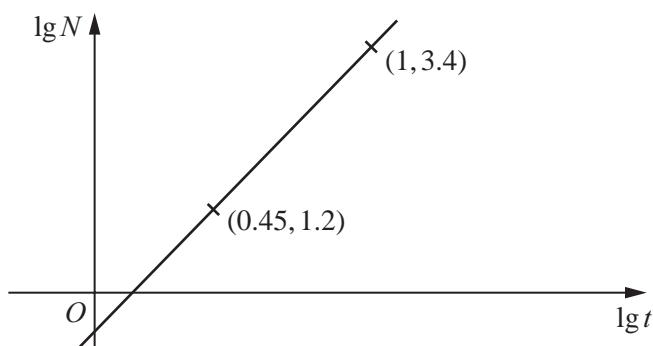
(i) Express the equation of the straight line graph in the form $\lg N = m \lg t + \lg c$, where m and c are constants to be found. [4]

(ii) Hence express N in terms of t. [1]

4 Six-digit numbers are to be formed using the digits 3, 4, 5, 6, 7 and 9. Each digit may only be used once in any number.

    (i) Find how many different six-digit numbers can be formed. [1]

   Find how many of these six-digit numbers are

    (ii) even, [1]

    (iii) greater than 500 000, [1]

    (iv) even and greater than 500 000. [3]

5 A particle moves in a straight line such that its displacement, x m, from a fixed point O at time t s, is given by $x = 3 + \sin 2t$, where $t \geqslant 0$.

    (i) Find the velocity of the particle when $t = 0$. [2]

    (ii) Find the value of t when the particle is first at rest. [2]

    (iii) Find the distance travelled by the particle before it first comes to rest. [2]

    (iv) Find the acceleration of the particle when $t = \dfrac{3\pi}{4}$. [2]

6 (a)
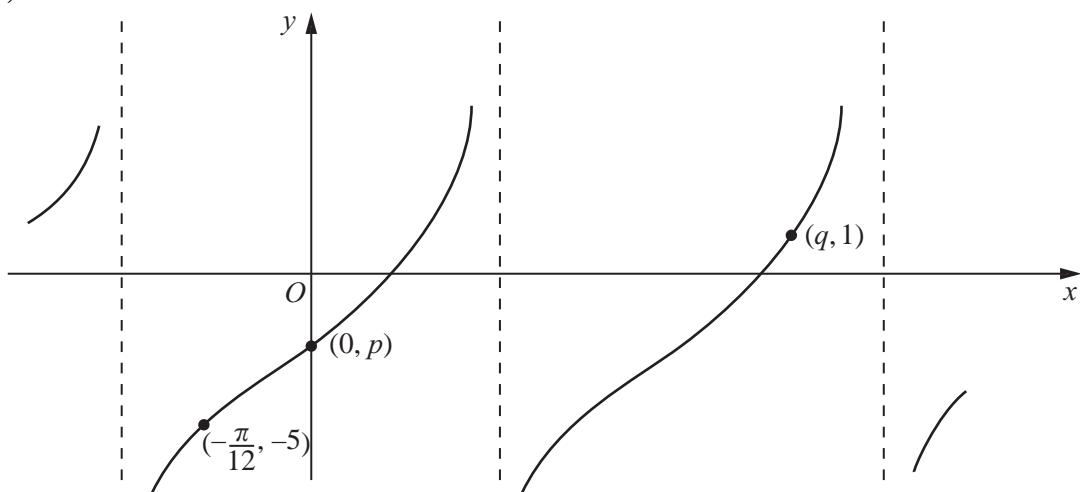
The diagram shows part of the graph $y = p + 3\tan 3x$ passing through the points $(-\frac{\pi}{12}, -5)$, $(0, p)$

and $(q, 1)$. Find the value of p and of q. [4]

(b) It is given that $f(x) = a\cos(bx) + c$, where a, b and c are integers. The maximum value of f is 11, the minimum value of f is 3 and the period of f is $72°$. Find the value of a, of b and of c. [4]

7 The coefficient of x^2 in the expansion of $\left(1 + \dfrac{x}{5}\right)^n$, where n is a positive integer, is $\dfrac{3}{5}$.

    (i) Find the value of n. [4]

    (ii) Using this value of n, find the term independent of x in the expansion of

$$\left(1 + \frac{x}{5}\right)^n \left(2 - \frac{3}{x}\right)^2.$$ [4]

8 **(a)** Find $\int (e^x + 1)^2 \, dx$ and hence evaluate $\int_0^2 (e^x + 1)^2 \, dx$. [6]

(b) A curve is such that $\frac{dy}{dx} = (4x + 1)^{-\frac{1}{2}}$. Given that the curve passes through the point with coordinates (2, 4.5), find the equation of the curve. [5]

9 **(i)** Solve $1 + \cot^2 x = 8 \sin x$ for $0° \leqslant x \leqslant 360°$. [5]

(ii) Solve $4 \sin(2y - 0.3) + 5 \cos(2y - 0.3) = 0$ for $0 \leqslant y \leqslant \pi$ radians. [5]

10 Answer only **one** of the following two alternatives.

EITHER
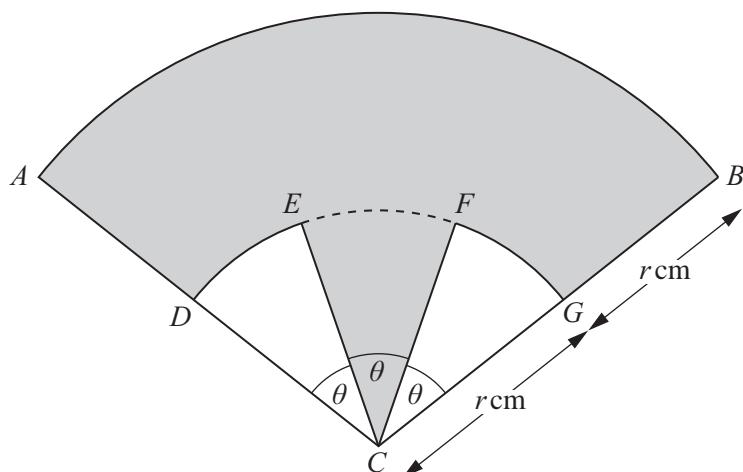
The figure shows a sector *ABC* of a circle centre *C*, radius $2r$ cm, where angle *ACB* is 3θ radians. The points *D*, *E*, *F* and *G* lie on an arc of a circle centre *C*, radius r cm. The points *D* and *G* are the midpoints of *CA* and *CB* respectively. Angles *DCE* and *FCG* are each θ radians. The area of the shaded region is $5\,\text{cm}^2$.

(i) By first expressing θ in terms of r, show that the perimeter, P cm, of the shaded region is given by $P = 4r + \dfrac{8}{r}$. [6]

(ii) Given that r can vary, show that the stationary value of P can be written in the form $k\sqrt{2}$, where k is a constant to be found. [4]

(iii) Determine the nature of this stationary value and find the value of θ for which it occurs. [2]

OR
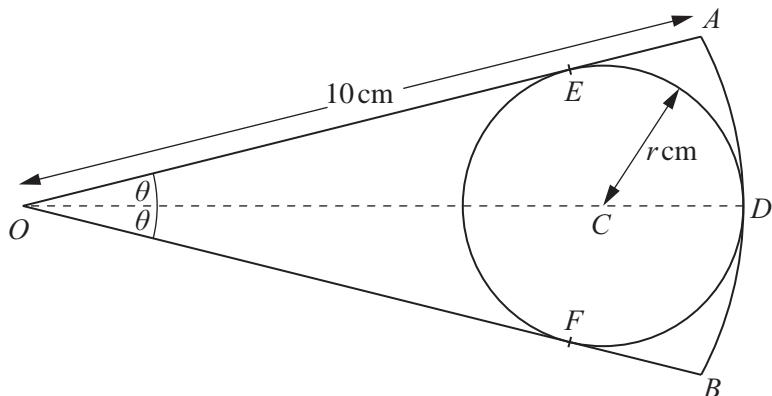
The figure shows a sector *OAB* of a circle, centre *O*, radius 10 cm. Angle $AOB = 2\theta$ radians where $0 < \theta < \dfrac{\pi}{2}$. A circle centre *C*, radius r cm, touches the arc *AB* at the point *D*. The lines *OA* and *OB* are tangents to the circle at the points *E* and *F* respectively.

(i) Write down, in terms of r, the length of *OC*. [1]

(ii) Hence show that $r = \dfrac{10 \sin \theta}{1 + \sin \theta}$. [2]

(iii) Given that θ can vary, find $\dfrac{\text{d}r}{\text{d}\theta}$ when $r = \dfrac{10}{3}$. [6]

(iv) Given that r is increasing at $2\,\text{cms}^{-1}$, find the rate at which θ is increasing when $\theta = \dfrac{\pi}{6}$. [3]

Start your answer to Question 10 here.

Indicate which question you are answering.

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