UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the October/November 2011 question paper for the guidance of teachers

4037 ADDITIONAL MATHEMATICS

4037/21 Paper 2, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW –1,2 This is deducted from A or B marks when essential working is omitted.

PA –1 This is deducted from A or B marks in the case of premature approximation.

S –1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX –1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

1		6.5 $4x - 5 = -21$ or $(4x - 5)^2 = 21^2$ -4	B1 M1 A1 [3]
2		Eliminates y $x^2 + 6x + k - c\ (= 0)$ Uses $b^2 = 4ac$ or completes square $k = c + 9$	M1 A1 M1 A1 [4]
	OR	$\frac{dy}{dx} = 2x + 9$ Equate to 3 and solve for x ($x = -3$) Substitute in both equations and equate $k = c + 9$	B1 M1 M1 A1
3		$\cos\theta = \frac{4 + \left(2 + \sqrt{3}\right)^2 - 9}{4\left(2 + \sqrt{3}\right)}$ or $9 = 4 + \left(2 + \sqrt{3}\right)^2 - 4\left(2 + \sqrt{3}\right)\cos\theta$ $\left(2 + \sqrt{3}\right)^2 = 7 + 4\sqrt{3}$ $\frac{2 + 4\sqrt{3}}{4\left(2 + \sqrt{3}\right)}$ Multiply top and bottom by $2 - \sqrt{3}$ $\frac{-4}{2} + \frac{3\sqrt{3}}{2}$ oe	M1 B1 A1 M1 A1 [5]
4	**(i)**	$\frac{kx}{\left(x^2 + 3\right)^2}$ $k = -2$	M1 A1
	(ii)	$\frac{6}{(-2)} \times \frac{1}{x^2 + 3}$ Correct use of limits in $\frac{C}{x^2 + 3}$ 0.5	M1 M1 A1 [5]
5	**(a)**	f(15) evaluated or $fg(x) = 2(x^2 - 1) + 3$ 33	M1 A1
	(b)	**(i)** kh **(ii)** h^2 or hh **(iii)** $h^{-1}k^{-1}$ or $(kh)^{-1}$	B1 B1 B2 [6]

6		$m_{AB} = 2$	B1
		Uses $m_1 m_2 = -1$ and point A	M1
		AD: $y - 4 = -\frac{1}{2}(x-1)$ or $x + 2y = 9$ or $y = -\frac{1}{2}x + \frac{9}{2}$	A1
		CD: $y - 13 = 2(x - 13)$ or $y = 2x - 13$	B1
		Solve equation AD with equation CD	M1
		(7,1)	A1
			[6]
7	**(a)**	$\cot^2 x = \frac{1}{\tan^2 x}$	B1
		$\text{cosec}^2 x = 1 + \cot^2 x$	B1
		$= 1 + \frac{1}{p^2}$ or $\frac{p^2+1}{p^2}$	B1
	OR	Draw triangle with 1, p and $p^2 + 1$ correct B1	
		$\text{cosec}\, x = \frac{\sqrt{p^2+1}}{p}$ B1 $\text{cosec}^2 x = \frac{p^2+1}{p^2}$ B1	
	(b)	$\sec\theta = \frac{1}{\cos\theta}$	B1
		Multiply out and correct use Pythagoras	M1
		$\frac{\sin^2\theta}{\cos\theta}$	A1
		$\frac{\sin\theta\sin\theta}{\cos\theta} = \sin\theta\tan\theta$	A1
			[7]
8	**(i)**	$\overrightarrow{OP} = \frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$ oe	M1 A1
		$\overrightarrow{OX} = \mu\left(\frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}\right)$	A1
	(ii)	$\overrightarrow{OX} = \mathbf{a} + \lambda\mathbf{b}$ or $\overrightarrow{AX} = \mu\left(\frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}\right) - \mathbf{a}$	B1
		Equates **a** components	M1
		$\mu = \frac{5}{3}$	A1
		Equates **b** components	M1
		$\lambda = \frac{2}{3}$	A1
			[8]

9	**(i)**	$x\sqrt{x}$ 1 2.83 5.20 8 11.18 $y\sqrt{x}$ 3.40 4.13 5.07 6.20 7.47	B1
	(ii)	Plot points on graph	B2, 1, 0
	(iii)	Calculates gradient $b = 0.4 \pm 0.001$ $a = 3 \pm 0.1$	M1 A1 B1
	(iv)	3.05	B1 [7]
10	**(i)**	$\begin{pmatrix} 5 & 0 \\ 4 & -13 \end{pmatrix}$	B2, 1, 0
	(ii)	Matrix multiplication $\begin{pmatrix} 7 & -18 \\ -3 & -19 \end{pmatrix}$	M1 A1
	(iii)	$-\frac{1}{17}\begin{pmatrix} -5 & -2 \\ -1 & 3 \end{pmatrix}$ or $\frac{1}{17}\begin{pmatrix} 5 & 2 \\ 1 & -3 \end{pmatrix}$	B1+ B1
	(iv)	evaluate $\begin{pmatrix} 23 \\ 19 \end{pmatrix}$ $x = 9$, $y = -2$ or $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ -2 \end{pmatrix}$	M1 A1 [8]
11	**(a)(i)**	Express in powers of 5. $\left(\frac{5^{2x+3}}{5^{4x}} = \frac{5^{2(2-x)}}{5^{3x}}\right)$ Use rules of indices $(2x + 3 - 4x = 2(2 - x) - 3x)$ $\frac{1}{3}$	M1 M1 A1
	(ii)	LHS = lg $y(y - 15)$ 2 = lg 100 Solve 3 term quadratic 20 only	B1 B1 M1 A1
	(b)	$\log_{12} 16 - \log_{12} 9 + \log_{12} 81$ Correctly combine 3 logarithms 2	B1 M1 A1 [10]

12E (i)	$\frac{dy}{dx} = \frac{1}{x+1} - \frac{1}{x}$	B1
	gradient tangent $= -\frac{1}{2}$	B1
	$y - \ln 2 = -\frac{1}{2}(x-1)$	M1
	$A(1 + 2\ln 2, 0)$	A1
	$B\left(0, \frac{1}{2} + \ln 2\right)$	A1
	Uses $m_1 m_2 = -1$ in equation of line $(y - \ln 2 = 2(x - 1))$	M1
	$C\left(1 - \frac{1}{2}\ln 2, 0\right)$	A1
	$D(0, -2 + \ln 2)$	A1
(ii)	Valid method for area of triangle	M1
	1.25 or $1.25 \times (\ln 2)^2$ or 0.601	A1
	$k = (\ln 2)^2$	A1
		[11]
12O (i)	Use product rule	M1
	$(x + 1)e^x$	A1
	Solve $\frac{dy}{dx} = 0$	M1
	$\left(-1, -\frac{1}{e}\right)$	A1
	Shows minimum	B1
(ii)	Gradient tangent = 2e	B1
	Use $m_1 m_2 = -1$ in equation of line $\left(y - e = -\frac{1}{2e}(x-1)\right)$	M1
	$R(1 + 2e^2, 0)$	A1
	$S\left(0, \frac{1 + 2e^2}{2e}\right)$	A1
	Area of triangle $= \frac{(1 + 2e^2)^2}{4e}$	M1 A1
		[11]