

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS  
GCE Ordinary Level

**MARK SCHEME for the October/November 2011 question paper  
for the guidance of teachers**

**4037 ADDITIONAL MATHEMATICS**

**4037/13**

Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

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<b>Page 2</b>	<b>Mark Scheme: Teachers' version</b>	<b>Syllabus</b>	<b>Paper</b>
	<b>GCE O LEVEL – October/November 2011</b>	<b>4037</b>	<b>13</b>

## Mark Scheme Notes

Marks are of the following three types:

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep\*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol  $\surd$  implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.  
B2, 1, 0 means that the candidate can earn anything from 0 to 2.

<b>Page 3</b>	<b>Mark Scheme: Teachers' version</b>	<b>Syllabus</b>	<b>Paper</b>
	<b>GCE O LEVEL – October/November 2011</b>	<b>4037</b>	<b>13</b>

The following abbreviations may be used in a mark scheme or used on the scripts:

AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)

### Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through $\sqrt{\phantom{x}}$ ” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.
OW –1,2	This is deducted from A or B marks when essential working is omitted.
PA –1	This is deducted from A or B marks in the case of premature approximation.
S –1	Occasionally used for persistent slackness – usually discussed at a meeting.
EX –1	Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

Page 4	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE O LEVEL – October/November 2011	4037	13

<p>1 <math>648x^{\frac{11}{2}}y^{\frac{21}{5}}</math></p>	<p>B1 B1 B1 [3]</p>	<p>B1 for each correct</p>
<p>2 <math>\sqrt{\frac{\sin^2 \theta}{4 \tan^2 \theta}}</math></p> <p><math>= \frac{\sin \theta}{2 \tan \theta}</math></p> <p><math>= \frac{\cos \theta}{2}, k = 0.5</math></p> <p>OR</p> <p><math>\sqrt{\frac{1 - \cos^2 \theta}{4 \cos^2 \theta} - 4}</math></p> <p><math>= \sqrt{\frac{1 - \cos^2 \theta}{4 - 4 \cos^2 \theta} \cos^2 \theta}</math></p> <p><math>= \frac{\cos \theta}{2}</math></p>	<p>B1 B1  M1  A1 [4]  B1   B1  M1 A1</p>	<p>B1 for numerator B1 for denominator</p> <p>M1 for rearrangement</p>
<p>3 (i) <math>A^{-1} = \frac{1}{16} \begin{pmatrix} -2 &amp; -3 \\ 8 &amp; 4 \end{pmatrix}</math></p> <p>(ii) <math>M = \frac{1}{16} \begin{pmatrix} -2 &amp; -2 \\ 8 &amp; 4 \end{pmatrix} \begin{pmatrix} 1 &amp; 4 \\ 2 &amp; 3 \end{pmatrix}</math></p> <p><math>= \frac{1}{16} \begin{pmatrix} -8 &amp; -17 \\ 16 &amp; 44 \end{pmatrix}</math></p>	<p>B2, 1, 0</p> <p>M1 DM1</p> <p>A1 [5]</p>	<p>-1 each error</p> <p>M1 for pre-multiplication DM1 for attempt to multiply matrices, at least one element correct</p> <p>A1 all correct</p>
<p>4 (a) <math>11 - x + x + 13 - x = 18</math> <math>x = 6</math></p> <p>(b) (i) <math>X: \frac{7\pi}{6}, \frac{11\pi}{6}</math></p> <p>(ii) <math>Y: \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}</math></p> <p>(iii) <math>X \subset Y, X \cap Y = X</math> or <math>X \cup Y = Y</math></p>	<p>M1 A1</p> <p>B1</p> <p>B1, B1</p> <p>B1 [6]</p>	<p>M1 for a valid method</p> <p>B1 for both</p> <p>B1 for each pair</p>

Page 5	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE O LEVEL – October/November 2011	4037	13

<p>5 (i) <math>\lg p^3 + \lg q = 10a</math> or  <math>\lg p - \lg q^2 = a</math>  <math>3\lg p + \lg q = 10a</math>  <math>\lg p - 2\lg q = a</math>  <math>\lg p = 3\lg q</math>  leading to <math>\log p = 3a</math> and  <math>\log q = a</math></p> <p>(ii) <math>\frac{\log q}{\log p} = \frac{1}{3}</math></p>	<p>M1 A1 A1 M1 A1  <math>\sqrt{B1}</math> [6]</p>	<p>M1 for attempt to simplify logs as shown A1 for each of any 2 correct M1 for attempt to solve simultaneous equations A1 for both ft on their <math>\log p</math> and <math>\log q</math>, both need to be simple functions of <math>a</math></p>
<p>6 (i) <math>\frac{dy}{dx} = -3\sin\frac{x}{2} + 2\cos\frac{x}{2}</math>   When <math>\frac{dy}{dx} = 0</math>, <math>\tan\frac{x}{2} = \frac{2}{3}</math>   <math>x = 1.18</math></p> <p>(ii) <math>\frac{d^2y}{dx^2} = -\frac{3}{2}\cos\frac{x}{2} - \sin\frac{x}{2}</math>   When <math>x = 1.18</math> <math>\frac{d^2y}{dx^2}</math> is -ve (-1.8)  Maximum</p>	<p>M1 A1, A1  M1 A1 M1  A1 [7]</p>	<p>M1 for attempt to differentiate A1 for each correct term M1 for their <math>\frac{dy}{dx} = 0</math> A1 correct solution only M1 for a valid method – needs to be seen, A1 [7]</p>
<p>7 <math>B(6, 4)</math>   <math>\text{grad } AM = \frac{1}{5} \therefore \text{grad } BC = -5</math>   <math>BC</math> equation: <math>y - 4 = -5(x - 6)</math>   When <math>y = 0</math>, <math>x = 6.8</math>   Area = 20.8</p>	<p>B1 M1 M1 A1 <math>\sqrt{B1}</math> M1,A1 [7]</p>	<p>M1 for attempt at gradient of <math>BC</math> M1 for attempt at straight line equation A1 for correct equation in any form Ft on their <math>BC</math> equation M1 for a correct method for area of triangle</p>

Page 6	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE O LEVEL – October/November 2011	4037	13

<p><b>8 (i)</b> <math>12 + 3m = 3n</math>  <math>30 + 3m = 6n</math>  leading to <math>m = 2, n = 6</math></p> <p><b>(ii)</b> <math>\mathbf{b} = \begin{pmatrix} -8 \\ 16 \end{pmatrix},  \mathbf{b}  = 8\sqrt{5}</math></p> <p><b>(iii)</b> <math>\frac{1}{8\sqrt{5}} \begin{pmatrix} -8 \\ 16 \end{pmatrix}</math></p>	<p>M1  A1  M1, A1</p> <p>M1, A1</p> <p><math>\sqrt{B1}</math>  [7]</p>	<p>M1 for equating like vectors  A1 for both correct equations  M1 for solution of equations</p> <p>M1 for attempt at magnitude</p> <p>Ft on their <math>\mathbf{b}</math> and its magnitude  Allow decimals</p>
<p><b>9 (i)</b> Amplitude = 2  Period = <math>120^\circ</math></p> <p><b>(ii)</b> Max Value = 1  Occurs when <math>\sin 3x = 1</math>  <math>x = 30^\circ, 150^\circ, 270^\circ</math></p> <p><b>(iii)</b> Sketch</p>	<p>B1  B1</p> <p>B1</p> <p>B1, B1</p> <p>B1  B1  [7]</p>	<p>B1 one cycle correct – ignore <math>x</math> values  B1 all correct</p>
<p><b>10 (a)</b> <math>3\sec^2(3x + 2)</math></p> <p><b>(b)</b> <math>\frac{2}{3} \times \frac{1}{2} x^{-\frac{1}{2}} \left( x^{\frac{1}{2}} + 1 \right)^{-\frac{1}{3}}</math></p> <p><b>(c)</b> <math>\frac{(2x-3) \frac{3x^2}{(x^3-1)} - 2 \ln(x^3-1)}{(2x+3)^2}</math></p>	<p>B1, B1</p> <p>B1  B1  B1</p> <p>M1, B1  A1  [8]</p>	<p>B1 for 3, B1 for <math>3\sec^2(3x + 2)</math></p> <p>B1 for <math>\frac{2}{3}</math>, B1 for <math>\frac{1}{2} x^{-\frac{1}{2}}</math>  B1 for <math>\left( x^{\frac{1}{2}} + 1 \right)^{-\frac{1}{3}}</math></p> <p>M1 for differentiation of a quotient  B1 for differentiation of <math>\ln</math> term  A1 for everything else correct</p>

Page 7	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE O LEVEL – October/November 2011	4037	13

<p><b>11 (i)</b> 3</p> <p><b>(ii)</b> <math>a = 6e^{2t} + 4</math> When <math>t = 0</math>, <math>a = 10</math></p> <p><b>(iii)</b> <math>x = \frac{3}{2}e^{2t} + 2t^2 (+ c)</math></p> <p><math>x_3 = 623.14 + c</math>,      <math>x_2 = 89.90 + c</math> or 621.64                      or 88.40</p> <p>Distance travelled = 533</p>	<p>B1</p> <p>M1, A1 A1</p> <p>M1, A1</p> <p>M1</p> <p>A1 [8]</p>	<p>M1 for attempt at differentiation</p> <p>M1 for attempt at integration</p> <p>M1 for attempt to find difference between <math>x_3</math> and <math>x_2</math> – condone omission of or incorrect <math>c</math></p>
<p><b>12 EITHER</b></p> <p><b>(i)</b> <math>\mathbb{R}</math> or equivalent</p> <p><b>(ii)</b> <math>e^y = 5x - 10</math>, <math>\frac{e^y + 10}{5} = x</math></p> <p><math>f^{-1}(x) = \frac{e^x + 10}{5}</math></p> <p><b>(iii)</b> <math>f^{-1}(x) &gt; 2</math> or <math>y &gt; 2</math></p> <p><b>(iv)</b> <math>1 = 5x - 10</math> <math>x = 2.2</math></p> <p><b>(v)</b> <math>g(\ln(5x - 10)) = \ln(5x^2 - 10)</math> <math>2\ln(5x - 10) - \ln 2 = \ln(5x^2 - 10)</math> <math>25x^2 - 100x - 100 = 10x^2 - 20</math> <math>3x^2 - 20x + 24 = 0</math>, leading to <math>x = 5.10</math> only</p>	<p>B1</p> <p>M1</p> <p>DM1 A1</p> <p>B1</p> <p>B1 B1</p> <p>M1 M1 A1 M1</p> <p>A1 [12]</p>	<p>M1 rearrangement to <math>x</math> in terms of <math>y</math></p> <p>DM1 for interchange of <math>x</math> and <math>y</math> A1 for correct form</p> <p>M1 for correct order gf M1 for dealing with <math>x^2</math> correctly A1 correct quadratic– allow unsimplified M1 for correct attempt at solution of a 3 term quadratic A1 for valid solution only</p>

<b>Page 8</b>	<b>Mark Scheme: Teachers' version</b>	<b>Syllabus</b>	<b>Paper</b>
	<b>GCE O LEVEL – October/November 2011</b>	<b>4037</b>	<b>13</b>

<p><b>12 OR</b></p> <p>(i) <math>f(x) &gt; 2</math></p> <p>(ii) <math>26 = 4e^{-x} + 2</math></p> <p><math>6 = e^{-x}</math> so <math>x = -\ln 6</math>, <math>\ln \frac{1}{6}</math> or <math>-1.79</math></p> <p>(iii) <math>\frac{(y-2)}{4} = e^{-x}</math>, <math>\ln \frac{(y-2)}{4} = -x</math></p> <p><math>f^{-1}(x) = \ln \frac{4}{x-2}</math> or <math>-\ln \frac{x-2}{4}</math></p> <p>(iv) <math>f^{-1}(x)</math> or <math>y &gt; 2</math></p> <p>(v) <math>2e^x - 4 = 4e^{-x} + 2</math>  <math>(2t - 4 = 4t^{-1} + 2)</math></p> <p><math>e^{2x} - 3e^x - 2 = 0</math>  <math>(t^2 - 3t - 2 = 0)</math>  <math>e^x = 3.56</math> so <math>x = 1.27</math></p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>M1 A1</p> <p>B1</p> <p>M1 A1</p> <p>A1</p> <p>M1 M1 A1</p> <p>[12]</p>	<p>M1 rearrangement to <math>x</math> in terms of <math>y</math></p> <p>M1 for interchange of <math>x</math> and <math>y</math> A1 for correct form</p> <p>M1 for attempt to deal with <math>t^{-1}</math> or <math>e^{-x}</math></p> <p>A1 for correct quadratic equation</p> <p>M1 for solution of quadratic M1 for correct attempt to obtain <math>x</math> A1 for 1 solution only</p>
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