UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the May/June 2009 question paper

for the guidance of teachers

4037 ADDITIONAL MATHEMATICS

4037/01 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2009 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW –1,2 This is deducted from A or B marks when essential working is omitted.

PA –1 This is deducted from A or B marks in the case of premature approximation.

S –1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX –1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

1 **(i)** $12 = 15\theta$, $\theta = 0.8$ rads **(ii)** Area = $\frac{1}{2}15^2(0.8)$ leading to 90 (cm^2)	M1, A1 [2] M1 A1 [2]	M1 for use of $s = r\theta$ M1 for use of $A = \frac{1}{2}r^2\theta$
2 $x^3 = 8$, leading to $x = 2$ $\frac{dy}{dx} = 3x^2$ leading to grad of $-\frac{1}{12}$ for normal $y - 0 = -\frac{1}{12}(x-2)$ $\left(y = -\frac{1}{12}x + \frac{1}{6}\right)$	B1 M1 DM1 A1 [4]	B1 for finding where curve crosses the x axis M1 for attempt to differentiate and use of $m_1m_2 = -1$ DM1 for attempt at equation of normal Allow unsimplified
3 $\frac{1-\cos^2\theta}{\sec^2\theta - 1} = \frac{\sin^2\theta}{\tan^2\theta}$ $= \cos^2\theta$ $= 1 - \sin^2\theta$ **Alt Scheme** $\frac{1-\cos^2\theta}{\sec^2\theta - 1} = \frac{\sin^2\theta}{1-\cos^2\theta / \cos^2\theta}$ $= \frac{\sin^2\theta\cos^2\theta}{\sin^2\theta}$ $= \cos^2\theta$ $= 1 - \sin^2\theta$	 M1 M1 M1 A1 [4] M1 M1 M1 A1	 M1 for use of $1-\cos^2\theta = \sin^2\theta$ M1 for use of $\sec^2\theta - 1 = \tan^2\theta$ M1 for attempt to simplify M1 for use of $1-\cos^2\theta = \sin^2\theta$ M1 for attempting to get all in terms of cos M1 for attempt to simplify
4 **(i)** $5x - 3 = kx^2 - 3x + 5$ $kx^2 - 8x + 8 = 0$ using $b^2 - 4ac = 0$, $k = 2$ (Alt scheme: $5 = 2kx - 3$, $x = \frac{4}{k}$ $\frac{20}{k} - 3 = \frac{16}{k} - \frac{12}{k} + 5$ leading to $k = 2$) **(ii)** leading to $x = 2, y = 7$	M1 DM1, A1 [3] M1, A1 [2]	M1 for equating line and curve equations DM1 for use of $b^2 - 4ac$ on resulting quadratic (Alt scheme: M1 for attempt to differentiate quadratic and equate to 5 DM1 for simplification and solution using resulting quadratic M1 for obtaining x and y coords

5 **(a)** $3^{2(2x-1)} = 3^{3x}$ $4x-2=3x$ $x=2$	B1 B1 B1 [3]	B1 for $3^{2(2x-1)}$ B1 for 3^{3x} B1 for $x=2$
(b) $a^{-2}b$ or $\dfrac{b}{a^2}$ (allow here) $p=-2, q=1$	B1 B1 [2]	B1 for each
6 $\mathrm{f}(3)$, $\mathrm{f}(-5)$ or $\mathrm{f}(0.5)=0$ spotted Either $(2x-1)(x^2+2x-15)$ Or $(x+5)(2x^2-7x+3)$ Or $(x-3)(2x^2+9x-5)$ $x = 3, -5, 0.5$	B1 M1 A1 M1 A2,1,0 [6]	B1 for spotting one root M1 for attempt to obtain quadratic factor A1 all correct M1 for solution of quadratic A2 for all 3 solutions (–1 each error) Correct factors only – lose 1 A mark
7 **(i)** $3x\mathrm{e}^{3x}+\mathrm{e}^{3x}-\mathrm{e}^{3x}$ $=3x\mathrm{e}^{3x}$	M1, A1, B1 [3]	M1 for attempt to differentiate a product. A1 for correct product. B1 for $-\mathrm{e}^{3x}$
(ii) $\int x\mathrm{e}^{3x}\mathrm{d}x = \dfrac{1}{3}\left(x\mathrm{e}^{3x}-\dfrac{\mathrm{e}^{3x}}{3}\right)$	DM1 DM1 A1 [3]	DM1 for recognition of the 'reverse' to **(i)** DM1 for dealing with '3' A1 all correct (condone omission of *c*)
8 **(i)** $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{(x^2+9)2-2x(2x)}{(x^2+9)^2}$ $=\dfrac{18-2x^2}{(x^2+9)^2}$, turning points, $x = \pm 3$	B2,1,0 M1 A1 [4]	Attempt to differentiate a quotient –1 each error M1 for correct attempt to find the turning points. A1 for both
(ii) $\dfrac{\mathrm{d}x}{\mathrm{d}t}=2$ $\dfrac{\mathrm{d}y}{\mathrm{d}t}=2\times\left(\dfrac{16}{100}\right)$ $= 0.32$ or $\dfrac{8}{25}$	B1 M1 A1 [3]	B1 for use of $\dfrac{\mathrm{d}x}{\mathrm{d}t}=2$ M1 for use of rates of change

9 **(i)** $10\sqrt{2}\left(\frac{1}{\sqrt{2}}\mathbf{i}+\frac{1}{\sqrt{2}}\mathbf{j}\right)=10\mathbf{i}+10\mathbf{j}$	M1 A1 [2]	M1 for attempt at a correct direction vector A1 all correct
(ii) $(-4\mathbf{i}+8\mathbf{j})+(20\mathbf{i}+20\mathbf{j})=16\mathbf{i}+28\mathbf{j}$	M1 A1 [2]	M1 for valid attempt A1 all correct
(iii) $(10\mathbf{i}+10\mathbf{j})-(8\mathbf{i}+6\mathbf{j})=2\mathbf{i}+4\mathbf{j}$	M1 A1 [2]	M1 for attempt at vector difference A1 condone negative
(iv) displacement of $(19\mathbf{i}+34\mathbf{j})-(16\mathbf{i}+28\mathbf{j})=3\mathbf{i}+6\mathbf{j}$ time =1330 hours (accept 1.5 hours) at $31\mathbf{i}+43\mathbf{j}$	M1 A1 A1 [3]	M1 for displacement and attempt to obtain time A1 for correct time A1 for correct position vector
Alternative scheme: $(19\mathbf{i}+34\mathbf{j})+(8\mathbf{i}+6\mathbf{j})t=$ $(16\mathbf{i}+28\mathbf{j})+(10\mathbf{i}+10\mathbf{j})t$ or equivalent		M1 for attempt to equate like vectors A marks as above
10 **(i)** $m_{AB} = 0.75$ line AB $y-0=0.75(x+4)$ $m_{PQ}=-\frac{4}{3}$ line PQ $y-10=-\frac{4}{3}(x-1)$ intersection at C (4, 6) Q (8.5 0)	M1 A1 M1 A1 M1 A1 √ B1 [7]	M1 for attempt at m_{AB} and line AB M1 for use of '$m_1m_2=-1$' and attempt at line PQ M1 for attempt at solving simultaneous equations Ft on their line PQ
(ii) $AC = 10$, $CQ = 7.5$ Area = 37.5	M1 A1 [2]	M1 for attempt at lengths and area

11 **(i)** $\ln s = n \ln t + \ln k$	M1, A1	M1 for attempt to take logs
	M1	A1 for correct form
	A1	M1 for attempt to plot correct graph
		A1 for a reasonable straight line

$\ln t$	1.6	2.7	3.4	4.2	4.6
$\ln s$	7.2	5.9	5	4	3.6

Plot $\ln s$ against $\ln t$ [4]

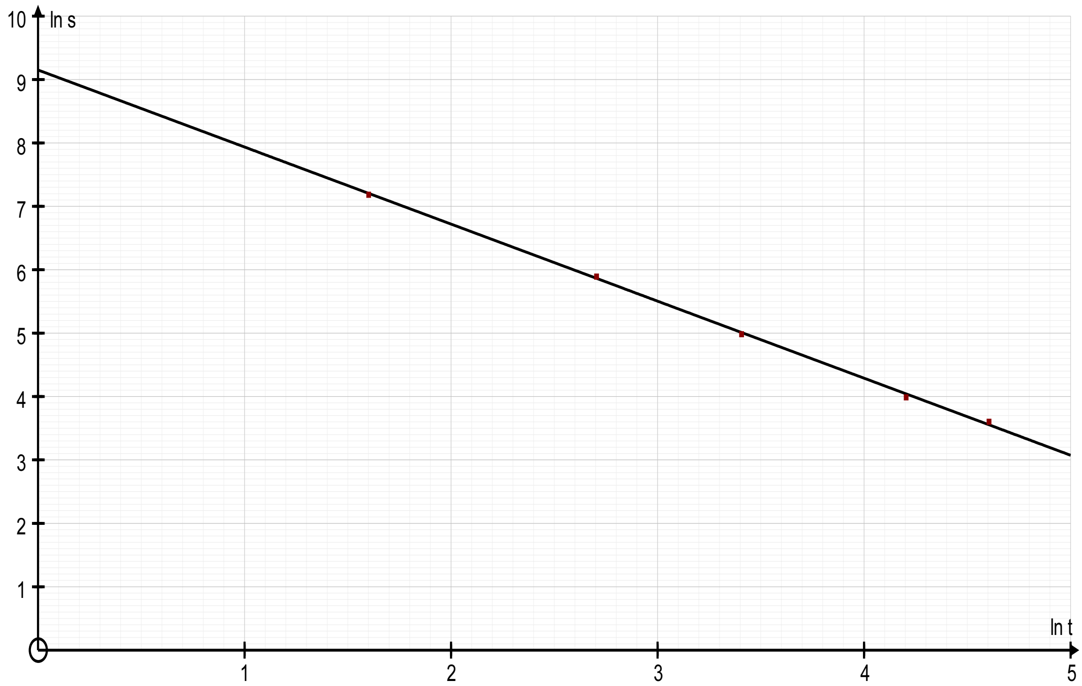
(ii) grad $n = -1.2$ (-1.4 to -1.0)	M1, A1	M1 for use of grad = n
Intercept = ln k, leading to	M1, A1	M1 for use of intercept = ln k
$k = 7900 - 10\,000$	[4]	
(iii) when $t = 50$, ln $t = 4.4$	M1	M1 for attempt to obtain s
leading to $s = 80$ ($72 - 92$)	A1	
	[2]	

Alternative method

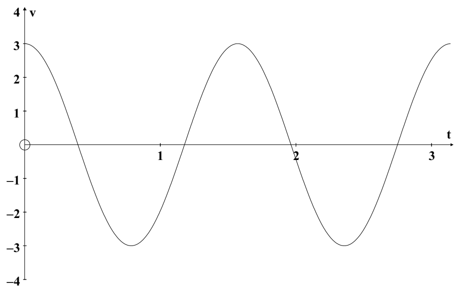
(i) $\lg s = n \lg t + \lg k$

$\lg t$	0.7	1.2	1.5	1.8	2
$\lg s$	3.1	2.5	2.2	1.7	1.6

Same scheme applies



12 EITHER		
(i) amplitude = 1	B1 [1]	
(ii) period = 6π, 18.8	B1 [1]	
(iii) $\sin\left(\frac{x}{3}\right)=\frac{1}{2},\ x=\frac{\pi}{2},\frac{5\pi}{2}$	M1 A1, A1 [3]	M1 for attempt to solve correctly A1 for each (allow degrees here)
(iv) Area under curve		
$\int_{\frac{\pi}{2}}^{\frac{5\pi}{2}}\left(1+\sin\frac{x}{3}\right)\mathrm{d}x = \left[x-3\cos\frac{x}{3}\right]_{\frac{\pi}{2}}^{\frac{5\pi}{2}}$	M1 B1, B1	M1 for attempt to integrate B1 for x, B1 for $-3\cos\frac{x}{3}$
leading to $2\pi + 3\sqrt{3}$	DM1	DM1 for **correct** use of limits
Area of rectangle $=\left(\frac{5\pi}{2}-\frac{\pi}{2}\right)\times\frac{3}{2}$ $= 3\pi$	M1	M1 for attempt at rectangle plus subtraction – must be working in radians
Shaded area = $3\sqrt{3} - \pi$ (2.05)	A1 [6]	
Alternative solution: Shaded area	M1 M1	M1 for subtraction (must be using radians) M1 for attempt to integrate
$\int_{\frac{\pi}{2}}^{\frac{5\pi}{2}}\left(\sin\frac{x}{3}-0.5\right)\mathrm{d}x=\left[-0.5x-3\cos\frac{x}{3}\right]_{\frac{\pi}{2}}^{\frac{5\pi}{2}}$	B1, B1 DM1, A1	B1 for $-0.5x$, B1 for $-3\cos\frac{x}{3}$ DM1 for correct use of limits

OR		
(i) $t = \dfrac{\pi}{8}$	B1 [1]	
(ii) $a = -4k\sin 4t$	M1, A1 [2]	M1 for attempt to differentiate
(iii) $12 = -4k\sin\dfrac{3\pi}{2}$ leading to	M1	M1 for attempt to substitute into their acceleration equation
$k = 3$	A1 [2]	
(iv) 	B1 $\sqrt{}$ B1 [2]	B1 for correct shape B1 ft on their value for *k*
(v) $s = \displaystyle\int_0^{\frac{\pi}{24}} 3\cos 4t.\mathrm{d}t$	M1, $\sqrt{}$ A1	M1 for attempt to integrate Ft on their value for *k*
$= \left[\dfrac{3}{4}\sin 4t\right]_0^{\frac{\pi}{24}}$ leading to $\dfrac{3}{8}$	DM1, A1 [4]	DM1 for application of limits or equivalent