UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the October/November 2008 question paper

4037 MATHEMATICS

4037/01 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2008 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW -1,2 This is deducted from A or B marks when essential working is omitted.

PA -1 This is deducted from A or B marks in the case of premature approximation.

S -1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX -1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

1 **(i)** correct diagram **(ii)** correct diagram **(iii)** correct diagram	B1 B1 B1 [3]	
2 $(2x+1)^2 > 8x+9$ $4x^2 - 4x - 8 > 0$ $x^2 - x - 2 > 0$ $(x+1)(x-2) > 0$ Leads to critical values $x = -1, 2$ $x < -1$ and $x > 2$	 M1 DM1 A1 √A1 [4]	 M1 for simplification to 3 term quadratic DM1 for factorisation A1 for critical values Follow through on their critical values.
3 LHS = $\dfrac{\sin^2 A + 1 + \cos^2 A + 2\cos A}{(1+\cos A)\sin A}$ $= \dfrac{2 + 2\cos A}{(1+\cos A)\sin A}$ $= \dfrac{2}{\sin A}$ leading to $2\operatorname{cosec} A$	 M1 A1 M1 A1 [4]	 M1 for attempt to deal with fractions and attempt to obtain numerator A1 correct M1 for use of $\sin^2 A + \cos^2 A = 1$
4 Substitution of $x = 1$ leading to $a + b + 4 = 0$ Substitution of $x = -\dfrac{1}{2}$ leading to $-a + 2b - 28 = 0$ Leading to $a = -12$, $b = 8$	M1 M1 A1 M1 A1 [5]	M1 for substitution of $x = 1$ and equated to 3 M1 for substitution of $x = -\dfrac{1}{2}$ and equated to 6 A1 for both correct M1 for solution A1 for both
5 **(i)** $\mathbf{a} = \dfrac{1}{13}(5\mathbf{i} - 12\mathbf{j})$ **(ii)** $q(5\mathbf{i} - 12\mathbf{j}) + p\mathbf{i} + \mathbf{j} = 19\mathbf{i} - 23\mathbf{j}$ $5q + p = 19$ $-12q + 1 = -23$ Leading to $q = 2$, $p = 9$	M1, A1 [2] M1 M1 A1 [3]	M1 for a valid attempt to obtain magnitude. M1 for equating like vectors M1 for solution of (simultaneous) equations A1 for both
6 **(i)** $2t^2 - 9t - 5 = 0$ $(2t+1)(t-5) = 0$ $t = \dfrac{1}{2}$, $t = 5$ **(ii)** $x^{\frac{1}{2}} = -0.5, 5$ $x = 0.25, 25$	M1 DM1 A1 [3] M1 A1,A1 [3]	M1 for attempting to form a quadratic in t DM1 for attempt to solve a 3 term quadratic A1 for both M1 for realising that $x^{0.5}$ is equivalent to t (or valid attempt at solution)

7 **(i)** $y = 4x^2 - 12x + 3$ $y = (2x - 3)^2 - 6$	B1 B1 B1 [3]	B1 for 2 (part of linear factor) B1 for –3 (part of linear factor) B1 for –6
(ii) $\left(\frac{3}{2}, -6\right)$	√B1, √B1 [2]	Follow through on their a, b and c Allow calculus method.
(iii) $f \geq -6$	√B1 [1]	Follow through on their c
8 $\frac{dy}{dx} = -2e^{-2x}(+c)$ When $\frac{dy}{dx} = 3, x = 0, \therefore c_1 = 5$ $\frac{dy}{dx} = -2e^{-2x} + 5$ $y = e^{-2x} + 5x(+c_2)$ When $x = 2$, $y = e^{-4}$ $\therefore c_2 = -10$ $y = e^{-2x} + 5x - 10$	B1 M1 A1 B1 M1 √A1 [6]	B1 for $-2e^{-2x}$ M1 for attempt to find c_1 B1 for $-2e^{-2x}$ M1 for attempt to find c_2 √ –2 times their c_1
9 **(i)** $2^5 + {}^5C_1 2^4(-3x) + {}^5C_2 2^3(-3x)^2$ $32 - 240x + 720x^2$	B1 B1 B1 [3]	B1 for 32 or 2^5 B1 for –240 B1 for 720.
(ii) $32a = 64$, $a = 2$ $32b - 240a = -192$, $b = 9$ $-240b + 720a = c$ $c = -720$	B1 M1 A1 M1 A1 [5]	B1 for $a = 2$ M1 for equation in a and b equated to ±192 A1 for $b = 9$ M1 for equation in a and b equated to c A1 for $c = -720$
10 (a) **(i)** $fg(x) = f\left(\frac{x}{x+2}\right)$ $= 3 - \frac{x}{x+2}$	M1 A1 [2]	M1 for order
(ii) $3 - \frac{x}{x+2} = 10$ leading to $x = -1.75$	DM1 A1 [2]	DM1 for dealing with fractions sensibly
(b) **(i)** $h(x) > 4$	B1 [1]	
(ii) $h^{-1}(x) = e^{x-4}$ $h^{-1}(9) = e^5$ (≈ 148) or $4 + \ln x = 9$, leading to $x = e^5$	M1 A1 [2]	M1 for attempting to obtain inverse function
(iii) correct graphs	B1 B1 B1 [3]	B1 for each curve B1 for idea of symmetry

11 (i) $\tan^2 2x = 3$	M1	M1 for an equation in $\tan^2 2x$
$\tan 2x = (\pm)\sqrt{3}$	DM1	M1 for attempt to solve using $2x$ correctly
$2x = 60°, 120°, 240°, 300°$ $x = 30°, 60°, 120°, 150°$	A1, A1 [4]	A1 for any pair
(ii) $2\text{cosec}^2 y + \text{cosec } y - 3 = 0$ $(2\text{cosec } y + 3)(\text{cosec } y - 1) = 0$	M1, A1	M1 for correct use of identity or other valid method A1 for a correct quadratic
$\text{cosec } y = -\frac{3}{2}, 1$ $\sin y = -\frac{2}{3}, 1$	M1	M1 for solution of quadratic and attempt to solve correctly
$y = 221.8°, 318.2°, y = 90°$	A1, A1 [5]	A1 for 221.8°, 318.2°, A1 for 90°
(iii) $\cos\left(z + \frac{\pi}{2}\right) = -\frac{1}{2}$	M1	M1 for dealing with sec and order of operations
$z + \frac{\pi}{2} = \frac{2\pi}{3}, \frac{4\pi}{3}$		
$z = \frac{\pi}{6}, \frac{5\pi}{6}$, allow 0.52, 2.62 rads	A1,A1 [3]	A1 for each
12 EITHER		
(i) $\frac{dy}{dx} = \frac{(x+1)2x - x^2}{(x+1)^2}$	M1 A1	M1 for attempt to differentiate a quotient A1 correct allow unsimplified
$= \frac{x(x+2)}{(x+1)^2}$		
$\frac{dy}{dx} = 0$, $x = 0, -2$	DM1	DM1 for equating to zero and an attempt to solve
$y = 0, -4$	A1,A1 [5]	A1 for each pair (could be $x = 0$ and $x = -2$)
(ii) gradient of normal $= -\frac{4}{3}$	M1	M1 for attempt to obtain gradient of the normal
normal $y = -\frac{4}{3}x + \frac{11}{6}$, leads to	A1	A1 for a correct (unsimplified) normal equation
M (1.375,0) N (0, –4)	√ B1 B1	Follow through on their normal B1 for N
Area = 2.75	M1 √ A1 [6]	M1 for attempt to get area of triangle Ft on their M and N (must be on axes)

12 OR		
(i) $\frac{dy}{dx} = e^{x-2} - 2$	B1 B1	B1 for e^{x-2} B1 for –2 only
$\frac{dy}{dx} = 0, e^{x-2} = 2$	M1	M1 for equating to zero and attempt to solve
$x = 2 + \ln 2$ (2.69)	A1	A1 for x
$y = 4 - 2\ln 2$ (2.61)	A1	A1 for y
$\frac{d^2y}{dx^2} = e^{x-2}$, always +ve ∴ min	B1 [6]	B1 for conclusion from a valid method
(ii)		
$\int_0^3 (e^{x-2} - 2x + 6)dx = \left[e^{x-2} - x^2 + 6x\right]_0^3$	M1, A1	M1 for attempt to integrate
$= (e - 9 + 18) - (e^{-2})$ $= e - e^{-2} + 9$	M1 A1	M1 for correctly applying limits A1 for $e - e^{-2}$
$k = 9$	B1 [5]	B1 for k