UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS **4037/01**

Paper 1 **October/November 2007**

2 hours

Additional Materials: Answer Paper
Graph paper (1 sheet)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **5** printed pages and **3** blank pages.

UNIVERSITY *of* CAMBRIDGE
International Examinations

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1} b + \binom{n}{2}a^{n-2} b^2 + \ldots + \binom{n}{r}a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

1 Given that $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$, find the value of each of the constants m and n for which

$$\mathbf{A}^2 + m\mathbf{A} = n\mathbf{I},$$

where $\mathbf{I}$ is the identity matrix. [4]

2 Show that

$$\frac{1}{1 - \cos\theta} - \frac{1}{1 + \cos\theta} \equiv 2\operatorname{cosec}\theta \cot\theta.$$

[4]

3 Given that $p = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$, express in its simplest surd form,

(i) p, [3]

(ii) $p - \frac{1}{p}$. [2]

4 A badminton team of 4 men and 4 women is to be selected from 9 men and 6 women.

(i) Find the total number of ways in which the team can be selected if there are no restrictions on the selection. [3]

Two of the men are twins.

(ii) Find the number of ways in which the team can be selected if exactly one of the twins is in the team. [3]

5 In this question, $\mathbf{i}$ is a unit vector due east, and $\mathbf{j}$ is a unit vector due north.

A plane flies from P to Q where $\overrightarrow{PQ} = (960\mathbf{i} + 400\mathbf{j})$ km. A constant wind is blowing with velocity $(-60\mathbf{i} + 60\mathbf{j})\,\text{km h}^{-1}$. Given that the plane takes 4 hours to travel from P to Q, find

(i) the velocity, in still air, of the plane, giving your answer in the form $(a\mathbf{i} + b\mathbf{j})\,\text{km h}^{-1}$, [4]

(ii) the bearing, to the nearest degree, on which the plane must be directed. [2]

6 A curve is such that $\frac{dy}{dx} = \frac{6}{\sqrt{4x + 1}}$, and (6, 20) is a point on the curve.

(i) Find the equation of the curve. [4]

A line with gradient $-\frac{1}{2}$ is a normal to the curve.

(ii) Find the coordinates of the points at which this normal meets the coordinate axes. [4]

7 **(i)** Use the substitution $u = 2^x$ to solve the equation $2^{2x} = 2^{x+2} + 5$. [5]

(ii) Solve the equation $2\log_9 3 + \log_5(7y - 3) = \log_2 8$. [4]

8 **(a)** The remainder when the expression $x^3 - 11x^2 + kx - 30$ is divided by $x - 1$ is 4 times the remainder when this expression is divided by $x - 2$. Find the value of the constant k. [4]

(b) Solve the equation $x^3 - 4x^2 - 8x + 8 = 0$, expressing non-integer solutions in the form $a \pm \sqrt{b}$, where a and b are integers. [5]

9

x	2	4	6	8	10
y	14.4	10.8	11.2	12.6	14.4

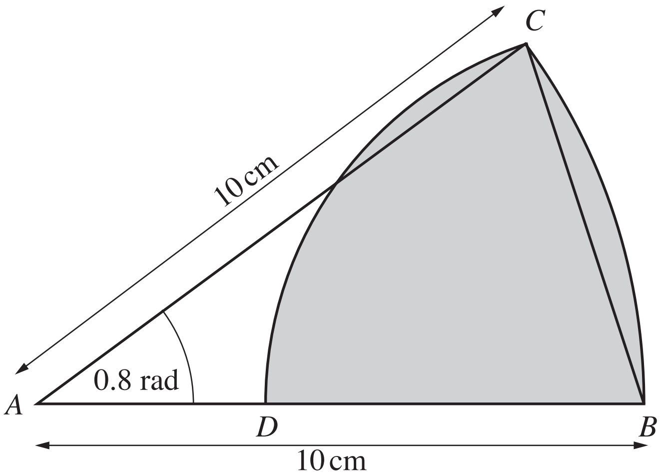
The table shows experimental values of two variables, x and y.

(i) Using graph paper, plot xy against x^2. [2]

(ii) Use the graph of xy against x^2 to express y in terms of x. [4]

(iii) Find the value of y for which $y = \dfrac{83}{x}$. [3]

10



The diagram shows a sector ABC of the circle, centre A and radius 10 cm, in which angle $BAC = 0.8$ radians. The arc CD of a circle has centre B and the point D lies on AB.

(i) Show that the length of the straight line BC is 7.79 cm, correct to 2 decimal places. [2]

(ii) Find the perimeter of the shaded region. [4]

(iii) Find the area of the shaded region. [4]

11 Answer only **one** of the following two alternatives.

EITHER

A curve has the equation $y = xe^{2x}$.

(i) Obtain expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [5]

(ii) Show that the y-coordinate of the stationary point of the curve is $-\frac{1}{2e}$. [3]

(iii) Determine the nature of this stationary point. [2]

OR

(i) Show that $\frac{d}{dx}\left(\frac{\ln x}{x^2}\right) = \frac{1 - 2\ln x}{x^3}$. [3]

(ii) Show that the y-coordinate of the stationary point of the curve $y = \frac{\ln x}{x^2}$ is $\frac{1}{2e}$. [3]

(iii) Use the result from part **(i)** to find $\int\left(\frac{\ln x}{x^3}\right)dx$. [4]

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