UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS **4037/02**

Paper 2 **May/June 2007**

2 hours

Additional Materials: Answer Booklet/Paper
Mathematical tables

3231119358

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Booklet/Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **5** printed pages and **3** blank pages.

SP (SLM/CGW) T19062/6

UNIVERSITY *of* CAMBRIDGE
International Examinations

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1} b + \binom{n}{2}a^{n-2} b^2 + \ldots + \binom{n}{r}a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\text{cosec}^2 A = 1 + \cot^2 A.$$

Formulae for ΔABC

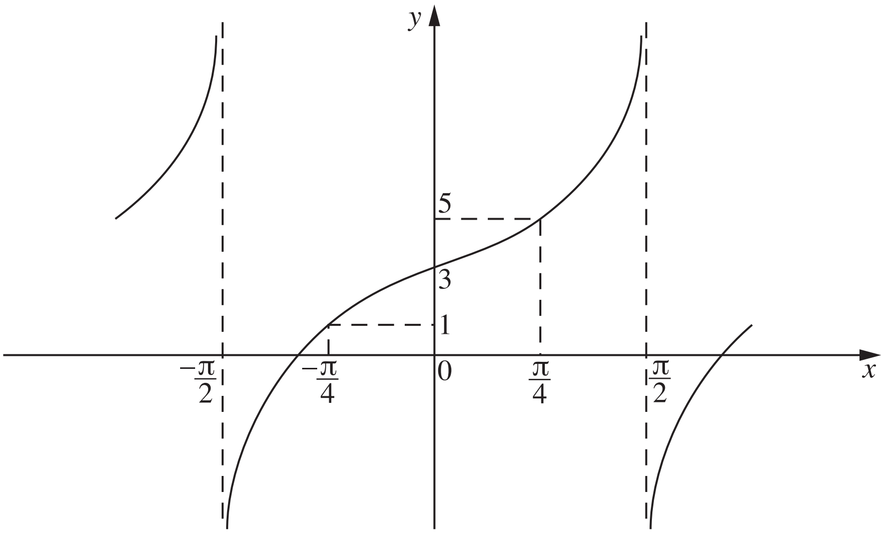
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

1 A triangle has a base of length $(13 - 2x)$ m and a perpendicular height of x m. Calculate the range of values of x for which the area of the triangle is greater than $3\,\text{m}^2$. [3]

2



The diagram shows part of the graph of $y = a \tan (bx) + c$. Find the value of

(i) c, **(ii)** b, **(iii)** a. [3]

3 The roots of the equation $x^2 - \sqrt{28}x + 2 = 0$ are p and q, where $p > q$. Without using a calculator, express $\frac{p}{q}$ in the form $m + \sqrt{n}$, where m and n are integers. [5]

4 An artist has 6 watercolour paintings and 4 oil paintings. She wishes to select 4 of these 10 paintings for an exhibition.

(i) Find the number of different selections she can make. [2]

(ii) In how many of these selections will there be more watercolour paintings than oil paintings? [3]

5 **(i)** Express $\frac{1}{\sqrt{32}}$ as a power of 2. [1]

(ii) Express $(64)^{\frac{1}{x}}$ as a power of 2. [1]

(iii) Hence solve the equation $\frac{(64)^{\frac{1}{x}}}{2^x} = \frac{1}{\sqrt{32}}$. [3]

6 **(i)** Differentiate $x^2 \ln x$ with respect to x. [2]

(ii) Use your result to show that $\int_1^{e} 4x \ln x \, dx = e^2 + 1$. [4]

7 Given that $\mathbf{A} = \begin{pmatrix} 2 & 3 \\ -2 & -1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 8 & 10 \\ -4 & 2 \end{pmatrix}$, find the matrices $\mathbf{X}$ and $\mathbf{Y}$ such that

(i) $\mathbf{X} = \mathbf{A}^2 + 2\mathbf{B}$, [3]

(ii) $\mathbf{YA} = \mathbf{B}$. [4]

8 The equation of the curve C is $2y = x^2 + 4$. The equation of the line L is $y = 3x - k$, where k is an integer.

(i) Find the largest value of the integer k for which L intersects C. [4]

(ii) In the case where $k = -2$, show that the line joining the points of intersection of L and C is bisected by the line $y = 2x + 5$. [4]

9 The position vectors, relative to an origin O, of three points P, Q and R are $\mathbf{i} + 3\mathbf{j}$, $5\mathbf{i} + 11\mathbf{j}$ and $9\mathbf{i} + 9\mathbf{j}$ respectively.

(i) By finding the magnitude of the vectors $\overrightarrow{PR}$, $\overrightarrow{RQ}$ and $\overrightarrow{QP}$, show that angle PQR is 90°. [4]

(ii) Find the unit vector parallel to $\overrightarrow{PR}$. [2]

(iii) Given that $\overrightarrow{OQ} = m\overrightarrow{OP} + n\overrightarrow{OR}$, where m and n are constants, find the value of m and of n. [3]

10 The functions f and g are defined, for $x \in \mathbb{R}$, by

$$\mathrm{f} : x \mapsto 3x - 2,$$

$$\mathrm{g} : x \mapsto \frac{7x - a}{x + 1}, \text{ where } x \neq -1 \text{ and } a \text{ is a positive constant.}$$

(i) Obtain expressions for f^{-1} and g^{-1}. [3]

(ii) Determine the value of a for which $\mathrm{f}^{-1}\mathrm{g}(4) = 2$. [3]

(iii) If $a = 9$, show that there is only one value of x for which $\mathrm{g}(x) = \mathrm{g}^{-1}(x)$. [3]

11 A particle, moving in a straight line, passes through a fixed point O with velocity $14\,\mathrm{ms}^{-1}$. The acceleration, $a\,\mathrm{ms}^{-2}$, of the particle, t seconds after passing through O, is given by $a = 2t - 9$. The particle subsequently comes to instantaneous rest, firstly at A and later at B. Find

(i) the acceleration of the particle at A and at B, [4]

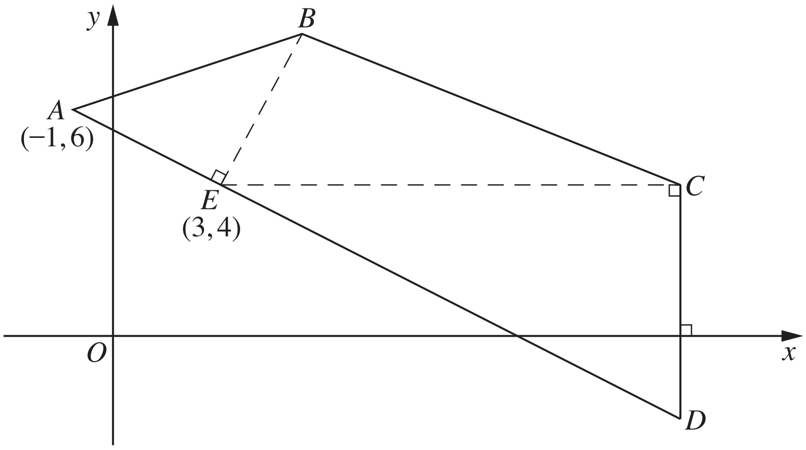
(ii) the greatest speed of the particle as it travels from A to B, [2]

(iii) the distance AB. [4]

12 Answer only **one** of the following two alternatives.

EITHER

Solutions to this question by accurate drawing will not be accepted.



The diagram shows a quadrilateral *ABCD*. The point *E* lies on *AD* such that angle *AEB* = 90°. The line *EC* is parallel to the *x*-axis and the line *CD* is parallel to the *y*-axis. The points *A* and *E* are (– 1, 6) and (3, 4) respectively. Given that the gradient of *AB* is $\frac{1}{3}$,

(i) find the coordinates of *B*. [5]

Given also that the area of triangle *EBC* is 24 units^2,

(ii) find the coordinates of *C*, [3]

(iii) find the coordinates of *D*. [2]

OR

(a) The expression $f(x) = x^3 + ax^2 + bx + c$ leaves the same remainder, *R*, when it is divided by $x + 2$ and when it is divided by $x - 2$.

(i) Evaluate *b*. [2]

f(*x*) also leaves the same remainder, *R*, when divided by $x - 1$.

(ii) Evaluate *a*. [2]

f(*x*) leaves a remainder of 4 when divided by $x - 3$.

(iii) Evaluate *c*. [1]

(b) Solve the equation $x^3 + 3x^2 = 2$, giving your answers to 2 decimal places where necessary. [5]

BLANK PAGE

BLANK PAGE

BLANK PAGE