UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE O Level

MARK SCHEME for the November 2005 question paper

4037 ADDITIONAL MATHEMATICS

4037/01 Paper 1 maximum raw mark 80

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

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UNIVERSITY of CAMBRIDGE
International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

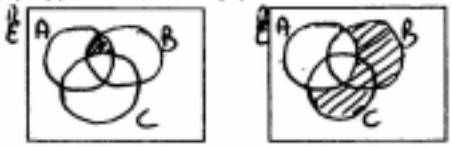
MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW –1,2 This is deducted from A or B marks when essential working is omitted.

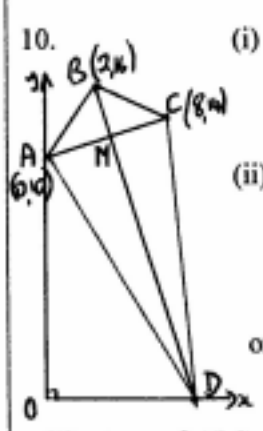
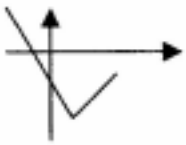
PA –1 This is deducted from A or B marks in the case of premature approximation.

S –1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX –1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

1 (a) Squares and sets to 0 → $x^2 - 13x + 36$ → $x = 4$ and 9 → $x < 4$ and $x > 9$	M1 A1 A1 [3]	Condone alg errors in squaring but eqn must be set to 0 and attempt at solution. Independent of <, = or > Co – not for ≤ or ≥.
2 (a) (i) $A' \cap B$, (ii) $A' \cup B$ or $(A \cap B')'$ (b) (i) (ii)  	B1 B1 [2] B1 [1] B1 [1]	Co Co Co Co
3 $2(2x + 6/x) = x + c$ → $3x^2 - cx + 12 = 0$ Use of $b^2-4ac=0$ → $c = 12$ and → $c = -12$	M1 M1 A1 A1√ [4]	Eliminates y and forms a quadratic eqn. Uses $b^2-4ac=0$ on quadratic=0. Co For the −ve root of $c^2=k$.
4 Length = $2-\sqrt{3}$. Area = $(2-\sqrt{3})^2 = 7 - 4\sqrt{3}$ Height = volume ÷ area = $(2\sqrt{3} - 3) \div (7 - 4\sqrt{3})$ × top and bottom by $7 + 4\sqrt{3}$ → height = $3 + 2\sqrt{3}$ (or $(7-4\sqrt{3})(a+b\sqrt{3}) = 2\sqrt{3}-3$ Sim eqns B1 M1(as before) M1 forming + sol A1)	B1 M1 M1 A1 [4]	Co Used with volume and area Technique correct Co but beware decimal answers leading to correct answer – this gets 0/4
5 F(i+12j) at (3i+2j). S (85i+5j) at -5i+kj At time t, $r_F = (1+3t)i+(12+2t)j$ $r_S = (85-5t)i +(5+kt)j$ Equate i's $1 + 3t = 85 - 5t$ $t = 10.5$ Equate j's $12+2t = 5+kt$ $k = 2\frac{2}{3}$	M1 A1 M1 A1 M1A1√ [6]	M1 for one x or y component. A1 for both x components correct. M1 for equating x components. A1 Co M1 for equating y components. A1√ for his t and his components.

6 $v = 6 - 6e^{-3t}$ (i) $a = dv/dt = 18e^{-3t}$ $t = \ln 2$ $e^t = 2 \rightarrow e^{-3t} = 1/8$ $\rightarrow a = 18/8$ or 2.25 (ii) $s = \int v dt = 6t + 2e^{-3t}$ [+c] but t=0 when s=0, $\rightarrow c = -2$ Put $t = \ln 2 \rightarrow 2.41$	M1 A1 A1 [3] M1 A1 DM1 A1 [4]	Attempt at differentiation. Co. Co. Attempt at integration. Co. (ignore c) Don't allow if c automatically = 0. Co.
7 (a) $2 = \log_7 49$ Combines two logs correctly Forms equation and solves $\rightarrow y = 2$ (b) $\log_p 8 \times \log_{16} p$ $\log_p 8 = \log_2 8 \div \log_2 p = 3/\log_2 p$ $\log_{16} p = \frac{1}{4}\log_2 p$ $\rightarrow \frac{3}{4}$ or 0.75	B1 M1 DM1 A1 [4] M1 M1 A1 [3]	Anywhere Allow even if first B1 not given Needs to have got rid of all logs correctly. Co Nb change to lg is same scheme –same work needed. Change of base once Same base – 2,8,16,10 – so that p cancels. Co.
8 $y = (x+2)\sqrt{(x-1)}$ (i) $dy/dx = \sqrt{x-1} + (x+2) \times \frac{1}{2}(x-1)^{-\frac{1}{2}}$ $= (x - 1 + \frac{1}{2}x + 1) \div \sqrt{x-1}$ $= \frac{3x}{2\sqrt{x-1}}$ $k = 1.5$ or $1\frac{1}{2}$ (ii) $\int_2^5 \frac{x dx}{\sqrt{x-1}} = \frac{2}{3} \times \sqrt{(x-1)}(x+2)$ evaluated from 2 to 5 = $\frac{2}{3} \times (14 - 4)$ $\rightarrow 20/3$	B1 M1 M1 A1 [4] M1 A1√ DM1 A1 [4]	B1 for correct diff of $\sqrt{x-1}$ Use of "uv" for M. co Reasonable attempt at algebra co Use of $\int$ = reverse of differentiation. For 1 ÷ "his k" Value at 25" – "value at 2". Co for A.
9 (a) $3\cos x = 8\tan x = 8\sin x/\cos x$ $\rightarrow 3\cos^2 x = 8\sin x = 3(1 - \sin^2 x)$ $\rightarrow 3s^2 + 8s - 3 = 0$ $\rightarrow s = -3$ or $\frac{1}{3}$ $\rightarrow x = 19.5°$ or $160.5°$ (b) $\cos(\frac{2}{3}y) = -\sqrt{3}/2$ $\rightarrow \frac{2}{3}y = 5\pi/6$ or 2π – (answer) $\rightarrow y = 5.50$ or $7\pi/4$	M1 M1 DM1 A1 A1√ [5] M1 DM1 A1 [3]	Use of t=s/c Use of $s^2+c=1$ Correct attempt at quadratic = 0 Co. for 180° – 1st answer. For $\cos^{-1}(\pm\sqrt{3}/2)$ For 2π – answer Allow if other answer (3.93) is given.

10.  (i) Pythagoras → $AB=\sqrt{40}$. $BC=\sqrt{40}$	M1 A1 [2]	Or by vectors
(ii) m of $AC = ½$. m of $BD = -2$ eqn BD → $y+2x=20$ → $D\,(10, 0)$	B1 M1 M1 A1 [4]	Anywhere Use of $m_1m_2 = -1$ Not necessary to have eqn since $y=0$ may be used.
or $M(4,12)$ → $m = -2$		Finds M→m of -2 equivalent to B1M1.
(iii) Area of ABC : Area of ACD $BM : MD$ $= \sqrt{20} : \sqrt{180} = 1:3$	M1 M1 A1 [3]	Realises that only heights are needed. Pythagoras – any form ok for A mark.
(or finds each area by "matrix" or $½bh$)		M1 ABC (40) M1 ACD (120) A1 1:3.
11 $f : x \to 2x-3-4 \quad -2 \le x \le 3$ (i) 	B2,1 [2]	Must be "V" shaped to get any marks. Must cross −ve x and −ve y axes. Endpoint −ve y. Start point + ve y.
(ii) Range of f −4 to 3	B1 B1 [2]	Independent of graph. −4 on own ok. 3 on its own.
(iii) $2x - 3 = 2 \to x = 2½$ or 2.5 $2x - 3 = -2 \to x = ½$ or 0.5	B1 M1A1 [3]	Co – answer only Correct method of other solution. co
(iv) Largest value is x value at "V" = 1½	B1√ [1]	From his graph – or any other method
(v) Equation of left hand part of "V". $m = -2 \to -2x - 1$.	M1 A1 [2]	Realises that one line only is needed + correct method ($y=mx+c$ etc). Or $-(2x-3) - 4 = -2x - 1$ Doesn't need a or b implicitly mentioned

12 EITHER

x	1.5	2	2.5	3	3.5
y	7.3	3.5	2.0	1.3	0.9
lg$x$	0.18	0.30	0.40	0.48	0.54
lg$y$	0.86	0.54	0.30	0.11	−0.05

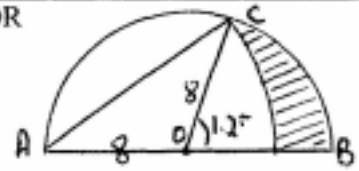
(i) Draws graph of lg$y$ against lg$x$. Accuracy of points and line. — M1, A2,1,0 [3] — Knows what to do. Within ½ square.

(ii) n = 2.45 to 2.60; a = 19.5 to 21.0 — M1 A1, M1 A1 [4] — Needs $m = \pm n$ for M. Co for +ve only. Needs lg$a$ = intercept on y axis.

(iii) $y = x^2 \rightarrow \lg y = 2\lg x$. → Line of gradient 2. $y = x^2$ intersects $yx^n = a$ where the lines meet. → x = 1.90 to 2.00 — M1, A1, A1 [3] — Allow M1 for statement in log form. Reasonable attempt at line of m = 2 through (0,0). Co.

(or solves $y = x^2$ with $yx^{2.5} = 20$ alg)

12 OR



(i) Area sector COB = ½×8²×1.2 = 38.4 — M1 A1 [2] — Use of ½$r^2\theta$ with radians.

(ii) $AOC = \pi - 1.2$ rad or OAC = 0.6rad; $AC^2 = 8^2 + 8^2 - 2\times8\times8\times\cos(\pi - 1.2)$ or $AC = 2\times8\times\cos0.6 = 13.2$; area = ½×13.2²×0.6 = 52.27 — B1, M1 A1, M1 A1√ [5] — Anywhere. Cosine rule or splitting into two 90° triangles. Use of ½$r^2\theta$ with radians.

(iii) Sector ACD + Shaded = AOC + Sector OBC — M1 — Plan mark linking the 4 regions.

Triangle AOC = ½×8×8×sin(π−1.2) = 29.8(3) — M1 — Independent mark –for triangle AOC.

Shaded = 29.83+38.4−52.27 = 15.9 (allow 16.0) — A1, [3] — Co for either 15.9 or 16.0.

DM1 for quadratic equation. Equation must be set to 0 if using formula or factors.

Formula.
Must be correct
– ignore arithmetic and algebraic slips.

Factors
Must attempt to put quadratic into 2 factors.
Each factor then equated to 0.