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FOREWORD

This booklet contains reports written by Examiners on the work of candidates in certain papers. **Its contents are primarily for the information of the subject teachers concerned**.

ADDITIONAL MATHEMATICS

GCE Ordinary Level

Paper 4037/01

Paper 1

General comments

There were many very good attempts and the overall standard seemed slightly better than in recent years. It would seem that candidates are becoming more familiar and confident. Standards of presentation and of algebraic and numerical manipulation remain very variable. Candidates need reminding that they should not divide script pages into two columns. It makes the candidate's work very difficult to mark and to be able to accurately allocate marks.

Comments on specific questions

Question 1

This was generally well answered with the majority of candidates squaring **A** prior to finding the inverse. Candidates were more comfortable with evaluating the inverse of a matrix than with squaring a matrix, for a significant number of attempts assumed that $\begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}^2 = \begin{pmatrix} 4 & 1 \\ 1 & 1 \end{pmatrix}$.

Answer: $\frac{1}{9}\begin{pmatrix} 0 & -3 \\ 3 & 3 \end{pmatrix}$.

Question 2

Part **(i)** was poorly answered with only a small minority of candidates realising that if the selection is to contain one particular CD, only 8 CDs remain, from which 3 must be selected. Part **(ii)** was more successfully attempted with the majority of candidates realising that there were three cases to be considered – and that each case resulted in the product of two '${}_nC_r$ s'.

Answers: **(i)** 56; **(ii)** 27.

Question 3

This presented many difficulties with a minority of candidates realising the need to evaluate $\cos\theta$, either by using $\sin^2\theta + \cos^2\theta = 1$ or by using an appropriate right-angled triangle. Of those using a triangle, '$x^2 = (\sqrt{3})^2 - 1^2 \Rightarrow x = 2$' was a common error. Most candidates realised the need to rationalise the denominator by multiplying numerator and denominator by (√2 + 1), though many attempted this algebraically without a numerical value for $\cos\theta$.

Answer: 1 + √2.

Question 4

Although use of $\overrightarrow{AB}$ **=** ***a*** + **b** or **a** − **b** were seen on many scripts, most candidates showed confidence in manipulating vectors and the majority obtained a correct expression for vector $\overrightarrow{OC}$. Only about a half of all candidates understood the term 'unit vector' and although there were some perfectly correct proofs, far too many were seen in which '$3^2 + 4^2 = 5^2$' seemed to be sufficient for the final answer.

Answer: $\overrightarrow{OC} = \begin{pmatrix} -\frac{3}{5} \\ \frac{4}{5} \end{pmatrix}$.

Question 5

Only a few candidates realised that the maximum value of 5cos*Bx* is 5 and that this leads directly to *A* = −2. The terms 'amplitude' and 'period' were only sketchily understood. Although only about a half of all candidates realised that the graph oscillated between 3 and −7, follow-though marks were often scored for graphs oscillating between '*a*' and '*b*' providing that the curve had exactly $1\frac{1}{2}$ oscillations between 0º and 180º (candidates sketching 3 oscillations between 0º and 360º were not penalised) and that the curve started at a maximum point and finished at a minimum point.

Answers: **(i)** −2; **(ii)** 5; **(iii)** 3.

Question 6

Many candidates showed a vague understanding of the modulus function. The majority of answers to parts **(ii)** and **(iii)** used just the end-points of the interval, thereby failing to realise that the function could take either higher or lower values in the interval. It was particularly common to see the answer '$1 \leqslant g(x) \leqslant 8$' for part **(ii)**. The majority of candidates however did realise that the functions g and h (the modulus functions) were not one-one and therefore did not have an inverse.

Answers: **(i)** $-7 \leqslant f(x) \leqslant 8$; **(ii)** $0 \leqslant g(x) \leqslant 8$; **(iii)** $-7 \leqslant h(x) \leqslant 2$. Only f has an inverse.

Question 7

The majority of candidates realised the need to take logarithms and that $t = \log\left(\frac{0.66}{0.64}\right) \div \log 1.0025$. Unfortunately, final answers were affected by premature approximation, by incorrect evaluation of 2.5×10^{-3} and more seriously by assuming that $t = \log\left(\frac{0.66}{0.64} \div 1.0025\right)$. Part **(ii)** was well answered by many candidates, though others failed to realise that log 10 = 1 and a depressingly high number took log (8 − *x*) to be log 8 − log *x*. The solution of 10(8 − *x*) = 3*x* + 2 was generally accurate, though both 13*x* = 82 and 7*x* = 78 were common errors.

Answers: **(a)** 12.3; **(b)** 6.

Question 8

This was well answered with many accurate well-drawn graphs. A majority of candidates realised that $y = kx^n$ transformed to $\log y = \log k + n\log x$ and that the gradient equated to *n* and the intercept to log *k*. Of the other attempts, taking $y = kx^n$ as '$\log y = kn\log x$' or as '$\log y = n\log kx$', were common errors. It was surprising that a lot of candidates, on stating that log *k* = 4.1, were unable to evaluate *k* or assumed that the value of *k* was 4.1.

Answers: **(ii)** *k* =14 000 to 16 000, *n* = −0.88 to −0.92.

Question 9

In part **(i)**, a pleasing number of candidates recognised that '$b^2 - 4ac < 0$' and correctly established that $b^2 - 4ac = 9k^2 - 16k$. Cancellation of k unfortunately was common, resulting in the answer $k < \frac{16}{9}$. Of those solving the quadratic to obtain $k = 0$ and $k = \frac{16}{9}$, it was very common to see the answer left as $k < 0$ and $k < \frac{16}{9}$. Only a small proportion of all attempts realised that part **(ii)** had a direct link to part **(i)** with $k = 1$ and that there were therefore no points of intersection of the line with the curve. Others ignored the word 'hence' and solved the equations simultaneously.

Answers: **(i)** $0 < k < \frac{16}{9}$; **(ii)** No points of intersection.

Question 10

This was well answered and a source of high marks. In part **(i)** the majority of candidates correctly realised that $f(-a) = 3f(a)$, though occasionally the '3' came on the wrong side. In part **(ii)**, most attempts realised the need to find a first solution (usually $a = 2$, though $a = -3$ was common). Division, synthetically or otherwise, was generally correct. A small minority interpreted the word 'solve' as 'factorise' and left the final answer as the product of three brackets.

Answers: **(ii)** $a = -3$, $\frac{1}{2}$ and 2.

Question 11

The question presented problems to most candidates and it was very rare to award full marks. In part **(i)**, most candidates realised the need to integrate, but most ignored the constant of integration. Even when '$+c$' was included, few realised that $v = 0$ when $t = 4$, thereby leading to $c = 24$. Integration in part **(ii)** was accurate, but correct final answers for the distance were rare through the omission of the term '$24t$'. Only a handful of candidates were able to offer a correct sketch in part **(iii)**.

Answers: **(i)** 24 ms^{-1}; **(ii)** $58\frac{2}{3}$ m.

Question 12 EITHER

This was the less commonly attempted of the two alternatives. Amongst good candidates however, it was a source of high marks. Most realised the need to integrate to find the area under the curve and generally the integration was accurate. Obtaining the coordinates of the point of intersection of the curve with the x-axis was generally correct, though $\frac{1}{2}\ln 8 = 1.03$ was a common error. Evaluating the coordinates of B presented more difficulty with only about a half of all attempts realising the need to differentiate to find the gradient, and then the equation of the tangent. Taking A as (8, 0) instead of (7, 0) was another common error. A surprising number of candidates wrongly assumed that the shaded area could be obtained from a single integral i.e. $\int_{1.04}^{3.5} \left[7 - 2x - \left(8 - e^{2x}\right)\right] dx$, whilst many others completely ignored the value of e^{2x} when $x = 0$.

Answer: 7.43.

Question 12 OR

There were many accurate attempts at part **(i)** and the standard of algebra was pleasing. Most realised the need to obtain an expression for r in terms of x from the equation $4x + 2\pi r = 2$, though often the $4x$ came as $2x$ or even x and the $2\pi r$ was seen as πr. Virtually all candidates realised the need to differentiate for parts **(ii)** and **(iii)**. Unfortunately the 'π' in the denominator led many candidates to use the quotient rule and in at least half of all attempts $\frac{\mathrm{d}}{\mathrm{d}x}(\pi)$ was taken as 1. Of those differentiating directly, most were accurate. Setting $\frac{\mathrm{d}A}{\mathrm{d}x}$ to zero in part **(ii)** and using the second differential in part **(iii)** were nearly always used. The most common error was to evaluate x and to assume that this was the stationary value of A. Method marks for parts **(ii)** and **(iii)** were nearly always gained and there were many very good solutions.

Answers: **(ii)** 0.14; **(iii)** Minimum.

Paper 4037/02

Paper 2

General comments

The overall standard of performance appeared to be slightly better than last year. As always many candidates found the topic of relative velocity particularly difficult. The question on sets also caused problems for many, although some of the difficulty may have been linguistic in some cases.

Many marks were lost through a failure to understand when degrees or radians are the appropriate angle measure and again through using an insufficient level of accuracy when conducting calculations involving decimals.

Comments on specific questions

Question 1

The quotient rule was the favoured approach although the product rule was seen quite often. Both these methods frequently failed due to the inability to deal correctly with $\frac{\mathrm{d}}{\mathrm{d}x}(8)$ Rather than 0, $\frac{\mathrm{d}}{\mathrm{d}x}(8)$ was often taken to be 1 although this value was usually implied rather than stated explicitly. Some candidates using the quotient method committed the expected and common error of interchanging the terms in the numerator. The value of –0.05 was frequently obtained as a small increment rather than a rate of change. Common errors were to lose the negative sign or to offer $0.2 \times (-4)$ rather than $0.2 \div (-4)$.

Answers: **(i)** $\frac{-16}{(2x-1)^2}$; **(ii)** – 0.05 units s^{-1}.

Question 2

The arithmetic required in part **(i)** was undemanding and the total was usually correct. The point of the exercise was to write down two simple matrices which were conformable for multiplication, a task which many candidates were unable to accomplish. The majority of candidates were unable top answer part **(ii)** correctly, their attempts being based on $\begin{pmatrix}10 & 10\\ 4 & 4\end{pmatrix}\begin{pmatrix}180 & 40\\ 400 & 150\end{pmatrix}$ or on dealing with Friday and Saturday separately.

Answers: **(i)** $\begin{pmatrix}300 & 40\end{pmatrix}\begin{pmatrix}12\\ 5\end{pmatrix}$ or $\begin{pmatrix}12 & 5\end{pmatrix}\begin{pmatrix}300\\ 40\end{pmatrix} = \begin{pmatrix}3800\end{pmatrix}$;

(ii) $\begin{pmatrix}180 & 40\\ 400 & 150\end{pmatrix}\begin{pmatrix}10\\ 4\end{pmatrix} = \begin{pmatrix}1960\\ 4600\end{pmatrix}$ or $\begin{pmatrix}10 & 4\end{pmatrix}\begin{pmatrix}180 & 400\\ 40 & 150\end{pmatrix} = \begin{pmatrix}1960 & 4600\end{pmatrix}$; **(iii)** \$10 360.

Question 3

Nearly all candidates obtained $x + y = 12$. Many candidates were able to obtain the quadratic equation $x^2 - y^2 = 60$ and solve, reaching the required value for *BP*. Weaker candidates became confused and assumed $xy = 60$ or even $x^2 = 60$. Some correctly took *AB* to be $\sqrt{60}$ which was then converted to a decimal; in quite a number of instances this did not become 60 when squared. A fallacious argument followed the lines $60 = x^2 - y^2 \Rightarrow \sqrt{60} = x - y \Rightarrow 60 = (x - y)^2$.

Answer: 3.5 m.

Question 4

A majority of the candidates were able to obtain $\sin x = 1$ but a considerable of these then gave the required value of x as 90 rather than $\frac{\pi}{2}$ or 1.57. Most were able to find $g^{-1}(x)$ but for weaker candidates $g^{-1}f(x)$ became $\left(\frac{x+3}{2}\right)\sin x$. Successful evaluation of $g^2(2.75)$ was easily achieved by substituting $x = 2.75$ in $g^2(x)$, few candidates opting to find g{g(2.75)}; weak candidates took $g^2(x)$ to be $(2x - 3)^2$ and for them $g^2(2.75)$ became $(2x - 3)^2(2.75)$. There were a few pleasing solutions where candidates solved $f(x) = g^3(2.75)$.

Answer: $\frac{\pi}{2}$ or 1.57.

Question 5

(i) Differentiation was frequently incorrect due to a failure to realise that a product was involved. Quite a number of candidates took the factors of the product to be x and $\ln x - x$, rather than $\ln x - 1$. A common error was to cancel the x's in $x \times \frac{1}{x}$ with the subsequent disappearance of this term instead of it becoming 1.

(ii) Most candidates understood that $\int \ln x \, dx$ was required. Candidates who had correctly completed the differentiation in part **(i)** were almost always able to reverse the process. Those unable to differentiate correctly were rarely able to integrate $\ln x$, although a few guessed that part **(i)** was connected and so evaluated $[x\ln x - x]_1^3$. Weak candidates produced nonsensical attempts at integration or simply evaluated $[\ln x]_1^3$. Those candidates for whom $x \times \frac{1}{x}$ disappeared in part **(i)** often arrived at $\int \ln x \, dx = x\ln x$ by reversing their result, $\frac{d}{dx}(x\ln x - x) = \ln x - 1$. The lower limit, $x = 1$, was usually found although some candidates took the lower limit to be $x = 0$, dismissing any difficulty with $\ln x$ at this value. Although 1.3 was accepted, the truncated value 1.29, which occasionally occurred, was not.

Answer: **(ii)** 1.30 units^2.

Question 6

Most candidates were able to find $\frac{dy}{dx}$ correctly. The commonest errors occurred in finding $\frac{d}{dx}\left(e^{2x}\right)$ e.g. $2xe^{2x}$ or $2xe^2$, in taking $\frac{d}{dx}(\sin x)$ to be $-\cos x$, and in interchanging the terms in the numerator of the quotient rule. Despite being specifically asked for $\frac{dy}{dx}$ some candidates did not include the denominator of the quotient rule. Most obtained $\tan x = 0.5$ but there were a few cases of $\cot x = 0.5$, and of $\tan x = 0$ through taking $\cos x \times \frac{1}{\cos x}$ to be zero. As in **Question 4** the value of x was frequently given in degrees (26.6) rather than in radians.

Answer: **(ii)** 0.464.

Question 7

There were many completely correct solutions with very few candidates attempting to employ logarithms. Common errors were $125^x = 25(5^y) = 125^y \Rightarrow x = y$, $5^{3x} = 5^2(5^y) \Rightarrow 3x = 2y$ and $7^x \div 49^y = 1 \Rightarrow x - 2y = 1$.

Answers: $x = 0.8$, $y = 0.4$.

Question 8

This question was very poorly answered and there were very few completely correct solutions. Although k was usually placed in $C \cap D$, about half the candidates then put $7k$, rather than $6k$, in $C \cap D'$ and $4k$, rather than $3k$, in $C' \cap D$. The statement $n(\mathscr{E}) = 6 \times n(C' \cap D')$ was rarely interpreted correctly so that $2k$ was hardly ever seen in the region corresponding to $C' \cap D'$. In part **(ii)**, 'homes which do not have both a computer and a dishwasher' was almost always taken to indicate $C \cup D$ or $(C \cap D') \cup (C' \cap D)$ rather than $(C \cap D)'$.

Answer: **(ii)** 180 000.

Question 9

This question also produced relatively few correct solutions. The basic difficulty was that most candidates were unable to produce a correct triangle of velocities. The speed of 300 km h^{-1} was commonly shown as lying along XY, whilst others indicated that the speeds of 120 and 300 km h^{-1} were inclined at 60° or 120° to each other. Some diagrams merely consisted of a right-angled triangle and in others the direction of the wind was, in effect, from the west rather that towards the west. Many of the weakest candidates omitted this question while others simply divided 720 by 300, or 300 + 120, or 300 – 120.

Answer: 3.25 hours.

Question 10

(a) Despite the formulae given on page 2 of the question paper, some candidates replaced $\tan^2 x$ by $1 - \sec^2 x$ or $1 + \sec^2 x$. Most managed to substitute correctly, arriving at a quadratic equation in $\sec x$ or, alternatively, $\cos x$. The solution of this equation was usually correct except for those candidates who proceeded via $\sec x(4\sec x + 15) = 4 \Rightarrow \sec x = 4$ or $4\sec x + 15 = 4$. A few candidates arriving at the correct values of $\sec x$ then took $\sec x$ to be $\frac{1}{\sin x}$ or $\frac{1}{\tan x}$. The angles given as the answers to $\cos x = -0.25$ frequently contained errors. Some candidates included the basic angle of 75.5° with their answers, others took the basic angle to be 104.5° resulting in answers of 75.5° and 284.5°, whilst some added 180° to arrive at a second solution, thus obtaining 104.5° and 284.5°.

(b) This part of the question proved more difficult than part **(a)** with weak candidates taking $\tan(3x - 2)$ to be $\tan 3x - \tan 2$, and with very many candidates only able to work in degrees. Others attempted to convert to radians at too late a stage e.g. $3y - 2 \approx 101.3° \Rightarrow y = 103.3° \div 3 \approx 34.4 \times \frac{\pi}{180}$ radians. Quite a number of those who worked correctly in radians did not proceed far enough, adding π and 2π to the basic angle of 1.37 when 3π was necessary. Candidates who successfully obtained the required value of y often made an initial estimate of the necessary value of $3y - 2$ by arguing that if $y > 3$ then $3y - 2 > 7$.

Answers: **(a)** 104.5°, 255.5°; **(b)** 3.35.

Question 11

(a)(i) There were very many correct answers and failure to obtain full marks was almost always caused by incorrect calculation of one of the coefficients, stating the first term to be 1 or 2, or omitting the last one or two terms.

(ii) Less than half the candidates who answered part **(i)** correctly were able to deal with this. Some candidates just considered the first two terms of the expansion, quoting *a* as 32 and *b* as 80. Many were unable to deal with $(\sqrt{3})^3$ and $(\sqrt{3})^5$; some were unable to make any attempt whilst others evaluated these powers as decimals and added to 32 in order to obtain *a*.

(b) Most of the better candidates considered powers and solved $r - (7 - r) = 1$ or $(7 - r) - r = 1$ to determine which term was required; some candidates attempting this were defeated by using 0 rather than 1 on the right-hand side of the equation. Many candidates simply wrote the whole expansion; some picked out the required term, others selected an incorrect term and some were unable to make any selection. The negative sign was frequently omitted and 2240 was given as the answer by some candidates whose expansion contained the term $-2240x$. A common error was to use 4 rather than 4^3 leading to an answer of – 140.

Answers: **(a)(i)** $32 + 80x + 80x^2 + 40x^3 + 10x^4 + x^5$, **(ii)** $a = 362$, $b = 209$; **(b)** – 2240.

Question 12 EITHER

This was the more popular of the two alternatives. The question provided opportunity for a variety of methods to be used. The most usual method was to find the coordinates of *C* by using *D* as the mid-point of *BC*, obtain simultaneous equations and solve them to find the coordinates of *E*, calculate lengths or use the array method to find the area of each triangle or, alternatively, apply the array method to the whole figure. The ideas necessary to complete this process were rarely incorrect and loss of marks was generally due to miscalculation. Some candidates attempted to find *E* by the solution of a linear equation with the quadratic equation obtained from $\frac{y-4}{x-2} \times \frac{y-6}{x-5} = -1$ but the algebra involved usually proved to be too much. Those applying the array method for area to the whole figure did not always understand that it is necessary to proceed round the figure taking the coordinates in order. Other methods involved finding lengths then comparing similar triangles to find the ratio of lengths of sides or, more directly, the ratio of areas; alternatively, the lengths of *AB* and *BC* were used to find the angle *ACB*, hence angle *CDE* and, by trigonometry, the lengths of *CE* and *ED*. Candidates using the trigonometric approach and those using approximations to lengths were not always accurate enough to obtain an answer approximating to 15.6. Weak candidates frequently had no clear overall plan and so wasted time and effort finding, and sometimes solving, the equations of *AB* and *BC*. The application of $\frac{1}{2}$ base × height e.g. to triangle *ABC* sometimes resulted in $\frac{1}{2}(8 - 2)(11 - 8)$ i.e. $\frac{1}{2}$(difference in *x*-coordinates of *B* and *C*)×(difference in *y*-coordinates of *A* and *B*).

Answer: 15.6 units^2.

Question 12 OR

All those attempting this question realised that, in order to answer parts **(ii)** and **(iii)**, it was necessary to find angles *BOD* and *BAD*. Some candidates found both these angles en route to showing *AB* ≈ 17.9, usually through employing an elaborate method e.g. $AB^2 = 10^2 + 10^2 - 2\times10\times10\times\cos AOB$, whereas others achieved the same result quite simply from $AB = \sqrt{8^2 + (6+10)^2}$. The cosine rule was also used to find angle *BOD* via $\cos BOD = \frac{10^2 + 10^2 - 16^2}{2\times10\times10}$. Some candidates used the idea of area in an attempt to find angle *BOD* via $\frac{1}{2} \times 16 \times 6 = \frac{1}{2} \times 10\times10\times\sin BOD$ but this invariably led to angle *BOD* ≈ 73.7° rather than 106.3°. Only a few candidates used degrees when calculating arc length or area of sector, and the perimeter required in part **(ii)** was frequently correct. Candidates were far less successful in part **(iii)** where relatively few had a clear idea of how to find the required area, which was frequently taken to be the difference between the areas of the sectors *ABED* and *OBCD*. Many calculations were spoiled by premature approximation.

Answers: **(ii)** 35.1 cm; **(iii)** 24.4 cm^2.