CAMBRIDGE INTERNATIONAL EXAMINATIONS

Joint Examination for the School Certificate and General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS 4037/2

PAPER 2

OCTOBER/NOVEMBER SESSION 2002

2 hours

Additional materials:
- Answer paper
- Graph paper
- Mathematical tables

TIME 2 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions.

Write your answers on the separate answer paper provided.

If you use more than one sheet of paper, fasten the sheets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 5 printed pages and 3 blank pages.

SP (NH/TC) S40050/2

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

1 Write down the inverse of the matrix $\begin{pmatrix} 4 & 3 \\ 7 & 6 \end{pmatrix}$ and use this to solve the simultaneous equations

$$4x + 3y + 7 = 0,$$
$$7x + 6y + 16 = 0.$$ [4]

2 Find the first three terms in the expansion, in ascending powers of x, of $(2 + x)^6$ and hence obtain the coefficient of x^2 in the expansion of $(2 + x - x^2)^6$. [4]

3 Given that $k = \frac{1}{\sqrt{3}}$ and that $p = \frac{1 + k}{1 - k}$, express in its simplest surd form

(i) p,

(ii) $p - \frac{1}{p}$.

[5]

4 Given that $\mathscr{E} = \{x: -5 < x < 5\}$,
$A = \{x: 8 > 2x + 1\}$,
$B = \{x: x^2 > x + 2\}$,

find the values of x which define the set $A \cap B$. [6]

5 **(a)** The producer of a play requires a total cast of 5, of which 3 are actors and 2 are actresses. He auditions 5 actors and 4 actresses for the cast. Find the total number of ways in which the cast can be obtained. [3]

(b) Find how many different odd 4-digit numbers less than 4000 can be made from the digits 1, 2, 3, 4, 5, 6, 7 if no digit may be repeated. [3]

6 The cubic polynomial f(x) is such that the coefficient of x^3 is –1 and the roots of the equation f(x) = 0 are 1, 2 and k. Given that f(x) has a remainder of 8 when divided by $x - 3$, find

(i) the value of k,

(ii) the remainder when f(x) is divided by $x + 3$.

[6]

7 **(i)** Differentiate $x \sin x$ with respect to x. [2]

(ii) Hence evaluate $\int_0^{\frac{\pi}{2}} x \cos x \, dx$. [4]

8 **(i)** Sketch the graph of $y = \ln x$. [2]

(ii) Determine the equation of the straight line which would need to be drawn on the graph of $y = \ln x$ in order to obtain a graphical solution of the equation $x^2 e^{x-2} = 1$. [4]

9 **(a)** Find, in its simplest form, the product of $a^{\frac{1}{3}} + b^{\frac{2}{3}}$ and $a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}$. [3]

(b) Given that $2^{2x+2} \times 5^{x-1} = 8^x \times 5^{2x}$, evaluate 10^x. [4]

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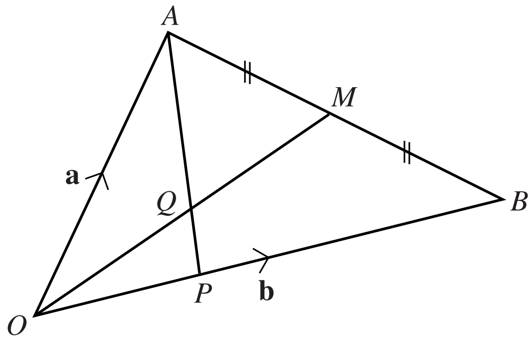


In the diagram, $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{AM} = \overrightarrow{MB}$ and $\overrightarrow{OP} = \frac{1}{3}\overrightarrow{OB}$.

(i) Express $\overrightarrow{AP}$ and $\overrightarrow{OM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [3]

(ii) Given that $\overrightarrow{OQ} = \lambda\overrightarrow{OM}$, express $\overrightarrow{OQ}$ in terms of λ, $\mathbf{a}$ and $\mathbf{b}$. [1]

(iii) Given that $\overrightarrow{AQ} = \mu\overrightarrow{AP}$, express $\overrightarrow{OQ}$ in terms of μ, $\mathbf{a}$ and $\mathbf{b}$. [2]

(iv) Hence find the value of λ and of μ. [3]

11 A car moves on a straight road. As the driver passes a point A on the road with a speed of $20\,\text{ms}^{-1}$, he notices an accident ahead at a point B. He immediately applies the brakes and the car moves with an acceleration of $a\,\text{ms}^{-2}$, where $a = \frac{3t}{2} - 6$ and t s is the time after passing A. When $t = 4$, the car passes the accident at B. The car then moves with a constant acceleration of $2\,\text{ms}^{-2}$ until the original speed of $20\,\text{ms}^{-1}$ is regained at a point C. Find

(i) the speed of the car at B, [4]

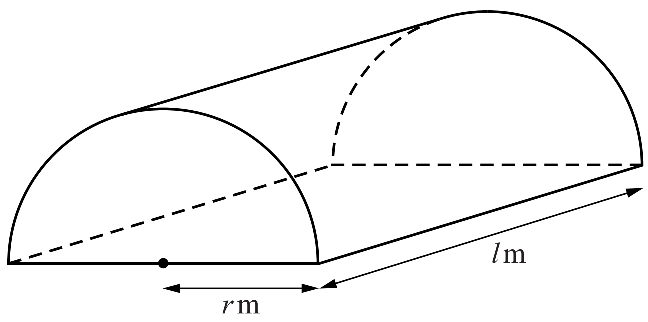
(ii) the distance AB, [3]

(iii) the time taken for the car to travel from B to C. [2]

Sketch the velocity-time graph for the journey from A to C. [2]

12 Answer only **one** of the following two alternatives.

EITHER



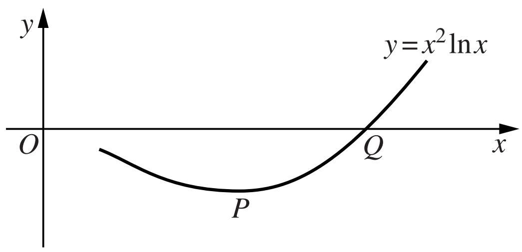
The diagram shows a greenhouse standing on a horizontal rectangular base. The vertical semicircular ends and the curved roof are made from polythene sheeting. The radius of each semicircle is r m and the length of the greenhouse is l m. Given that 120 m^2 of polythene sheeting is used for the greenhouse, express l in terms of r and show that the volume, V m^3, of the greenhouse is given by

$$V = 60r - \frac{\pi r^3}{2}.$$ [4]

Given that r can vary, find, to 2 decimal places, the value of r for which V has a stationary value. [3]

Find this value of V and determine whether it is a maximum or a minimum. [3]

OR



The diagram shows part of the curve $y = x^2 \ln x$, crossing the x-axis at Q and having a minimum point at P.

(i) Find the value of $\frac{dy}{dx}$ at Q. [4]

(ii) Show that the x-coordinate of P is $\frac{1}{\sqrt{e}}$. [3]

(iii) Find the value of $\frac{d^2y}{dx^2}$ at P. [3]

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