CAMBRIDGE INTERNATIONAL EXAMINATIONS

Joint Examination for the School Certificate and General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS 4037/1

PAPER 1

OCTOBER/NOVEMBER SESSION 2002

2 hours

Additional materials:
- Answer paper
- Graph paper
- Mathematical tables

TIME 2 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions.

Write your answers on the separate answer paper provided.

If you use more than one sheet of paper, fasten the sheets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 5 printed pages and 3 blank pages.

University of Cambridge Local Examinations Syndicate

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$
$$\sec^2 A = 1 + \tan^2 A.$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

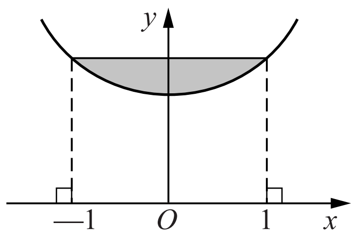
$$\Delta = \frac{1}{2} bc \sin A.$$

1 Solve, for $0° \leqslant \theta \leqslant 360°$, the equation $4\sin\theta + 3\cos\theta = 0$. [3]

2 Find the values of m for which the line $y = mx - 9$ is a tangent to the curve $x^2 = 4y$. [4]

3 The speed v ms^{-1} of a particle travelling from A to B, at time t s after leaving A, is given by $v = 10t - t^2$. The particle starts from rest at A and comes to rest at B. Show that the particle has a speed of 5 ms^{-1} or greater for exactly $4\sqrt{5}$ s. [5]

4



The diagram shows part of the curve $y = e^x + e^{-x}$ for $-1 \leqslant x \leqslant 1$. Find, to 2 decimal places, the area of the shaded region. [6]

5 A company produces 4 types of central heating radiator, known as types A, B, C and D. A builder buys radiators for all the houses on a new estate. There are 20 small houses, 30 medium-sized houses and 15 large houses.

A small house needs 3 radiators of type A, 2 of type B and 2 of type C.
A medium-sized house needs 2 radiators of type A, 3 of type C and 3 of type D.
A large house needs 1 radiator of type B, 6 of type C and 3 of type D.

The costs of the radiators are \$30 for type A, \$40 for B, \$50 for C and \$80 for D.

Using matrix multiplication twice, find the total cost to the builder of all the radiators for the estate. [6]

6 A curve is such that $\dfrac{dy}{dx} = \dfrac{6}{(2x-3)^2}$. Given that the curve passes through the point (3, 5), find the coordinates of the point where the curve crosses the x-axis. [6]

7 The function f is given by $f: x \mapsto x^3 + x - 1,\ x \in \mathbb{R}$.

(i) Determine whether or not the curve $y = f(x)$ has any turning points and hence explain why the function f has an inverse. [4]

(ii) Evaluate $f^{-1}(9)$. [3]

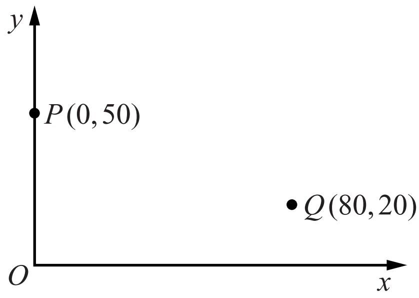
8 Solve the equation

(i) $e^x(2e^x - 1) = 10$, [3]

(ii) $\log_5(8y - 6) - \log_5(y - 5) = \log_4 16$. [4]

9 The line $2y = 3x - 6$ intersects the curve $xy = 12$ at the points P and Q. Find the equation of the perpendicular bisector of PQ. [8]

10



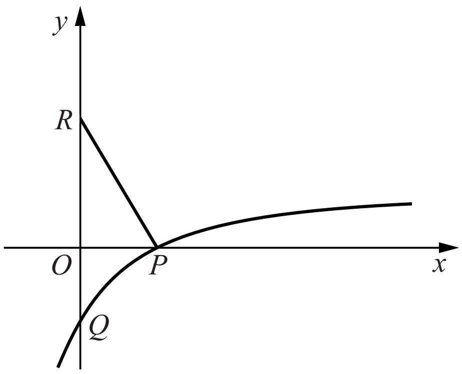
At 1200 hours, ship P is at the point with position vector $50\mathbf{j}$ km and ship Q is at the point with position vector $(80\mathbf{i} + 20\mathbf{j})$ km, as shown in the diagram. Ship P is travelling with velocity $(20\mathbf{i} + 10\mathbf{j})$ km h^{-1} and ship Q is travelling with velocity $(-10\mathbf{i} + 30\mathbf{j})$ km h^{-1}.

(i) Find an expression for the position vector of P and of Q at time t hours after 1200 hours. [3]

(ii) Use your answers to part (**i**) to determine the distance apart of P and Q at 1400 hours. [3]

(iii) Determine, with full working, whether or not P and Q will meet. [2]

11



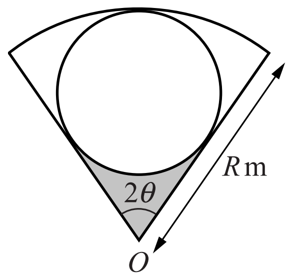
The diagram shows part of the curve $y = \dfrac{2x-6}{x+2}$ crossing the x-axis at P and the y-axis at Q. The normal to the curve at P meets the y-axis at R.

(i) Given that $\dfrac{dy}{dx} = \dfrac{k}{(x+2)^2}$, evaluate k. [3]

(ii) Find the length of RQ. [6]

12 Answer only **one** of the following two alternatives.

EITHER



The diagram shows a garden in the form of a sector of a circle, centre O, radius R m and angle 2θ. Within this garden a circular plot of the largest possible size is to be planted with roses. Given that the radius of this plot is r m,

(i) show that $R = r\left(1 + \dfrac{1}{\sin\theta}\right)$. [4]

Given also that $\theta = 30°$,

(ii) calculate the fraction of the garden that is to be planted with roses. [4]

When the circular plot has been constructed, the remainder of the garden consists of three regions. Given further that $R = 15$,

(iii) calculate, to 1 decimal place, the length of fencing required to fence along the perimeter of the shaded region. [3]

OR

A rectangle of area y m^2 has sides of length x m and $(Ax + B)$ m, where A and B are constants and x and y are variables. Values of x and y are given in the table below.

x	50	100	150	200	250
y	3700	11 000	21 600	36 000	53 500

(i) Use the data above in order to draw, on graph paper, the straight line graph of $\dfrac{y}{x}$ against x. [3]

(ii) Use your graph to estimate the value of A and of B. [4]

(iii) On the same diagram, draw the straight line representing the equation $y = x^2$ and explain the significance of the value of x given by the point of intersection of the two lines. [3]

(iv) State the value approached by the ratio of the two sides of the rectangle as x becomes increasingly large. [1]

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