

CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level
ADDITIONAL MATHEMATICS
PAPER 2

4037/2

MAY/JUNE SESSION 2002

2 hours

Additional materials:
Answer paper
Graph paper
Mathematical tables

TIME 2 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions.

Write your answers on the separate answer paper provided.

If you use more than one sheet of paper, fasten the sheets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 5 printed and pages 3 blank pages.



*Mathematical Formulae***1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

1 It is given that $\mathbf{A} = \begin{pmatrix} 5 & 7 \\ 4 & 5 \end{pmatrix}$ and that $\mathbf{A} - 3\mathbf{A}^{-1} - k\mathbf{I} = \mathbf{0}$, where \mathbf{I} is the identity matrix and $\mathbf{0}$ is the zero matrix. Evaluate k . [4]

2 (i) Sketch, on the same diagram, the graphs of $y = |x| + 1$ and $y = |2x - 3|$.

(ii) State the number of solutions of the equation $|2x - 3| = |x| + 1$.

[4]

3 Sets H , M and P are defined by

$$\begin{aligned} H &= \{\text{students studying history}\}, \\ M &= \{\text{students studying mathematics}\}, \\ P &= \{\text{students studying physics}\}. \end{aligned}$$

Express the following statements in set notation.

(i) No student studies both history and physics.

(ii) All physics students also study mathematics.

Describe in words which students belong to the set

(iii) $H' \cap M \cap P'$,

(iv) $(H \cup M) \cap P'$.

[5]

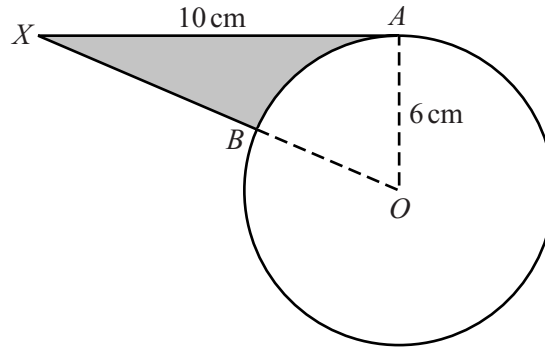
4 Solve the equation $x^3 - 4x^2 - 11x + 2 = 0$, expressing non-integer solutions in the form $a \pm b\sqrt{2}$, where a and b are integers. [6]

5 A plane flies from an airport A to an airport B . The position vector of B relative to A is $(1200\mathbf{i} + 240\mathbf{j})$ km, where \mathbf{i} is a unit vector due east and \mathbf{j} is a unit vector due north. Because of the constant wind which is blowing, the flight takes 4 hours. The velocity in still air of the plane is $(250\mathbf{i} + 160\mathbf{j})$ kmh⁻¹. Find the speed of the wind and the bearing of the direction from which the wind is blowing. [6]

6 (i) Show that $\frac{d}{dx} \left(\frac{\cos x}{1 - \sin x} \right)$ can be written in the form $\frac{k}{1 - \sin x}$ and state the value of k . [4]

(ii) Hence evaluate $\int_0^{\frac{\pi}{4}} \frac{\sqrt{2}}{1 - \sin x} dx$. [3]

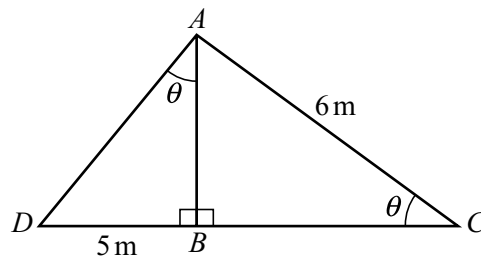
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The diagram shows a circle, centre O and radius 6 cm. The tangent from X touches the circle at A and $XA = 10$ cm. The line from X to O cuts the circle at B .

- (i) Show that angle AOB is approximately 1.03 radians. [1]
- (ii) Find the perimeter of the shaded region. [3]
- (iii) Find the area of the shaded region. [3]

8



In the diagram, angle $ABC = \text{angle } ABD = 90^\circ$, $AC = 6$ m, $BD = 5$ m and angle $ACB = \text{angle } DAB = \theta$.

- (i) Use each of the triangles ABC and ABD to express AB in terms of θ . [2]
- (ii) Hence evaluate θ . [5]

- 9 (a) Find the value of k for which the line $y = x + k$ is a tangent to the curve $y^2 = 4x + 8$. [4]
- (b) Find the value of a and of b for which the solution set of the quadratic inequality $x^2 + ax > b$ is $\{x: x > 2\} \cup \{x: x < -4\}$. [3]

10 Solve the equation

- (i) $\lg(2x) - \lg(x - 3) = 1$, [3]
- (ii) $\log_3 y + 4\log_y 3 = 4$. [4]

11 Functions f and g are defined, for $x \in \mathbb{R}$, by

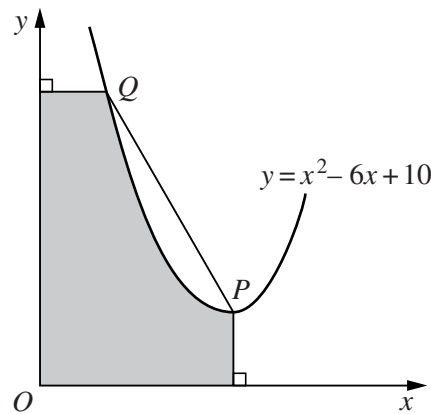
$$f : x \mapsto 3x - 7,$$

$$g : x \mapsto \frac{12}{x-2}, \quad x \neq 2.$$

- (i) Find f^{-1} and g^{-1} in terms of x , stating the value of x for which g^{-1} is not defined. [3]
- (ii) Find the values of x for which $fg(x) = x$. [3]
- (iii) Sketch the graphs of f and f^{-1} on the same diagram, giving the coordinates of the points of intersection of each graph with the axes. [3]

12 Answer only **one** of the following two alternatives.

EITHER



The diagram shows part of the curve $y = x^2 - 6x + 10$ passing through the points P and Q . The curve has a minimum point at P and the gradient of the line PQ is -2 . Calculate the area of the shaded region. [11]

OR

A particle travels in a straight line, starting from rest at point A , passing through point B and coming to rest again at point C . The particle takes 5 s to travel from A to B with constant acceleration. The motion of the particle from B to C is such that its speed, $v \text{ ms}^{-1}$, t seconds after leaving A , is given by

$$v = \frac{1}{225} (20 - t)^3 \quad \text{for } 5 \leq t \leq T.$$

- (i) Find the speed of the particle at B and the value of T .
- (ii) Find the acceleration of the particle when $t = 14$.
- (iii) Sketch the velocity-time curve for $0 \leq t \leq T$.
- (iv) Calculate the distance AC .

[11]

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