# THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS 

COURSE III

Friday, January 25, 2002 - 9:15 a.m. to 12:15 p.m., only

## Notice . . .

Scientific calculators must be available to all students taking this examination.

The formulas that you may need to answer some questions in this examination are found on page 2 . The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of the answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. The answer sheet cannot be accepted if you fail to sign this declaration.

## Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of $\neq$ or in radical form. [60]

1 Express $\frac{10 \pi}{3}$ radians in degree measure.

2 In the accompanying diagram, $\overline{B A}$ is a diameter and $\mathrm{m} \overparen{B C}=50$. Find $\mathrm{m} \angle C B A$.


3 Find the value of $27^{\frac{4}{3}}$.

4 Solve for $x: \quad \frac{5}{4 x}-\frac{6}{3 x}=\frac{1}{12}$

5 In $\triangle A B C, \mathrm{~m} \angle A=35, \mathrm{~m} \angle C=60$, and
$A C=12$ meters. Find the length of $\overline{B C}$ to the nearest meter.

6 In the accompanying diagram of a circle, chords $\overline{A B}$ and $\overline{C D}$ intersect at $E, C E=5, C D=13$, and $A E=4$. Find the length of $\overline{B E}$.


7 In a circle with a radius of 4 centimeters, what is the number of radians in a central angle that intercepts an arc of 24 centimeters?

8 If $x$ varies inversely as $y$, and $x=8$ when $y=3$, find the value of $x$ when $y=6$.

9 Express in simplest form: $\frac{\frac{1}{a}}{\frac{1}{a}-\frac{1}{b}}$
10 Evaluate: $\quad \sum_{k=3}^{6} k^{2}$
11 If $\mathrm{f}(x)=x^{2}+3$ and $\mathrm{g}(x)=x-2$, find $(\mathrm{f} \circ \mathrm{g})(2)$.

12 Express in simplest form: $\sqrt{48}-5 \sqrt{27}+2 \sqrt{75}$

Directions (13-35): For each question chosen, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

13 Which graph does not represent a function?

(1)

(2)

( 3 )

(4)

14 The amplitude of the graph of the equation $y=\frac{1}{2} \sin 3 x$ is
(1) $\frac{2 \pi}{3}$
(3) $\frac{1}{2}$
(2) $6 \pi$
(4) $\frac{3}{2}$

15 If $7.289 \times 10^{n}=0.007289$, what is the value of $n$ ?
(1) -2
(3) 3
(2) 2
(4) -3

16 The value of $\sin 170^{\circ} \cos 20^{\circ}-\cos 170^{\circ} \sin 20^{\circ}$ is
(1) $\frac{1}{2}$
(3) $\frac{\sqrt{3}}{2}$
(2) $-\frac{1}{2}$
(4) $-\frac{\sqrt{3}}{2}$

17 If $\sin A=-\frac{5}{13}$ and $\cos A>0$, angle $A$ terminates in Quadrant
(1) I
(3) III
(2) II
(4) IV

18 Which expression is equivalent to $\frac{x^{3}}{x+3}-\frac{9 x}{x+3}$ ?
(1) $\frac{-9 x}{x+3}$
(3) $\frac{x^{2}}{x+3}$
(2) $\frac{x}{x+3}$
(4) $x(x-3)$

19 What is the best approximation for the area of a triangle with consecutive sides of 4 and 5 and an included angle of $59^{\circ}$ ?
(1) 5.0
(3) 10.0
(2) 8.6
(4) 17.1

20 If $A=\pi r^{2}, \log A$ equals
(1) $2 \log \pi+\log r$
(3) $2 \log \pi+2 \log r$
(2) $\log \pi+2 \log r$
(4) $2 \pi \log r$

21 The value of $\tan (\operatorname{Arcsin} 1)$ is
(1) 1
(3) 90
(2) -1
(4) undefined

22 The solution set of the equation $3^{x^{2}+x}=9$ is
(1) $\{1\}$
(3) $\{-2,1\}$
(2) $\{-2\}$
(4) $\{-1,2\}$

23 The solution set of the equation $|2 x-1|+4=8$ is
(1) $\left\{\frac{5}{2}\right\}$
(3) $\left\{-\frac{3}{2}\right\}$
(2) $\left\{\frac{5}{2},-\frac{3}{2}\right\}$
(4) $\}$

24 What is the image of point $(-1,3)$ after a reflection in the line $x=2$ ?
(1) $(5,3)$
(3) $(-1,1)$
(2) $(3,3)$
(4) $(-1,-3)$

25 For all values of $\theta$ for which the expression is defined, $\frac{\sec \theta}{\csc \theta}$ is equivalent to
(1) $\sin \theta$
(3) $\tan \theta$
(2) $\cos \theta$
(4) $\cot \theta$

26 In the accompanying diagram of a unit circle, the ordered pair $(x, y)$ represents the point where the terminal side of $\theta$ intersects the unit circle.


If $\theta=150^{\circ}$, what is the value of $x$ ?
(1) 1
(3) $-\frac{1}{2}$
(2) $-\frac{\sqrt{3}}{2}$
(4) $-\frac{\sqrt{2}}{2}$

27 Which equation has roots $3-i$ and $3+i$ ?
(1) $x^{2}-10 x+6=0$
(3) $x^{2}+6 x+10=0$
(2) $x^{2}+10 x-6=0$
(4) $x^{2}-6 x+10=0$

28 What is the domain of the function $\mathrm{f}(x)=\frac{4}{\sqrt{x+5}}$ over the set of real numbers?
(1) $\{x \mid x>-5\}$
(3) $\{x \mid x \geq-5\}$
(2) $\{x \mid x<-5\}$
(4) $\{x \mid x=-5\}$

29 What is one possible value of $\theta$ in the equation $\cot \theta=\cos \theta$ ?
(1) $0^{\circ}$
(3) $90^{\circ}$
(2) $45^{\circ}$
(4) $180^{\circ}$

30 The graph of the equation $4 x^{2}+3 y=8$ forms
(1) a straight line
(3) a hyperbola
(2) an ellipse
(4) a parabola

31 If $\mathrm{m} \angle A=28^{\circ} 10^{\prime}, a=20$, and $b=25$, what is the maximum number of distinct triangles that can be constructed?
(1) 1
(3) 3
(2) 2
(4) 0

32 The solution set for the inequality $x^{2}+4 x-5 \geq 0$ is
(1) $-5 \leq x \leq 1$
(3) $x \leq-5$ or $x \geq 1$
(2) $x \leq-1$ or $x \geq 5$
(4) $-1 \leq x \leq 5$

33 To the nearest degree, what is the measure of the largest angle in a triangle with sides measuring 10,12 , and 18 centimeters?
(1) 109
(3) 71
(2) 81
(4) 32

34 The roots of the equation $x^{2}-6 x+7=0$ are
(1) imaginary
(2) real and irrational
(3) real, rational, and unequal
(4) real, rational, and equal

35 The expression $\left(i^{3}-1\right)\left(i^{3}+1\right)$ is equivalent to
(1) -2
(3) $2 i+1$
(2) $2 i-1$
(4) $-2 i$

## Answers to the following questions are to be written on paper provided by the school.

## Part II

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [40]

36 Find, to the nearest ten minutes or nearest tenth of $a$ degree, all values of $x$ in the interval $0^{\circ} \leq x<360^{\circ}$ that satisfy the equation
$6 \sin x+3=2 \csc x$. [10]
$37 a$ On the same set of axes, sketch and label the graphs of the equations $y=-3 \cos x$ and $y=\tan x$ in the interval $-\pi \leq x \leq \pi . \quad[8]$
$b$ Using the graph sketched in part $a$, find the number of values of $x$ in the interval $-\pi \leq x \leq \pi$ that satisfy the equation $-3 \cos x=\tan x$. [2]
$38 a$ On the same set of axes, sketch and label the graphs of the equations below:
(1) $x y=9$ in the interval $-9 \leq x \leq 9$
[3]
(2) $y=3^{x}$ in the interval $-2 \leq x \leq 2$
$b$ On the same set of axes, sketch the reflection of $y=3^{x}$ in the $x$-axis in the interval $-2 \leq x \leq 2$. Label the reflection $b$. [2]
$c$ Write an equation of the graph sketched in part b. [2]

39 In the accompanying diagram of circle $O$, tangent $\overline{A B}$ and chord $\overline{B C}$ are drawn, secant $\overline{A C D}$ intersects diameter $\overline{E B}$ at $F, \mathrm{~m} \overparen{B D}=160$, and $\mathrm{m} \overparen{B C}=80$.


Find:

| $a \mathrm{~m} \angle A$ | $[2]$ |
| :--- | :--- |
| $b \mathrm{~m} \angle A B E$ | $[2]$ |
| $c \mathrm{~m} \angle A B C$ | $[2]$ |
| $d \mathrm{~m} \angle E F C$ | $[2]$ |
| $e \mathrm{~m} \angle A C B$ | $[2]$ |

$40 a$ The playground at a day-care center has a triangular-shaped sandbox. Two of the sides measure 20 feet and 14.5 feet and form an included angle of $45^{\circ}$. Find the length of the third side of the sandbox to the nearest tenth of a foot. [6]
$b$ Given: $y=4.1^{x}$
Find $x$, to the nearest tenth, when $y=26$. [4]
$41 a$ Christina participated in 20 basketball games this season. The scorekeeper recorded the number of "shots" she attempted in each game. The table below shows the number of shots she attempted in the number of games she played.

| Shots Attempted | Number of Games |
| :---: | :---: |
| 10 | 4 |
| 13 | 3 |
| 17 | 5 |
| 23 | 6 |
| 33 | 2 |

(1) Find the mean number of shots that Christina attempted. [1]
(2) Find the standard deviation of the shots attempted to the nearest tenth. [3]
(3) What is the total number of games in which the number of shots attempted fell outside one standard deviation of the mean? [1]
$b$ The probability that bus A will arrive on time is $\frac{5}{6}$. Yolanda takes this bus on 4 consecutive days. Find the probability that this bus will arrive on time:
(1) all 4 days [2]
(2) at least 3 days [3]
$42 a$ Solve for $x$ and express your answer in simplest $a+b i$ form:

$$
\frac{x^{2}}{4}=x-2
$$

$b$ Prove the following identity:

$$
\begin{equation*}
\frac{\sin \theta}{\sin ^{2} \theta+\cos 2 \theta}=\frac{\sec \theta}{\cot \theta} \tag{5}
\end{equation*}
$$

| The University of the State of New York |  |  |
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| ination | Part I Score |  |
|  | Part II Score | ........... |
| SEQUENTIAL MATH - COURSE III | Total Score |  |
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Part I Score
Part II Score
    Total Score
Rater's Initials:
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## ANSWER SHEET



Your answers to Part I should be recorded on this answer sheet.

## Part I

Answer 30 questions from this part.

| 1 | 11 | 21 | 31 |
| :---: | :---: | :---: | :---: |
| 2 | 12 | 22 | 32 |
| 3 | 13 | 23 | 33 |
| 4 | 14 | 24 | 34 |
| 5 | 15 | 25 | 35 |
| 6 | 16 | 26 |  |
| 7 | 17 | 27 |  |
| 8 | 18 | 28 |  |
| 9 | 19 | 29 |  |
| 10 | 20 | 30 |  |

Your answers for Part II should be placed on paper provided by the school. The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that $I$ have neither given nor received assistance in answering any of the questions during the examination.

