

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Thursday, January 27, 2000 — 9:15 a.m. to 12:15 p.m., only

Notice . . .

Scientific calculators must be available to all students taking this examination.

The formulas which you may need to answer some questions in this examination are found on page 2. The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Formulas

Pythagorean and Quotient Identities

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 & \tan A &= \frac{\sin A}{\cos A} \\ \tan^2 A + 1 &= \sec^2 A & \cot A &= \frac{\cos A}{\sin A} \\ \cot^2 A + 1 &= \csc^2 A\end{aligned}$$

Functions of the Sum of Two Angles

$$\begin{aligned}\sin(A + B) &= \sin A \cos B + \cos A \sin B \\ \cos(A + B) &= \cos A \cos B - \sin A \sin B \\ \tan(A + B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B}\end{aligned}$$

Functions of the Difference of Two Angles

$$\begin{aligned}\sin(A - B) &= \sin A \cos B - \cos A \sin B \\ \cos(A - B) &= \cos A \cos B + \sin A \sin B \\ \tan(A - B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B}\end{aligned}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Functions of the Double Angle

$$\begin{aligned}\sin 2A &= 2 \sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ \cos 2A &= 2 \cos^2 A - 1 \\ \cos 2A &= 1 - 2 \sin^2 A \\ \tan 2A &= \frac{2 \tan A}{1 - \tan^2 A}\end{aligned}$$

Functions of the Half Angle

$$\begin{aligned}\sin \frac{1}{2}A &= \pm \sqrt{\frac{1 - \cos A}{2}} \\ \cos \frac{1}{2}A &= \pm \sqrt{\frac{1 + \cos A}{2}} \\ \tan \frac{1}{2}A &= \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}\end{aligned}$$

Area of Triangle

$$K = \frac{1}{2}ab \sin C$$

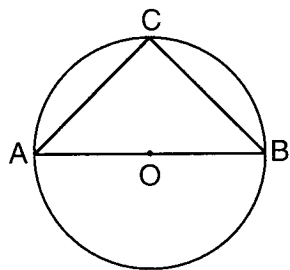
Standard Deviation

$$\text{S.D.} = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2}$$

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form. [60]

- 1 Express $\frac{7\pi}{6}$ radians in degrees.
- 2 Express the sum of $3 + \sqrt{-49}$ and $2 + \sqrt{-121}$ in simplest $a + bi$ form.
- 3 Solve for the positive value of x : $|2x - 3| = 11$
- 4 Solve for x : $3^{2x+1} = 27^x$
- 5 Which negative real number is *not* in the domain of $\frac{3}{x^2 - 4}$?
- 6 Evaluate: $\sum_{n=1}^3 n^2$
- 7 If $x + 2$ is a factor of $x^2 + bx + 10$, what is the value of b ?
- 8 In $\triangle ABC$, $a = 16$, $c = 14$, and $m\angle B = 30$. What is the area of $\triangle ABC$?
- 9 In the accompanying diagram, isosceles triangle ABC is inscribed in circle O with diameter \overline{AOB} . Find $m\angle CAB$.



- 10 In $\triangle ABC$, $a = 2$, $\sin A = \frac{2}{3}$, and $\sin B = \frac{5}{6}$. Find the length of side b .

- 11 If $f(x) = 2 \cos^2 x + \sin x - 1$, find the value of $f(\frac{\pi}{2})$.
- 12 Express $\frac{\frac{x}{3} - 1}{\frac{x^2}{3} - 3}$ in simplest form.
- 13 Find the value of $5x^0 + x^{-\frac{1}{2}} - x^{\frac{1}{2}}$ when $x = 16$.
- 14 Express $\frac{\cos 2A + \sin^2 A}{\cos A}$ as a single trigonometric function for all values of A for which the fraction is defined.
- 15 In circle O , chords \overline{AB} and \overline{CD} intersect at E , $AE = 3$ inches, $BE = 8$ inches, and CE is 2 inches longer than DE . What is the length of \overline{DE} , expressed in inches?
- 16 What is the probability of getting *exactly* two heads in three tosses of a fair coin?

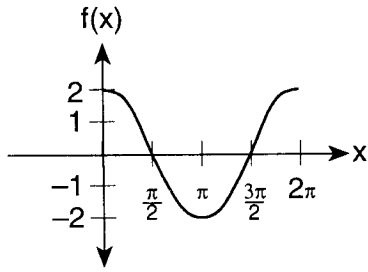
Directions (17–35): For each question chosen, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

- 17 The numerical value of $\sin \frac{3\pi}{2} + \cos \frac{\pi}{4}$ is

(1) $1 + \frac{\sqrt{2}}{2}$	(3) $-1 + \frac{\sqrt{2}}{2}$
(2) $\frac{\sqrt{2}}{2}$	(4) -1
- 18 If $\tan A = \frac{2}{3}$ and $\tan B = \frac{1}{2}$, what is the value of $\tan(A + B)$?

(1) $\frac{1}{8}$	(3) $\frac{1}{4}$
(2) $\frac{7}{8}$	(4) $\frac{7}{4}$

19 Which trigonometric function is shown in the graph below?



- (1) $f(x) = 2 \sin x$ (3) $f(x) = \cos 2x$
 (2) $f(x) = 2 \cos x$ (4) $f(x) = \sin 2x$

20 What is the solution set of the equation $\sqrt{x^2 - 3x + 3} = 1$?

- (1) {1} (3) {1,2}
 (2) {2} (4) { }

21 If $\sin \theta = -\frac{3}{5}$ and $\cos \theta > 0$, what is the value of $\tan \theta$?

- (1) $\frac{3}{4}$ (3) $\frac{4}{3}$
 (2) $-\frac{3}{4}$ (4) $-\frac{4}{3}$

22 The roots of the equation $x^2 - 7x + 15 = 0$ are

- (1) imaginary
 (2) real, rational, and equal
 (3) real, rational, and unequal
 (4) real, irrational, and unequal

23 How many distinct triangles can be formed if $a = 20$, $b = 30$, and $m\angle A = 30^\circ$?

- (1) 1 (3) 3
 (2) 2 (4) 0

24 Which expression is *not* an isometry?

- (1) $r_{y=x}$ (3) D_{-2}
 (2) $T_{-2,4}$ (4) $R_{0,90^\circ}$

25 What is one solution of the equation $(\sin x + \cos x)^2 = 2$?

- (1) $\frac{\pi}{4}$ (3) $\frac{\pi}{2}$
 (2) $\frac{\pi}{3}$ (4) 0

26 Which equation does *not* represent a function?

- (1) $y = 2x$ (3) $y = \frac{10}{x}$
 (2) $y = x^2 + 10$ (4) $x^2 + y^2 = 9$

27 In which quadrant does the sum of $2 + 3i$ and $3 - 5i$ lie?

- (1) I (3) III
 (2) II (4) IV

28 If the coordinates of point P are $(2, -3)$, then $(R_{90^\circ} \circ R_{180^\circ})(P)$ is

- (1) $(-2, 3)$ (3) $(3, -2)$
 (2) $(-2, -3)$ (4) $(-3, -2)$

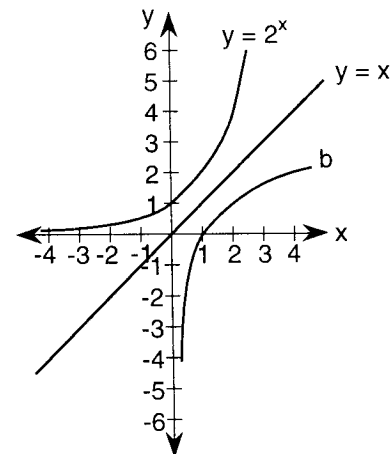
29 What is the value of $\sin\left(\text{Arc cos } \frac{1}{x}\right)$?

- (1) $\frac{\sqrt{1-x^2}}{x}$ (3) $\frac{\sqrt{x^2-1}}{x}$
 (2) $\frac{\sqrt{1+x^2}}{x}$ (4) $\frac{x}{\sqrt{x^2+1}}$

30 In a normal distribution, what is the greatest percent of the data that falls within 2 standard deviations of the mean?

- (1) 95 (3) 68
 (2) 81.5 (4) 34

31 In the diagram below, figure b is the reflection of $y = 2^x$ in the line $y = x$.



Which is an expression for the equation of figure b ?

- (1) $y = (-2)^x$ (3) $y = \log_2 x$
 (2) $y = 2^{-x}$ (4) $y = \log_x 2$

- 32 What is the solution set of the inequality $x^2 + 3x - 10 > 8$?
- (1) $\{x | -6 < x < 3\}$ (3) $\{x | -3 < x < 6\}$
 (2) $\{x | x < -6 \text{ or } x > 3\}$ (4) $\{x | x < -3 \text{ or } x > 6\}$
- 33 What is the third term in the expansion of $(x + 2y)^5$?
- (1) $10x^3y^2$ (3) $80x^2y^3$
 (2) $40x^3y^2$ (4) $20x^2y^3$

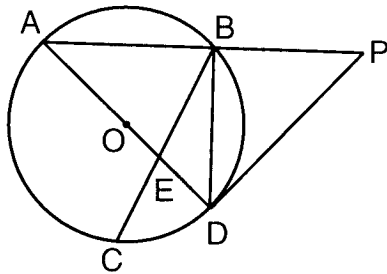
- 34 If $f(x) = x^2$, what is the value of $f(i^3)$?
- (1) 1 (3) i
 (2) -1 (4) $-i$
- 35 If x varies inversely as y and $x = 12$ when $y = 3$, what is the value of x when $y = 9$?
- (1) 36 (3) $\frac{1}{4}$
 (2) $\frac{1}{3}$ (4) 4

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [40]

- 36 In the accompanying diagram of circle O , \overline{AOED} is a diameter, \overline{PD} is a tangent, \overline{PBA} is a secant, chords \overline{BD} and \overline{BEC} are drawn, $m\angle DAB = 43$, and $m\angle DEC = 72$.



Find:

- a $m\angle BDP$ [2]
 b $m\widehat{AB}$ [2]
 c $m\widehat{AC}$ [2]
 d $m\angle P$ [2]
 e $m\angle CBD$ [2]

- 37 a In the month of February at a ski resort, the probability of snow on any day is $\frac{3}{4}$.
- (1) What is the probability that snow will fall on every day of a 5-day trip to that resort in February? [2]
 (2) What is the probability that snow will fall on *at least* 3 days of that 5-day trip in February? [4]
- b Using the scores in the table below, find the standard deviation to the *nearest tenth*. [4]

Scores	Frequency
60	2
65	6
70	4
75	8
80	5

GO RIGHT ON TO THE NEXT PAGE.

- 38 In parallelogram $ABCD$, $AD = 8$, $AB = 12$, and diagonal $BD = 15$.
- a* Find $m\angle BAD$ to the nearest degree. [6]
- b* Using the answer obtained in part *a*, find the area of parallelogram $ABCD$ to the nearest tenth. [4]
- 39 *a* Find the period of the graph of $y = 3 \sin 2x$. [2]
- b* On graph paper, sketch the graph of $y = 3 \sin 2x$ for one period. [4]
- c* On the same set of axes, sketch the image of the graph drawn in part *b* after it is reflected in the x -axis. Label the graph *c*. [2]
- d* Write an equation for the graph sketched in part *c*. [2]
- 40 Find, to the nearest degree, all positive values of θ less than 360° that satisfy the equation $2 \tan^2 \theta - 2 \tan \theta = 3$. [10]
- 41 *a* Given: $\log 2 = x$
 $\log 3 = y$
- Express in terms of x and y :
- (1) $\log \frac{2}{3}$ [2]
- (2) $\log 12$ [3]
- b* Using logarithms, find w to the nearest hundredth.
- $$5^{2w} + 9 = 40 \quad [5]$$
- 42 *a* For all values of θ for which the expressions are defined, prove the following is an identity:
- $$\frac{\tan \theta - \cot \theta}{\tan \theta + \cot \theta} = 2 \sin^2 \theta - 1 \quad [5]$$
- b* For all values of x for which the expression is defined, solve for x :
- $$\frac{3}{x+3} + \frac{2}{x-4} = \frac{4}{3} \quad [5]$$

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REGENTS HIGH SCHOOL EXAMINATION

SEQUENTIAL MATH – COURSE III

Thursday, January 27, 2000 — 9:15 a.m. to 12:15 p.m., only

Part I Score
Part II Score
Total Score
Rater's Initials:

ANSWER SHEET

Pupil Sex: Male Female Grade

Teacher School

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

- | | | | |
|----------|----------|----------|----------|
| 1 | 11 | 21 | 31 |
| 2 | 12 | 22 | 32 |
| 3 | 13 | 23 | 33 |
| 4 | 14 | 24 | 34 |
| 5 | 15 | 25 | 35 |
| 6 | 16 | 26 | |
| 7 | 17 | 27 | |
| 8 | 18 | 28 | |
| 9 | 19 | 29 | |
| 10 | 20 | 30 | |

Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

Signature

Tear Here

Tear Here

Tear Here

Tear Here