

POSSIBLE ANSWERS
FEB / MARCH 2007

Mathematics/P1/SG

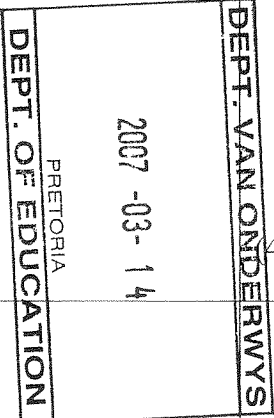
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Marking Guideline,

Senior Certificate Examination – Feb/Mar 2007

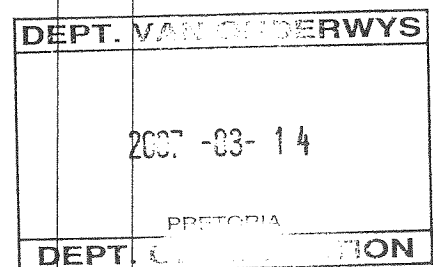
MATHEMATICS P1 SG FEB/MARCH 2007				
1.1	1.1.1	$x(x-3) = 4(x+2)$ $x^2 - 3x = 4x + 8$ $x^2 - 7x - 8 = 0$ $(x-8)(x+1) = 0$ $x = 8$ or $x = -1$	(3)	<ul style="list-style-type: none"> ✓ multiplication ✓ Factorization ✓ answer
	1.1.2.	$3x^2 + 5x - 4 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-5 \pm \sqrt{25 + 48}}{6}$ $= \frac{-5 \pm \sqrt{73}}{6}$ $= 0,59$ or $-2,26$	(4)	<ul style="list-style-type: none"> ✓ Formula ✓ substitution ✓ ✓ each value
	1.1.3	$\sqrt{2-7x} = 2-x$ $2-7x = 4-4x+x^2$ $x^2 + 3x + 2 = 0$ $(x+2)(x+1) = 0$ $x = -2$ or $x = -1$ Check: For $x = -2$ L.H.S = $\sqrt{16} = 2 - (-2) = 4 =$ R.H.S For $x = -1$: L.H.S = $\sqrt{9} = 2 - (-1) = 3 =$ R.H.S <p style="text-align: center;">OR</p> For the 2 sides to be defined we need $2-x \geq 0$ and $2-7x \geq 0 \therefore x \leq 2$ and $x \leq \frac{2}{7}$ $\therefore x \leq \frac{2}{7}$ Both $x = -2$ and $x = -1$ are $\leq \frac{2}{7}$ \therefore both are solutions	(5)	<ul style="list-style-type: none"> ✓ Squaring both sides ✓ simplifying ✓ Factorization ✓ x values ✓ checking

<p>1.2</p>	<p> $2x - y = 2$.....(1) $y = (x-1)(x-2)$.....(2) From equation (1): $y = 2x - 2$.....(3) Substitute equation (3) into equation (2): $2x - 2 = (x-1)(x-2)$ \Downarrow $x^2 - 3x + 2 = 2x - 2$ OR $2(x-1) = (x-1)(x-2)$ $x^2 - 5x + 4 = 0$ $x - 1 = 0$ or $x - 2 = 2$ $(x-1)(x-4) = 0$ $x = 1$ or $x = 4$ $x = 1$ or $x = 4$ For $x = 1; y = 0$ For $x = 4; y = 6$ Points of intersection are: (1;0) and (4;6) OR </p>	<p> ✓ subject of the formula ✓ substitution ✓ Simplification ✓ standard form ✓ factorization ✓ values for x ✓ value for y ✓ value for y </p>
	<p> $2x - y = 2$.....(1) $y = (x-1)(x-2)$.....(2) From equation (1): $x = \frac{y+2}{2}$.....(3) Substitute equation (3) into equation (2): $y = (\frac{y+2}{2} - 1)(\frac{y+2}{2} - 2)$ $y = (\frac{y+2-2}{2})(\frac{y+2-4}{2})$ $4y = (y)(y-2)$ \Downarrow $y^2 - 2y = 4y$ or $y = 0$ or $y - 2 = 4$ $y^2 - 6y = 0$ $y = 0$ or $y = 6$ $y(y-6) = 0$ $y = 0$ or $y = 6$ For $y = 0; x = 1$ For $y = 6; x = 4$ Points of intersection are: (1;0) and (4;6) </p>	<p> ✓ subject of the formula ✓ substitution ✓ simplification ✓ standard form ✓ factorization ✓ values for y ✓ value for x ✓ value for x </p> <p>(8)</p>
<p>1.3.</p>	<p> $6x^2 - 7mx - 5m^2 = 0$ $\Delta = b^2 - 4ac$ $= (-7m)^2 - 4(6)(-5m^2)$ $= 49m^2 + 120m^2$ $= 169m^2$ $= (13m)^2$ is a perfect square \therefore The roots are rational OR </p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;"> <p style="text-align: center;"> DEPT. OF EDUCATION PRETORIA 2007-03-14 DEPT. VAN ONDERWYS </p> </div>	<p> ✓ formula $b^2 - 4ac$ ✓ substitution ✓ simplification ✓ perfect square ✓ rational </p>

1.3	$6x^2 - 7mx - 5m^2 = 0$ $(2x + m)(3x - 5m) = 0$ $x = \frac{-m}{2} \text{ or } x = \frac{5m}{3}$ <p>\therefore Roots are rational since m is rational</p>	(5)	✓✓ factorization ✓✓ values of x ✓ rational
1.4	$3x^2 - x + k = 0$ $x = \frac{4}{3} : 3\left(\frac{4}{3}\right)^2 - \frac{4}{3} + k = 0$ $\frac{16}{3} - \frac{4}{3} + k = 0$ $\frac{12}{3} + k = 0$ $k = -4$ <p style="text-align: center;">OR</p> <p>For $x = \frac{4}{3} \therefore 3x - 4$ is a factor</p> <p>Then $3x^2 - x + k = (3x - 4)(x + 1) + (k + 4)$.</p> <p>But $(3x - 4)(x + 1) = 0$ for $x = \frac{4}{3}$</p> <p>$\therefore 0 = 0 + (k + 4)$</p> <p>$\therefore k = -4$</p>	(3)	✓ substitution ✓ simplification ✓ value of k ✓ factor ✓ simplifying ✓ value of k
1.5	<p>Let the original number be x and the new number be y</p> <p>$\therefore x = 10a + b$(1)</p> <p>and $y = 10b + a$(2)</p> <p>Difference = $x - y = 10a + b - (10b + a)$</p> $= 10a + b - 10b - a$ $= 9a - 9b$ $= 9(a - b)$	(4) [32]	✓ equation ✓ equation ✓ difference ✓ answer
2.1	<p>Let $f(x) = x^3 + px^2 + 2x + 3$</p> $f(3) = 0$ $3^3 + p(3)^2 + 2(3) + 3 = 0$ $27 + 9p + 6 + 3 = 0$ $9p = -36$ $p = -4$	 (4)	✓ factor theorem ✓ substitution ✓ simplification ✓ answer
2.2	$f(x) = 2x^3 - 5x^2 + 2ax + 48$ $g(x) = x^3 + 3ax^2 + 11x - 6$ $f(-2) = -16 - 20 - 4a + 48 = -4a + 12$(1) $g(-2) = -8 + 12a - 22 - 6 = 12a - 36$(2) $g(-2) = f(-2)$ $12a - 36 = -4a + 12$ $16a = 48$	(6)	✓✓ substitution ✓✓ simplification for f and g ✓ equation ✓ answer

		$a = 3$	[10]
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3.1.	3.1.1	<p>A(2;3) and B(0;1)</p> $\text{Gradient of AB} = m_{AB} = \frac{y_B - y_A}{x_B - x_A}$ $= \frac{1 - 3}{0 - 2} = 2$ <p>Equation of AB: $y - y_B = m_{AB}(x - x_B)$</p> $y - 1 = 2(x - 0)$ $y = 2x + 1$ <p style="text-align: center;">OR</p> <p>..... $c = 1$; and $m_{AB} = 2$</p> $y = m_{AB}x + c$ $= 2x + 1$	<p>✓✓ gradient</p> <p>✓ substitution</p> <p>✓ equation</p> <p>✓✓ gradient/ y-intercept</p> <p>✓ formula</p> <p>✓ equation</p> <p>(4)</p>
	3.1.2	<p>Form of the equation:</p> $y = a(x - p)^2 + q$ $y = a(x - 2)^2 + 5$ $1 = a(0 - 2)^2 + 5$ $4a + 5 = 1$ $a = \frac{-4}{4} = -1$ <p>Equation of the parabola:</p> $y = -1(x - 2)^2 + 5$ $= -1(x^2 - 4x + 4) + 5$ $= -x^2 + 4x - 4 + 5$ $= -x^2 + 4x + 1$	<p>✓✓ Substitution</p> <p>✓ substitution</p> <p>✓ value of a</p> <p>✓ substitution</p> <p>✓ form of the equation</p> <p>(6)</p>
	3.1.3	$1 \leq x \leq 2, x \in R$	<p>(2) ✓✓ Interval</p>
3.2	3.2.1	$f(x) = 4 - x$	<p>(2) f: ✓ intercepts with axes</p> <p>✓ line</p>
	3.2.2	$g(x) = \sqrt{16 - x^2}$	<p>(2) g: ✓ shape</p> <p>✓ radius [or any point]</p>



3.3		$x = 4$ or $x = 0$	(2)	✓✓ values of x
3.4.1	A(3;4)	Form of the equation: $xy = k$ $k = 3 \times 4 = 12$ Equation of the hyperbola: $xy = 12$	(3)	✓ form ✓ value of k ✓ Equation
3.4.2	B(4;3)		(1)	✓ answer
3.4.3.	C(-3;-4)		(1)	✓ answer
			<u>/23/</u>	

4.1	4.1.1	$\frac{4^{n-3} \cdot 10^{n+2}}{8^{n-1} \cdot 5^{1+n}}$ $= \frac{2^{2n-6} \cdot 2^{n+2} \cdot 5^{n+2}}{2^{3n-3} \cdot 5^{1+n}}$ $= 2^{2n-6+n+2-3n+3} \cdot 5^{n+2-1-n}$ $= 2^{-1} \cdot 5^1$ $= \frac{5}{2} \text{ or } 2,5$	(6)	✓✓✓ converting composite numbers to prime factors ✓ Application of laws ✓ simplification ✓ answer
	4.1.2	$2 \log x + 3 \log \sqrt{x}$ $= 2 \log x + 3 \log x^{\frac{1}{2}}$ $= 2 \log x + \frac{3}{2} \log x$ $= \frac{7}{2} \log x$ <p>or</p> $2 \log x + 3 \log \sqrt{x}$ $= \log x^2 + \log \left(x^{\frac{1}{2}} \right)^3$ $= \log \left(x^2 \right) \left(x^{\frac{3}{2}} \right)$ $= \log x^{\frac{4}{2} + \frac{3}{2}}$ $= \log x^{\frac{7}{2}} \quad \text{or} \quad \frac{7}{2} \log x$	(3)	✓ removing radical sign ✓ application of the law ✓ answer ✓ removing radical sign ✓ application of the laws ✓ answer

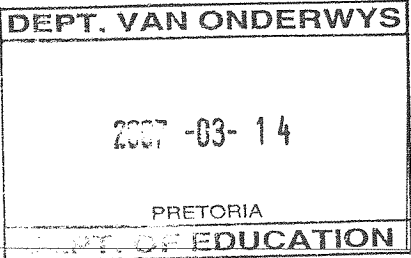
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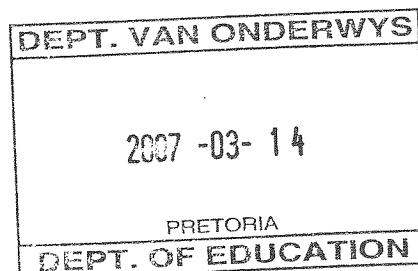
4.2	4.2.1	$2 \cdot 3^x + 5 \cdot 3^{x+1} - 17 = 0$ $2 \cdot 3^x + 5 \cdot 3^x \cdot 3^1 = 17$ $3^x(2 + 5 \cdot 3^1) = 17$ $17 \cdot 3^x = 17$ $3^x = 1 = 3^0$ $x = 0$	(4)	<ul style="list-style-type: none"> ✓ application of law ✓ common factor ✓ simplification ✓ answer
	4.2.2	$\frac{3}{27x^4} = 8$ $x^4 = \frac{8}{27}$ $x = \left(\frac{8}{27}\right)^{\frac{1}{4}} = \left(\frac{2^3}{3^3}\right)^{\frac{1}{4}} = \frac{2^{\frac{3}{4}}}{3^{\frac{3}{4}}}$ $= \frac{16}{81}$	(4)	<ul style="list-style-type: none"> ✓ subject of the formula ✓ reciprocal of exponent ✓ simplification ✓ answer
	4.2.3	$\log_2(x+1) - \log_2 x = 1$ $\log_2 \frac{x+1}{x} = 1$ $\frac{x+1}{x} = 2^1 = 2$ $2x = x+1$ $x = 1$	(3)	<ul style="list-style-type: none"> ✓ application of the laws ✓ Exponential form ✓ answer
	4.2.4	$2 \cdot 3^{x+2} = 9$ $2 \times 9 \times 3^x = 9$ $3^x = \frac{1}{2} = 2^{-1}$ $x \log 3 = -\log 2$ $x = \frac{-\log 2}{\log 3}$ $= -0,6224$ $= -0,62$ <p>OR</p> $3^{x+2} = \frac{9}{2}$ $(x+2) \log 3 = \log \frac{9}{2}$ $x+2 = \frac{\log 9 - \log 2}{\log 3}$ $= 1,38$ $x = 1,38 - 2$ $= -0,62$	(5)	<ul style="list-style-type: none"> ✓ simplification ✓ log both sides ✓ simplification ✓ answer ✓ subject of the formula ✓ logging both sides ✓ simplification ✓ simplification ✓ answer



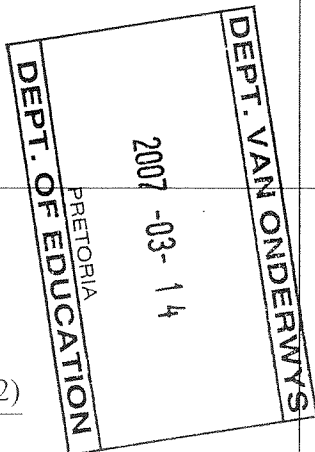
5.1.		$2x - 1; 4x - 5; 3x + 3$		
5.1.1		$d = T_2 - T_1 = T_3 - T_2$ $4x - 5 - (2x - 1) = 3x + 3 - (4x - 5)$ $4x - 5 - 2x + 1 = 3x + 3 - 4x + 5$ $2x - 4 = -x + 8$ $3x = 12$ $x = 4$ $\therefore T_1 = 7, T_2 = 11, T_3 = 15$ OR For $x = 4$ $T_1 = 2x - 1 = 2(4) - 1 = 7$ $T_2 = 4x - 5 = 4(4) - 5 = 11$ $T_3 = 3x + 3 = 3(4) + 3 = 15$ This is an Arithmetic sequence with common difference = $d = 4$	(5)	✓ common difference ✓ substitution ✓ simplification ✓ ✓ sequence ✓ substitution ✓ substitution ✓ substitution ✓ ✓ common difference
5.1.2		$T_n = a + (n-1)d$ $= 7 + (n-1)4$ $= 7 + 4n - 4$ $= 4n + 3$	(3)	✓ formula ✓ substitution ✓ simplification
5.1.3		$T_n = 43$ $4n + 3 = 43$ $4n = 40$ $n = 10$	(2)	✓ equation ✓ value of n
5.2.		$\sum_{k=1}^{10} (2k + 4)$ $= 6 + 8 + 10 + \dots + 24$ This is an Arithmetic sequence with $d = 2$ $S_n = \frac{n}{2}(a+l)$ $S_{10} = \frac{10}{2}(6+24)$ $= 5(30)$ $= 150$ OR $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{10} = \frac{10}{2}[2(6) + 9(2)]$ $= 5(12+18)$ $= 5(30)$ $= 150$	(5)	✓ Expanded form ✓ common difference ✓ formula ✓ substitution ✓ answer ✓ formula ✓ substitution ✓ answer

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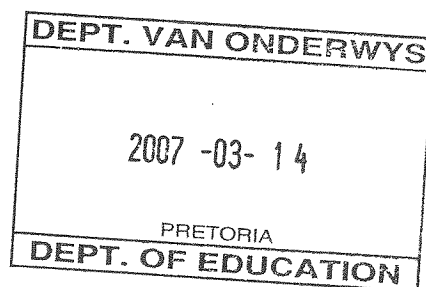
5.3	$T_1 = 48, \quad r = \frac{1}{2}, \quad n = 12$ $S_n = \frac{a(1-r^n)}{1-r}$ $= \frac{48 \left[1 - \left(\frac{1}{2} \right)^{12} \right]}{1 - \frac{1}{2}}$ $= 96 \left(1 - \frac{1}{4096} \right)$ $= 96 \left(\frac{4095}{4096} \right)$ $= 95,98$	(4)	✓ formula ✓ substitution ✓ simplification ✓ answer
5.4.	<p>$P = R150\,000$, $r = 5,6\%$ per year. $n = 5$ years</p> <p>$r = \frac{5,6}{4} = 1,4\%$ per quarter</p> <p>$n = 5 \times 4 = 20$ time periods when compounded quarterly</p> $A = P \left(1 + \frac{r}{100} \right)^n$ $= 150000 \left(1 + \frac{1,4}{100} \right)^{20}$ $= 150000 (1,014)^{20}$ $= 198\,084,4386$ <p>$A = R198\,084,44$</p>	(5) [24]	✓ rate per quarter ✓ number of time periods ✓ formula ✓ substitution ✓ answer



<p>6.1</p>	$f(x) = -2x + 3$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-2(x+h) + 3 - (-2x + 3)}{h}$ $= \lim_{h \rightarrow 0} \frac{-2x - 2h + 3 + 2x - 3}{h}$ $= \lim_{h \rightarrow 0} \frac{-2h}{h}$ $= \lim_{h \rightarrow 0} (-2)$ $= -2$ <p style="text-align: center;">OR</p> $f(x) = -2x + 3$ $f(x+h) = -2(x+h) + 3 = -2x - 2h + 3$ $f(x+h) - f(x) = -2x - 2h + 3 - (-2x + 3)$ $= -2x - 2h + 3 + 2x - 3 = -2h$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-2h}{h}$ $= \lim_{h \rightarrow 0} (-2)$ $= -2$	<p>(5)</p>	<ul style="list-style-type: none"> ✓ Definition ✓ substitution ✓ simplification ✓ simplification ✓ answer ✓ substitution ✓ substitution ✓ difference ✓ Definition ✓ answer
<p>6.2.</p>	$f(x) = 2x^3 - 3$ $f(2) = 2(2^3) - 3 = 16 - 3 = 13$ $f(3) = 2(3^3) - 3 = 54 - 3 = 51$ $\text{Average gradient} = \frac{f(3) - f(2)}{3 - 2}$ $= \frac{51 - 13}{1}$ $= 38$ <p style="text-align: center;">OR</p> $\text{Average gradient} = \frac{f(x+h) - f(x)}{h}$ $= \frac{2(x+h)^3 - 3 - (2x^3 - 3)}{h}$ $= \frac{6x^2h + 6xh^2 + 2h^3}{h}$ $= 6x^2 + 6xh + 2h^2$ <p>For $x = 2$ and $x = 3$</p> $\text{Average gradient} = 6(2)^2 + 6(2)(3-2) + 2(3-2)^2$ $= 24 + 12 + 2$	<p>(5)</p>	<ul style="list-style-type: none"> ✓ y value at 2 ✓ y value at 3 ✓ formula ✓ answer ✓ definition ✓ substitution ✓ simplification



		= 38	(4)	✓ answer
6.3	6.3.1	$f(x) = \sqrt[3]{x} - \frac{3}{x}$ $= x^{\frac{1}{3}} - 3x^{-1}$ $f'(x) = \frac{1}{3}x^{-\frac{2}{3}} + 3x^{-2}$	(4)	✓✓ Simplification ✓✓ each derivative
	6.3.2	$f(x) = \frac{x^3 + x}{x}$ $= \frac{(x)(x^2 + 1)}{(x)}$ $= x^2 + 1$ $f'(x) = 2x$	(3)	✓ factors ✓✓ each derivative
6.4.		$f(x) = -x^3 + 3x^2$		
	6.4.1	For x intercepts (roots): $y = 0$ $-x^3 + 3x^2 = 0$ $x^3 - 3x^2 = 0$ $x^2(x - 3) = 0$ $x = 0$ or $x = 3$ Coordinates: (0 ; 0) or (3 ; 0)	(4)	✓ substitution ✓ factors ✓ roots ✓ coordinates
	6.4.2	For turning points: $f'(x) = 0$ $-3x^2 + 6x = 0$ $-3x(x - 2) = 0$ $x = 0$ or $x = 2$ for $x = 0$ $y = 0$ for $x = 2$ $y = -8 + 12 = 4$ Turning points: (0;0) and (2;4)	(6)	✓ definition ✓ application of rules ✓ factors ✓ values of x ✓✓ turning points



<p>6.4.3</p>		<ul style="list-style-type: none"> ✓ shape ✓ TP (2; 4) ✓ TP (0; 0) ✓ x- intercept 3 <p>(4) [30]</p>
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7.1.	$f(x) = 24x - 3x^2 \quad 0 \leq x \leq 4.$ $f(2) = 24(2) - 3(2^2)$ $= 48 - 12$ $= 36 \text{ dm} = 3,6 \text{ m}$	(2)	✓ substitution ✓ height
7.2	It will reach its maximum height when $f'(x) = 0$. $\therefore 24 - 6x = 0$ $x = 4$ years <p style="text-align: center;">OR</p> $x = \frac{-b}{2a} = \frac{-24}{2(-3)} = 4 \text{ years}$ <p style="text-align: center;">OR</p> $f(x) = -3x^2 + 24x$ $= -3(x^2 - 8x)$ $= -3(x^2 - 8x + 16 - 16)$ $= -3(x - 4)^2 + 48$ $\therefore x = 4$ years for the tree to reach its maximum height	(2)	✓ derivative = 0 ✓ value of x
7.3	Maximum height = $f(4) = 24(4) - 3(4^2)$ $= 96 - 48$ $= 48 \text{ dm}$ $= 4,8 \text{ m}$ <p style="text-align: center;">OR</p> Maximum height = $\frac{-\Delta}{4a} = \frac{4ac - b^2}{4a}$ $= \frac{4(-3)(0) - 24^2}{4(-3)}$ $= \frac{-576}{-12}$ $= 48 \text{ dm}$ $= 4,8 \text{ m}$ <p style="text-align: center;">OR</p> From 7.2. (the third alternate solution) The maximum height is 48dm $= 4,8 \text{ m}$	(2)	✓ substitution ✓ value of height ✓ substitution ✓ value of height
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TOTAL			[150]

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