education

Department:
Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATION - 2007

MATHEMATICS P1

HIGHER GRADE

FEBRUARY/MARCH 2007

301-1/1

MARKS: 200

TIME: 3 hours

MATHEMATICS HG: Paper 1

301 1 1E HG

This question paper consists of 9 pages, 1 graph paper and 1 formula sheet.

X05

GAUTENG

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions:

1. This question paper consists of 8 questions. Answer ALL the questions.
2. Clearly show ALL calculations, diagrams graphs, et cetera that you have used in determining the answers.
3. An approved calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
5. The attached graph paper must be used only for QUESTION 8.2. It must be inserted inside the front cover of the answer book and handed in.
6. Number the answers EXACTLY as the questions are numbered.
7. Diagrams are not necessarily drawn to scale.
8. It is in your own interest to write legibly and present the work neatly.
9. A formula sheet is included at the end of the question paper.

QUESTION 1

1.1 Solve for x:

1.1.1 $\sqrt{2x+5} - x + 5 = 0$ (7)

1.1.2 $|x-3| = 2$ (2)

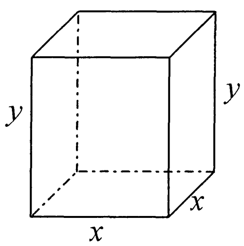
1.1.3 $(3x-2)^2 > 3x$ (6)

1.2 Given: $x^2 + p(x+1) - 2 = 0$

1.2.1 Prove that the equation has unequal, real roots for all real values of p. (5)

1.2.2 Determine the roots of the equation if $p = -5$. Round off the answers to TWO decimal places. (5)

1.3 A closed box has the shape of a rectangular prism with a square base. The sides of the base are x cm long. The height is y cm. The surface area of the box is 288 cm^2. The lengths of the edges are such that $2x + y = 21$.



1.3.1 Show that $x^2 + 2xy - 144 = 0$. (3)

1.3.2 Hence, calculate the values of x and y. (7)

[35]

QUESTION 2

2.1 Given: $f(x) = |x| - 2$ and $g(x) = -\frac{1}{2}(x-2)^2$

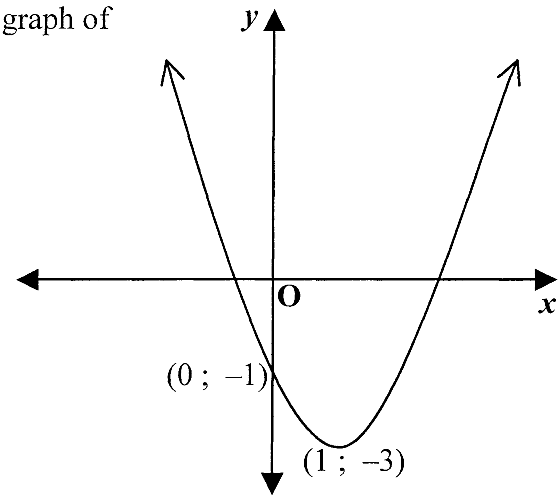
2.1.1 Draw sketch graphs of f and g on the same set of axes. Clearly indicate the co-ordinates of all intercepts with the axes. (7)

2.1.2 Use the graphs to determine the values of x for which $f(x) < g(x)$. (3)

2.1.3 Write down the range of f. (2)

2.2 The accompanying figure shows the graph of $y = ax^2 + bx + c$.

The turning point is at $(1\ ;-3)$ and the y-intercept is at $(0\ ;\ -1)$.



2.2.1 Determine the values of a, b and c. (5)

2.2.2 Determine, with the aid of the graph, the values of k for which the product of the roots of $ax^2 + bx + c = k$ is negative. (4)

2.3 Given: $f(x) = a^x$ and $h(x) = \frac{k}{x}$ where a is a positive constant, $a \neq 1$ and $k > 0$

2.3.1 Give the equation of the line about which the graphs of f and its inverse f^{-1} are symmetrical. (1)

2.3.2 If $0 < a < 1$, draw on separate sets of axes sketch graphs of the two functions h and f^{-1}, which is the inverse of f. Indicate the co-ordinates of any intercepts with the axes. (5)

2.3.3 Give the values of x which are common to the domains of both graphs drawn in QUESTION 2.3.2. (2)

[29]

QUESTION 3

When the third degree polynomial $p(x)$ is divided by $(x-3)$, the quotient is $2x^2+5x+10$ and the remainder is 28.

3.1 Determine the remainder when $p(x)$ is divided by $(x+1)$. (4)

3.2 Solve for x: $p(x)=0$ (7)

[11]

QUESTION 4

4.1 Simplify as far as possible:

$$\sqrt[n]{\frac{10^n+2^{n+2}}{5^{2n}+4.5^n}}$$ (5)

where $n \neq 0$.

4.2 Given that: $\log_2 5 = a$, express $\log_8 \sqrt{10}$ in terms of a. (5)

4.3 Solve for x, **without using a calculator:**

4.3.1 $3x^{\frac{2}{3}} - 13x^{\frac{1}{3}} - 10 = 0$ (4)

4.3.2 $\log x - \log(x-1) > 1$ (7)

4.3.3 $10^{-2\log x} = 8x$ (6)

[27]

QUESTION 5

5.1 Prove that the sum to n terms, S_n, of a geometric series with first term a and common ratio r ($r \neq 1$) is given by $S_n = \dfrac{a(r^n - 1)}{r-1}$. (4)

5.2 The first term of a geometric sequence is 1,5 and the n^{th} term is 192. The sum of the first n terms is 382,5. The common ratio is r.

5.2.1 Show that $r^n = 128r$. (2)

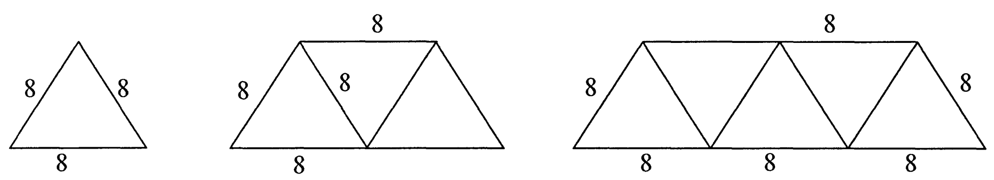
5.2.2 Calculate the value of n. (8)

5.3 The first two terms of a geometric series are: $x+3$ and x^2-9.

5.3.1 Calculate the values of x for which the series converges. (7)

5.3.2 Calculate the value of x if the sum to infinity is 13. (4)

5.4 The sides of a railway bridge are constructed using girders with a length of 8 m. The girders are made into sections in the form of equilateral triangles. A horizontal girder joins the top corners of the triangles.



The third sketch for example, shows a bridge having a length of 24 metres (3 times 8). It requires 11 girders.

Calculate the number of girders needed to construct a bridge having length 112 m. (5)

5.5 Calculate the value of x for which $\sum_{n=1}^{3} \log x^n = 12$. (5)

[35]

QUESTION 6

6.1 Given: $f(x) = 10x - x^2$

6.1.1 Determine:
$\lim_{x \to 0} \frac{f(x)}{x}$ (2)

6.1.2 Use **first principles** to prove that $f'(x) = -2x + 10$. (5)

6.1.3 Determine the equation of the tangent to the graph of f at the point where $f'(x) = -2$. (5)

6.2 Given $y = kx + k$ with k a constant.
Show that $\frac{dy}{dx} = \frac{y}{x+1}$. (3)

6.3 Determine $\frac{dy}{dx}$ in each of the following:

6.3.1 $y = x(x + x^{-1})^2$ (4)

6.3.2 $\sqrt[3]{x}\,.\,y = x - 3$ (4)

[23]

QUESTION 7

7.1 Given: $V(x) = 12x^2 - 2x^3$

7.1.1 Prove that V is increasing on the interval $0 < x < 4$. (3)

7.1.2 Draw a sketch graph of V. Indicate the co-ordinates of all turning points and intercepts with the axes. (9)

7.1.3 Refer to your sketch graph of V and write down the interval for which x values can represent the sides of the square base of a box with volume $V(x)$. (2)

7.2 The number of people in a certain area affected by a new strain of influenza t months after the time it was first detected is modelled by the function:
$N(t) = 10t^3 + 20t + 1$

7.2.1 Calculate the rate at which this flu is spreading after 4 months. (3)

7.2.2 Does the flu spread at a constant rate? Give a reason for your answer. (3)

7.3 When a person coughs, the trachea (windpipe) contracts causing air to flow faster through it. According to a mathematical model of coughing, the speed (v) of the airstream through the trachea is related to its radius (r) by the equation:

$v = k(n - r)r^2$ provided that $\frac{1}{2}n \leq r \leq n$.

In the equation k is a constant and n is the normal radius.

Determine to what fraction of its normal radius the trachea contracts when v is a maximum. (6)
[26]

QUESTION 8

8.1 A dressmaker can make a maximum of 20 dresses per week. She can make either silk dresses or cotton dresses. The material for a cotton dress costs R100 per dress and for a silk dress R200 per dress. She has R3 000 to spend toward the costs of material. She needs to make at least 5 of each type of dress. Let x be the number of cotton dresses and y the number of silk dresses.

Two of the constraint inequalities are:

$x \geq 5$ and
$y \geq 5$

8.1.1 Write down TWO more constraint inequalities that represent the above information. (3)

8.1.2 The dressmaker makes a profit of R50 on each silk dress and R40 on each cotton dress. Write down an equation for the profit, P. (1)

8.2 Given the following constraint inequalities:

$y \leq 20$
$x \leq 40$
$2y \leq 60 - x$

8.2.1 Represent the constraint inequalities on the graph paper provided and shade the feasible region. (5)

8.2.2 Given that: $P = 2x + 5y$ is the objective function.

(a) On your graph, show the optimal position of the search line (objective function) in order to maximise P. (2)

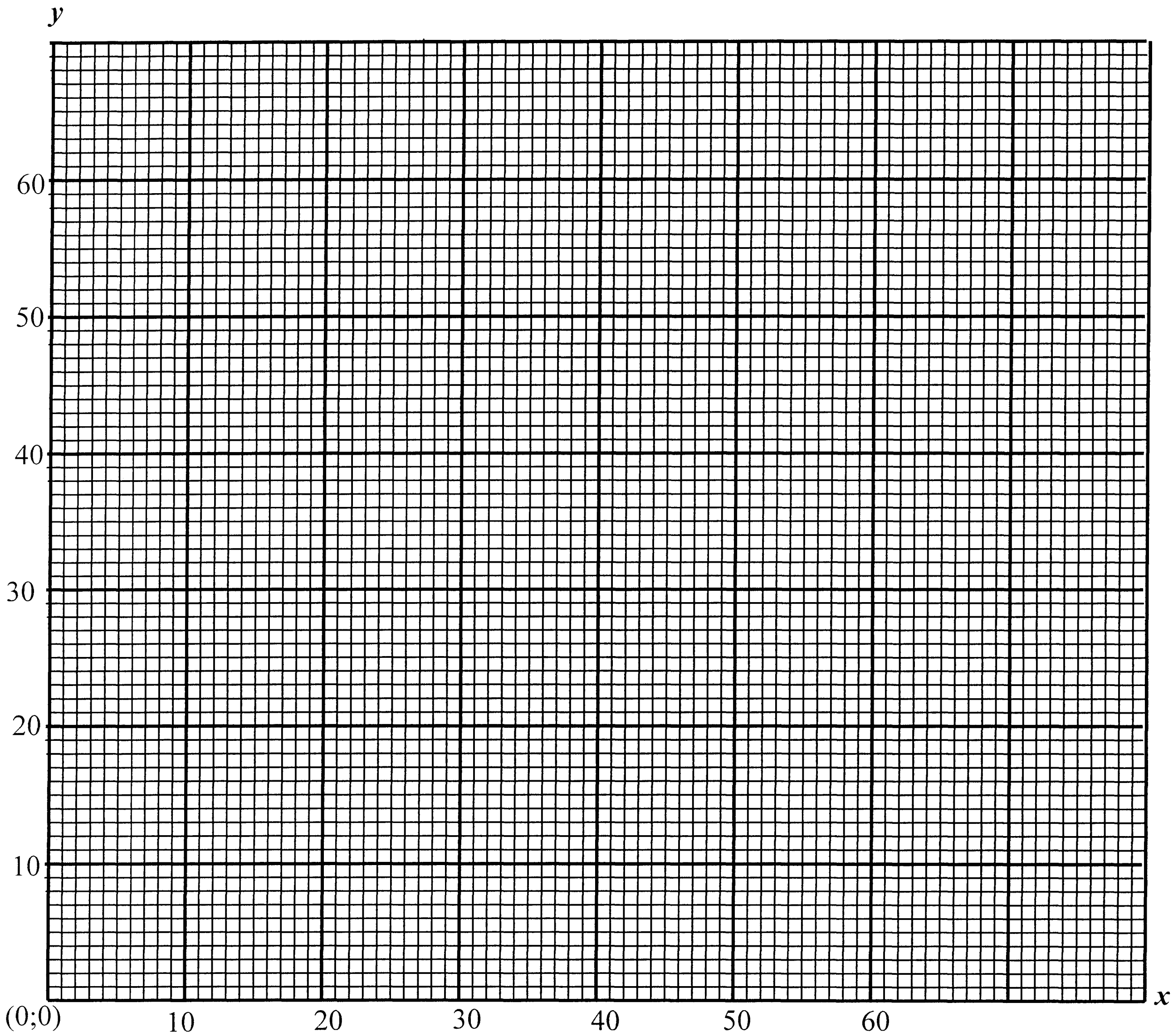
(b) Determine the maximum value of P. (3)

[14]

TOTAL: 200

GRAPH PAPER FOR QUESTION 8.2 : TO BE HANDED IN.

EXAMINATION NUMBER	
CENTRE NUMBER	



GAUTENG

Senior Certificate Examination

Mathematics Formula Sheet (HG and SG)
Wiskunde Formuleblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$T_n = a + (n - 1)d$ $\quad S_n = \frac{n}{2}(a + T_n)$ or / of $S_n = \frac{n}{2}(a + \ell)$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(1 - r^n)}{1 - r}\ (r \neq 1)$ $\quad S_n = \frac{a(r^n - 1)}{r - 1}\ (r \neq 1)$

$$S_\infty = \frac{a}{1 - r} \quad (|r| < 1)$$

$A = P\left(1 + \frac{r}{100}\right)^n$ or / of $A = P\left(1 - \frac{r}{100}\right)^n$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$(x_3; y_3) = \left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$x^2 + y^2 = r^2$$

$$(x - p)^2 + (y - q)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta ABC = \frac{1}{2}ab.\sin C$$