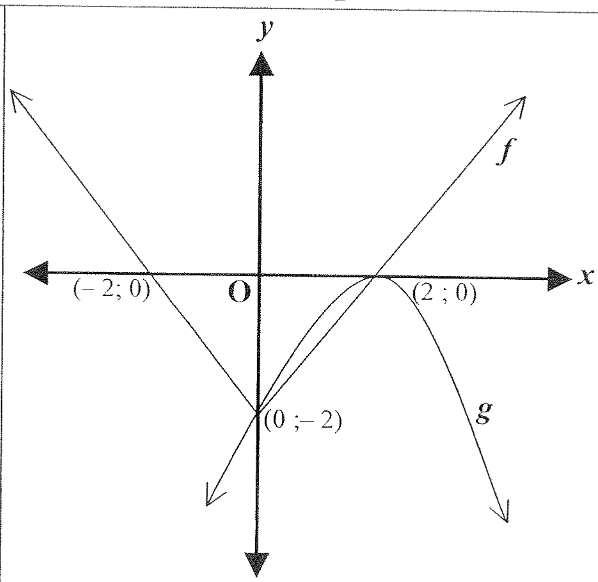
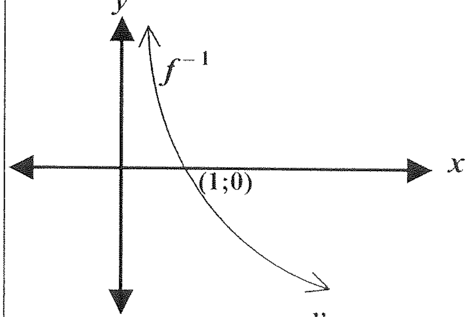
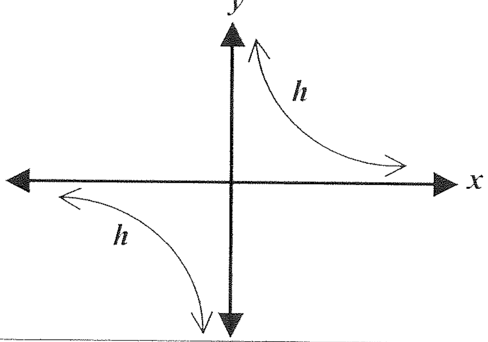
MATHEMATICS P1 HG MARCH 2007

1.1	1.1.1	$\sqrt{2x+5}-x+5=0$ $\sqrt{2x+5}=x-5$ $2x+5=x^2-10x+25$ $x^2-12x+20=0$ $(x-10)(x-2)=0$ $x=10$ or $x=2$ Check: $\sqrt{2(10)+5}-10+5=0$ $\therefore\ x=10$ $\sqrt{2(2)+5}-2+5=6\neq 0$ $\therefore\ x\neq 2$	(7)	✓ surd one side ✓ square both sides ✓ standard form ✓ factors ✓ both values ✓ checking ✓ answer
		OR Since $\sqrt{2x-5}=x-5$, $x\geq 5$. Then $2x+5=(x-5)^2$ $x^2-12x+20=0$ $(x-10)(x-2)=0$ $\therefore x=10$ or $x=2$ But only $x = 10$ is greater than 5. $\therefore\ x = 10$ is the only solution.	(7)	✓ statement ✓ $x\geq 5$ ✓ square both sides ✓ standard form ✓ factorisation ✓ both values ✓ solution
	1.1.2	$\lvert x-3\rvert=2$ $x-3=2$ or $-(x-3)=2$ $\therefore\ x=5$ or $x=1$ **OR** by inspection: $x=5$ or $x=1$	(2)	✓ both equations ✓ values of x ✓✓ each value (1 mark each)
	1.1.3	$(3x-2)^2>3x$ $9x^2-12x+4>3x$ $9x^2-15x+4>0$ $(3x-4)(3x-1)>0$ $x<\frac{1}{3}$ or $x>\frac{4}{3}$	(6)	✓ multiplying ✓ standard form ✓ factors ✓ $x<\frac{1}{3}$ ✓ or ✓ $x>\frac{4}{3}$
1.2		$x^2+p(x+1)-2=0$		
	1.2.1	$x^2+px+p-2=0$ $\Delta=p^2-4(p-2)$ $=p^2-4p+8$ $=p^2-4p+4+4$ $=(p-2)^2+4$ >0 $\therefore$ roots are real for all p.	(5)	✓ expansion / std form ✓ substitution / use delta ✓ completing a square ✓ write as perfect square + 4 ✓ conclusion

	1.2.2	$x^2 - 5x - 7 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{5 \pm \sqrt{25 + 28}}{2}$ $= 6{,}14$ or $-1{,}14$ **OR** substitute p = – 5 in 1.2.1 i.e. $\Delta = 53$	(5)	✓ standard form ✓ formula ✓ substitution ✓✓ values (1 mark each)
1.3	side = x ; height = y ; surface area = 288			
	1.3.1	surface area $= 2x^2 + 4xy$ $\therefore\ 2x^2 + 4xy = 288$ $x^2 + 2xy - 144 = 0$	(3)	✓✓ equation ✓ substitution
	1.3.2	$2x + y = 21$ $\therefore\ y = 21 - 2x$ Substituting yields: $x^2 + 2x(21 - 2x) - 144 = 0$ $\therefore\ x^2 + 42x - 4x^2 - 144 = 0$ $-3x^2 + 42x - 144 = 0$ $x^2 - 14x + 48 = 0$ $(x - 6)(x - 8) = 0$ $\therefore\ x = 6$ or $x = 8$ $y = 9$ or $y = 5$	(7)	✓ y-subject ✓ substitution ✓ expansion ✓ factors ✓ both values ✓✓ y-values (1 mark each)
			[35]	

2.1	$f(x) = \lvert x \rvert - 2$ and $g(x) = -\frac{1}{2}(x - 2)^2$			
	2.1.1		(7)	For f: ✓ shape ✓✓ x-intercepts ✓ y-intercept For g: ✓ shape ✓ x-intercept / TP ✓ y-intercept
	2.1.2	$0 < x < 2$ **OR** $x \in (0\,;2)$	(3)	✓ 0 ✓ 2 ✓ inequalities
	2.1.3	$y \geq -2$ **OR** $y \in [-2;\infty)$	(2)	✓✓ answer

2.2	$y = ax^2 + bx + c$; TP is $(1\,;-3)$; y-intercept $= -1$			
	2.2.1	$y = a(x-1)^2 - 3$ subst. $(0\,;-1)$: $-1 = a(0-1)^2 - 3$ $\therefore\ a = 2$ $\therefore\ y = 2(x-1)^2 - 3$ $= 2(x^2 - 2x + 1) - 3$ $= 2x^2 - 4x - 1$ $\therefore\ a = 2\ ;\ b = -4$ and $c = -1$	(5)	✓✓ substituting TP ✓ substituting $(0\,;-1)$ ✓ standard form ✓ answer
	2.2.1	**OR** From graph $c = -1$ and $f(1) = -3$ $\therefore\ a + b + c = -3$ i.e. $a + b = -2$ .... (1) Also $f'(x) = 2ax + b = 0$ when $x = 1$ $\therefore\ 2a + b = 0$ .... (2) (2) – (1): $a = 2$ Subst. in (1): $b = -4$	(5)	✓ $f(1) = -3$ ✓ eq. (1) ✓ eq. (2) ✓ value of a ✓ value of b
	2.2.2	$c - k < 0$ $-k < 1$ $k > -1$	(4)	✓✓ inequality ✓ substitution & trasposition ✓ solution
2.3	$f(x) = a^x$ and $h(x) = \dfrac{a}{x}$			
	2.3.1	$y = x$	(1)	✓ answer
	2.3.2		(5)	✓ shape ✓ intercept ✓ realising that f^{-1} is log graph ✓ shape ✓ correct quadrants
	2.3.3	$x > 0$ **OR** $x \in (0;\infty)$	(2)	✓✓ answer
			[29]	

	$p(x)=(x-3)(2x^2+5x+10)+28$		
3.1	$p(x)=(x-3)(2x^2+5x+10)+28$ $p(-1)=(-1-3)(2-5+10)+28$ $=0$ the remainder is 0	(4)	✓expression for $p(x)$ ✓use of $p(-1)$ ✓substitution ✓value of remainder
3.2	$p(x)=2x^3-x^2-5x-2=0$ $(x+1)(2x^2-3x-2)=0$ $(x+1)(2x+1)(x-2)=0$ $x=-1$ or $x=-\frac{1}{2}$ or $x=2$	(7)	✓standard form ✓using $(x+1)$ as a factor ✓✓trinomial factor ✓further factoring ✓✓ answers [minus 1 per error]
		[11]	

4.1	$\sqrt[n]{\frac{10^n+2^{n+2}}{5^{2n}+4\times5^n}}=\sqrt[n]{\frac{2^n(5^n+2^2)}{5^n(5^n+4)}}$ $=\sqrt[n]{\frac{2^n}{5^n}}$ $=\sqrt[n]{\left(\frac{2}{5}\right)^n}$ $=\frac{2}{5}$	(5)	✓ factorise numerator ✓ factorise denominator ✓ simplification ✓ exponential law ✓ exponential law
4.2	$\log_8\sqrt{10}=\log_8 10^{\frac{1}{2}}$ $=\frac{1}{2}\log_8 10$ $=\frac{1}{2}\left(\frac{\log_2 10}{\log_2 8}\right)$ $=\frac{1}{2}\left(\frac{\log_2 5+\log_2 2}{3}\right)$ $=\frac{a+1}{6}$ **OR** $5=2^a$ and $10=2\times2^a=2^{a+1}$ Let $y=\log_8\sqrt{10}$ $\therefore$ $10^{\frac{1}{2}}=8^y=2^{3y}$ $\therefore\ 10=2^{6y}$ $\therefore\ a+1=6y$ $\therefore\ y=\frac{a+1}{6}$	(5)	✓ log law ✓ log law (change of base) ✓ $\log_2 5+\log_2 2$ ✓ $\log_2 8=3$ ✓ substitution & simplification
4.3	4.3.1 $3x^{\frac{2}{3}}-13x^{\frac{1}{3}}-10=0$ $\left(3x^{\frac{1}{3}}+2\right)\left(x^{\frac{1}{3}}-5\right)=0$ $x^{\frac{1}{3}}=-\frac{2}{3}$ or $x^{\frac{1}{3}}=5$ $x=-\frac{8}{27}$ or $x=125$	(4)	✓ factors ✓ values of $x^{\frac{1}{3}}$ ✓ $x=-\frac{8}{27}$ ✓ $x=125$

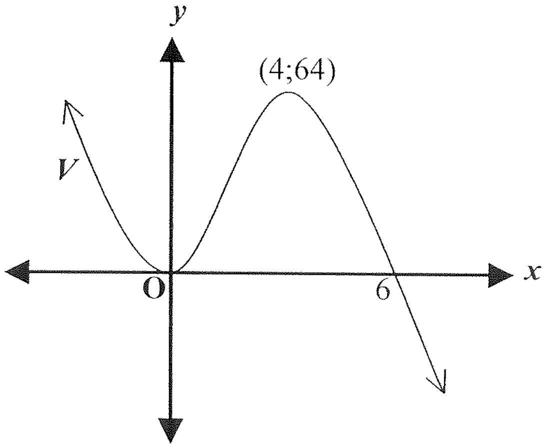
	4.3.2	$\log x - \log(x-1) > 1$ $\log \frac{x}{x-1} > 1$ by definition $x > 1$ $\frac{x}{x-1} > 10^1$ $x > 10x - 10$ $9x < 10$ $x < \frac{10}{9}$ $\therefore \quad 1 < x < \frac{10}{9}$	(7)	✓ single log ✓ use definition ✓ exponential form ✓ multiply by $x-1$ ✓ add like terms ✓ value of x ✓ solution
		OR $x > 1$ for log $(x-1)$ to be defined Also $\frac{x}{x-1} > 10$ $\therefore \; 1 + \frac{1}{x-1} > 10$ $\therefore \; \frac{1}{x-1} > 9$ But $x - 1 > 0 \; \therefore \; x - 1 < \frac{1}{9}$ $x < \frac{10}{9}$ $\therefore 1 < x < \frac{10}{9}$		✓ use definition ✓ ✓ single log/exponential form ✓ add like terms ✓ cross multiplication ✓ value of x ✓ solution
		OR $\frac{x}{x-1} - 10 > 0$ ✓ $\therefore \; \frac{x - 10(x-1)}{x-1} > 0$ ✓ $\therefore \; \frac{10 - 9x}{x-1} > 0$ ✓ But $x - 1 > 0 \; \therefore \; 10 - 9x > 0$ ✓ $\therefore \; x < \frac{10}{9}$ Finaly $1 < x < \frac{10}{9}$ ✓	(7)	
4.4		$10^{-2\log x} = 8x$ $\log 10^{-2\log x} = \log 8x$ $-2\log x \log 10 = \log 8x$ $\log x^{-2} = \log 8x$ $x^{-2} = 8x$ $\frac{1}{x^2} = 8x$ $1 = 8x^3$ $x^3 = \frac{1}{8}$ $x = \frac{1}{2}$ **OR** $10^{\log x^{-2}} = 8x$ ✓ $x^{-2} = 8x$ ✓✓ $\frac{1}{x^2} = 8x$ ✓ $1 = 8x^3$ ✓ $x^3 = \frac{1}{8}$ ✓ $x = \frac{1}{2}$	(6)	✓ apply logs both sides ✓ $-2 \log x \log 10$ ✓ log laws ✓ remove logs ✓ exponential law ✓ $x^3 = \frac{1}{8}$
			[27]	

5.1		$S_n = a + ar + ar^2 + \cdots + ar^{n-1}$ $rS_n = \quad ar + ar^2 + \cdots \quad + \quad ar^n$ $rS_n - S_n = ar^n - a$ $S_n(r-1) = a(r^n - 1)$ $S_n = \dfrac{a(r^n-1)}{r-1}$	(4)	✓ expansion ✓ multiplying by r ✓ subtract equations ✓ factors
5.2		$a = 1{,}5$; $T_n = 192$; $S_n = 382{,}5$ common ratio $= r$		
	5.2.1	$ar^{n-1} = 192$ $1{,}5 \times r^{n-1} = 192$ $r^{n-1} = 128$ $r^n = 128r$	(2)	✓ substitution ✓ dividing by 1,5
	5.2.2	$\dfrac{a(r^n-1)}{r-1} = S_n$ $\dfrac{1{,}5(r^n-1)}{r-1} = 382{,}5$ .... (1) $\dfrac{1{,}5(128r-1)}{r-1} = 382{,}5$ $192r - 1{,}5 = 382{,}5r - 382{,}5$ $190{,}5r = 381$ $r = 2$ $\therefore \quad 2^n = 256$ $= 2^8$ $\therefore \quad n = 8$	(8)	✓ substitute S_n and a ✓ substitute r^n ✓ multiplying ✓ add like terms ✓ value of r ✓ substitute r ✓ $256 = 2^8$ ✓ value of n
		OR From (1): $\dfrac{1{,}5(r^n-1)}{\frac{r^n}{128} - 1} = 382{,}5$ $1{,}5r^n - 1{,}5 = \dfrac{382{,}5}{128} r^n - 382{,}5$ $\dfrac{190{,}5}{128} r^n = 381$ $r^n = \dfrac{381 \times 128}{190{,}5}$ $= 256$ But $r^n = 128r$ $\therefore \quad 128r = 256$ $\therefore \quad r = 2$ $\therefore \quad 2^n = 2^8$ $\therefore \quad n = 8$	(8)	✓ substitute r by $\frac{r^n}{128}$ ✓ multiplying ✓ add like terms ✓ value of r^n ✓ substitute r^n ✓ value of r ✓ value of n

5.3	$T_1 = a = x+3$; $T_2 = ar = x^2 - 9$			
	5.3.1	$r = \frac{x^2-9}{x+3}$ $= \frac{(x+3)(x-3)}{x-3}$ $= x-3$ converges when : $-1 < x-3 < 1$ that is when $2 < x < 4$	(7)	✓ equation of r ✓ factors ✓ simplification ✓✓ inequality / reasoning ✓✓ solution
	5.3.2	$r = x-3$ and $a = x+3$ $S_\infty = \frac{a}{1-r}$ $13 = \frac{x+3}{1-(x-3)}$ $13 - 13x + 39 = x + 3$ $14x = 49$ $x = 3\frac{1}{2}$	(4)	✓ formula ✓ substitution ✓ simplification ✓ answer

5.4	3 ; 7 ; 11 ; . . . is an arithmetic sequence with $d = 4$. For 112 m need: $\frac{112}{8} = 14$ sections $T_n = a + (n-1)d$ $T_{14} = 3 + (14-1)4$ $= 55$ girders	(5)	✓ sequence ✓ no. sec. ✓ formula ✓ substitution ✓ number of girders **OR** a bridge with n triangles needs $3n + (n - 1) = 4n - 1$ girders. Here $n = \frac{112}{8} = 14$ $\therefore 4 \times 14 - 1 = 56 - 1 = 55$
5.5	$\sum_{n=1}^{3} \log x^n = 12$ $\therefore \log x + \log x^2 + \log x^3 = 12$ $\log x + 2\log x + 3\log x = 12$ $6\log x = 12$ $\log x = 2$ $x = 10^2 = 100$ **OR** $\log x + \log x^2 + \log x^3 = 12$ $\log(x \times x^2 \times x^3) = 12$ $\log x^6 = 12$ $6\log x = 12$ $\log x = 2$ $x = 10^2 = 100$	(5)	✓ expansion ✓ log law ✓ add like terms ✓ dividing by 6 ✓ answer ✓ expansion ✓ log law ✓ log law ✓ dividing by 6 ✓ answer
		[35]	

6.1		$f(x) = 10x - x^2$		
	6.1.1	$\lim_{x\to 0} \frac{f(x)}{x} = \lim_{x\to 0} \frac{10x - x^2}{x}$ $= \lim_{x\to 0}(10 - x)$ $= 10$	(2)	✓ $10 - x$ ✓ 10
	6.1.2	$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h\to 0} \frac{10(x+h) - (x+h)^2 - (10x - x^2)}{h}$ $= \lim_{h\to 0} \frac{10x + 10h - x^2 - 2xh - h^2 - 10x + x^2}{h}$ $= \lim_{h\to 0} \frac{10h - 2xh - h^2}{h}$ $= \lim_{h\to 0}(10 - 2x - h)$ $= 10 - 2x$	(5)	✓ formula ✓ substitution ✓ multiplying out ✓ simplification ✓ dividing by h
	6.1.3	$f'(x) = -2$ $\therefore\ 10 - 2x = -2$ $2x = 12$ $x = 6$ $\therefore\ y = 10(6) - (6)^2 = 24$ $\therefore\ y - 24 = -2(x - 6)$ $y = -2x + 36$	(5)	✓ equate derivative to gradient ✓ value of x ✓ y-value ✓ substitution into equation ✓ equation
6.2		$y = kx + k$ $\therefore\ \frac{dy}{dx} = k$ but $y = k(x+1)$ $\therefore\ k = \frac{y}{x+1}$ that is $\frac{dy}{dx} = \frac{y}{x+1}$	(3)	✓ derivative ✓ factors ✓ k subject
6.3	6.3.1	$y = x(x + x^{-1})^2$ $= x(x^2 + 2 + x^{-2})$ $= x^3 + 2x + x^{-1}$ $\frac{dy}{dx} = 3x^2 + 2 - x^{-2}$	(4)	✓ expansion ✓ simplification ✓ $3x^2 + 2$ ✓ x^{-2}
	6.3.2	$\sqrt[3]{x}\,y = x - 3$ $y = x^{\frac{2}{3}} - 3x^{-\frac{1}{3}}$ $\frac{dy}{dx} = \frac{2}{3}x^{-\frac{1}{3}} - \left(-\frac{1}{3}\right)3x^{-\frac{4}{3}}$	(4)	✓✓ eq. exp. form (1 per term) ✓✓ derivative (1 per term)
			[23]	

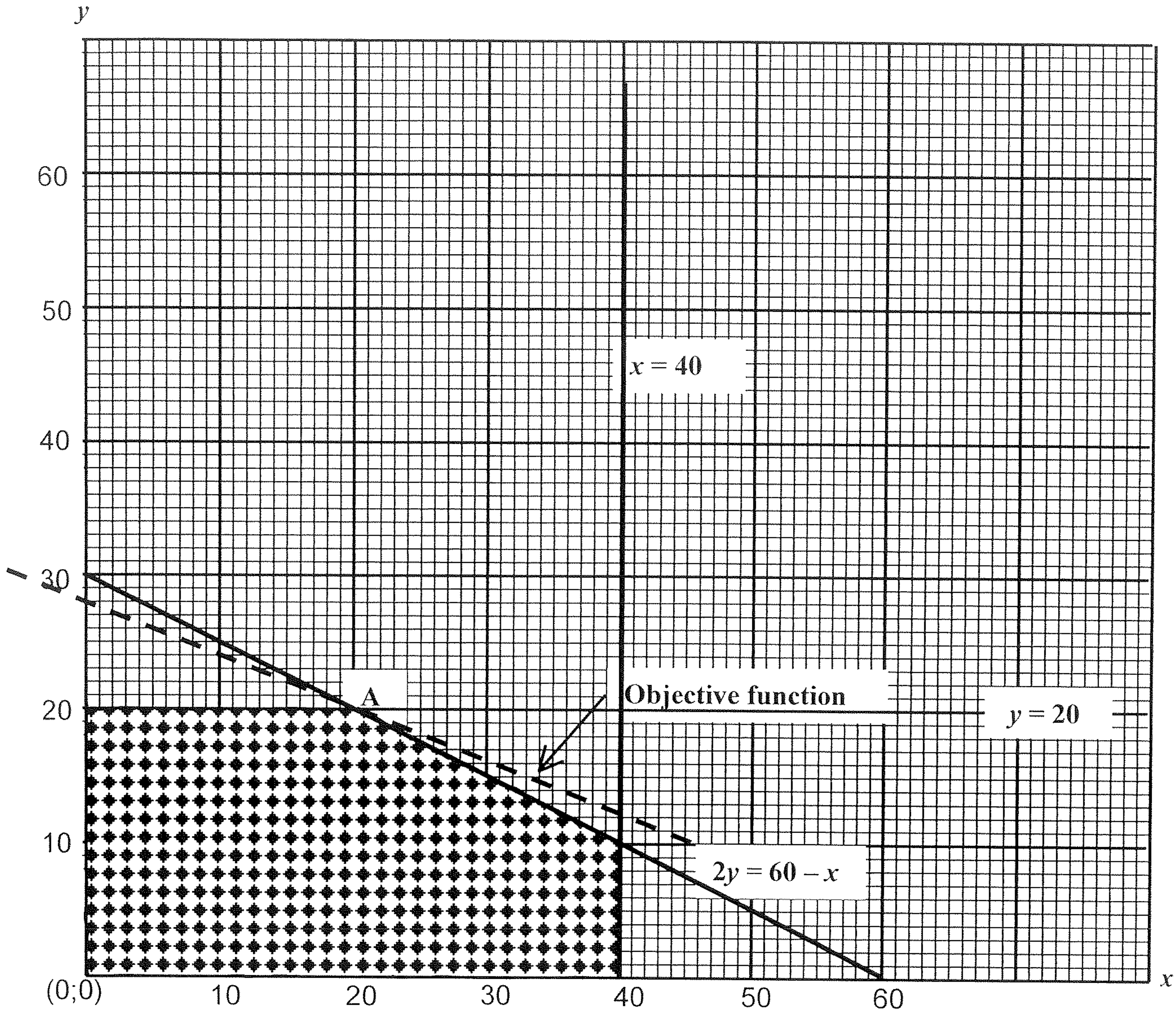
7.1	$V(x) = 12x^2 - 2x^3$			
	7.1.1	$V'(x) = 24x - 6x^2$ For $0 < x < 4$, $24x > 0$ and $-6x^2 < 0$ $\therefore\ V'(x) > 0$ for $0 < x < 4$ That is V is a **decreasing** function for $x < 0$. **OR** $V'(x) = 6x(4 - x)$ If $x > 0$ then $6x > 0$ and $4 - x > 0$ if $x < 4$ $\therefore\ V'(x) < 0$ for $0 < x < 4$. **OR** Using a sign table for $V'(x)$	(3)	✓ derivative ✓ reasoning ✓ conclusion ✓ factorisation ✓ reasoning ✓ conclusion
	7.1.2	Turning pts.: $24x - 6x^2 = 0$ $6x(4 - x) = 0$ $x = 0$ or $x = 4$ $y = 0$ or $y = 64$ x-intercepts: $12x^2 - 2x^3 = 0$ $2x^2(6 - x) = 0$ $x = 0$ or $x = 6$ 	(9)	✓ derivative $= 0$ ✓ both values of x ✓✓ values of y (1 each) ✓ both roots ✓ shape ✓✓ turning pints ✓ x-intercept
	7.1.3	$0 < x < 6$ **OR** $x \in (0\,;6)$	(2)	✓✓ solution
7.2		$N(t) = 10t^3 + 20t + 1$		
	7.2.1	$N'(t) = 30t^2 + 20$ $N'(4) = 30(4)^2 + 20$ $= 500$ people per month	(3)	✓ derivative ✓ substitution ✓ answer
	7.2.2	No. Derivative is not a constant function. **OR** $N'(2) = 140$; $N'(3) = 290$ & $N'(4) = 500$ i.e. rate always increasing. **OR** $\frac{dN}{dt}$ depends on t.	(3)	✓ answer ✓✓ explanation

7.3	$v(r) = k(n-r)r^2$; $\frac{1}{2}n \le r \le n$ $v(r) = knr^2 - kr^3$ $\frac{dv}{dr} = 2knr - 3kr^2$ For maximum v: $2nkr - 3kr^2 = 0$ $kr(2n-3r) = 0$ $3r = 2n$ $\therefore \quad r = \frac{2}{3}n$	(6)	✓ expansion ✓ derivative ✓ = 0 ✓ factors ✓ simplification ✓ answer
		[26]	

8.1	8.1.1	$x + y \le 20$ $x + 2y \le 30$	(3)	✓ answer ✓✓ answer
	8.1.2	$P = 40x + 50y$	(1)	✓ answer

8.2	8.2.1	See graph paper		(5)	✓ $x = 40$ ✓ $y = 20$ ✓✓ $2y = 60 - x$ ✓ feasible region
		(a)	Search line gradient : $-\frac{2}{5}$ Passing through A $(20;20)$	(2)	✓ gradient ✓ line through A
		(b)	$P = 2x + 5y$ $= 2(20) + 5(20)$ $= 140$	(3)	✓ values of x and y ✓ substitution ✓ answer
				[14]	

TOTAL :	**200**	


y
60
50
40
30
20
10
(0;0)
10
20
30
40
50
60
x
x = 40
A
Objective function
y = 20
2y = 60 – x