education

Department:
Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATION - 2006

MATHEMATICS P2 : GEOMETRY

STANDARD GRADE

FEBRUARY/MARCH 2006

301-2/2 E

Marks: 150

3 Hours

This question paper consists of 12 pages, 1 formula sheet and 6 diagram sheets.

MATHEMATICS SG: Paper 2

301 2 2E SG

X05

INSTRUCTIONS

1. This question paper consists of **9** questions, a formula sheet and diagram sheets.
2. Use the formula sheet to answer this question paper.
3. Detach the diagram sheets from the question paper and place them inside your **ANSWER BOOK.**
4. The diagrams are not drawn to scale.
5. Answer **ALL** the questions.
6. Number **ALL** the answers correctly and clearly.
7. **ALL** the necessary calculations must be shown.
8. Non-programmable calculators may be used, unless otherwise stated.
9. The number of decimal digits to which answers must be rounded off will be stated in the question where necessary.

ANALYTICAL GEOMETRY

NOTE:
- **USE ANALYTICAL METHODS IN THIS SECTION.**
- **CONSTRUCTION AND MEASUREMENT METHODS MAY NOT BE USED.**

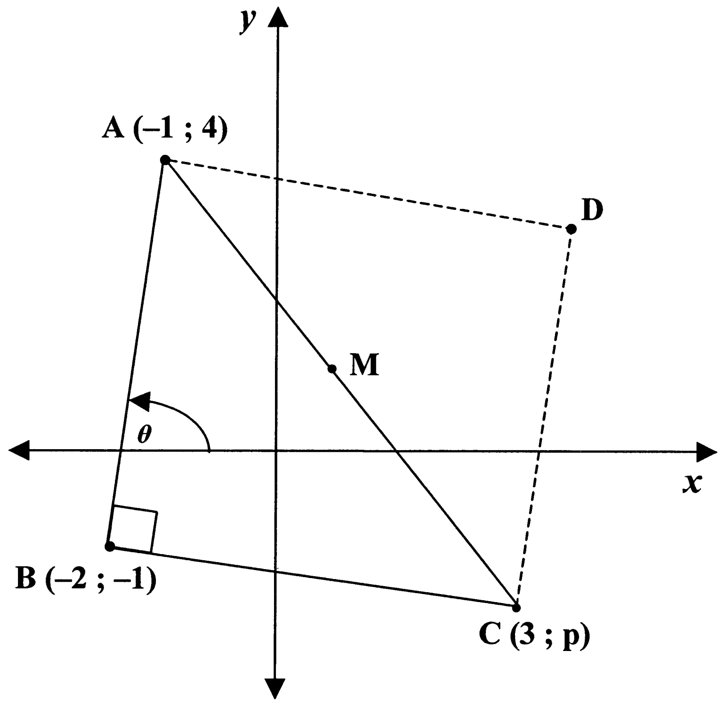
QUESTION 1

In the diagram alongside, A (– 1 ; 4) , B (– 2 ; –1) , C (3 ; p) and D are four points in a Cartesian plane.

M is the midpoint of AC.

$\hat{B} = 90°$

The angle of inclination of AB is θ.



1.1 Determine the size of θ, rounded off to ONE decimal digit. (4)

1.2 Prove that p = – 2 (4)

1.3 1.3.1 If p = – 2 calculate the coordinates of M. (3)

1.3.2 Hence, or otherwise, determine the coordinates of D if ABCD is a rectangle. (4)

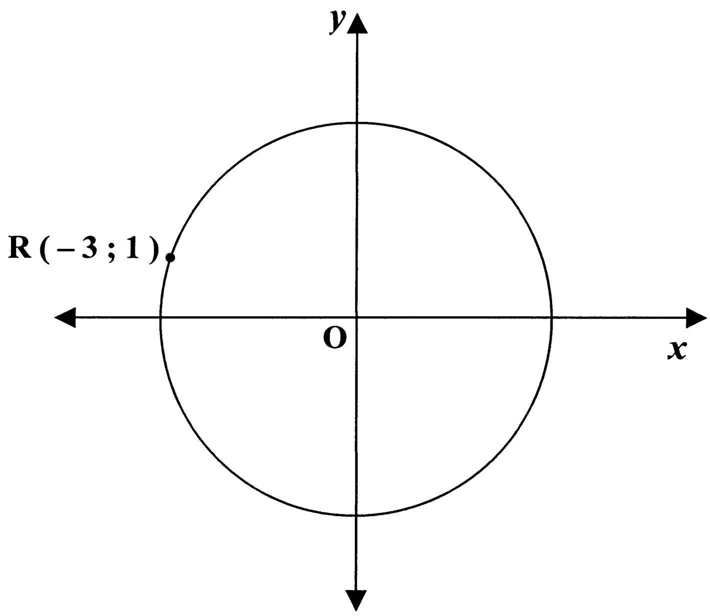
1.4 1.4.1 Determine the length of AB (give the answer in simplified surd form). (3)

1.4.2 Hence, prove analytically that ABCD is a square. (4)

[22]

QUESTION 2

2.1 In the diagram alongside, the circle with centre at the origin, passes through points R (– 3 ; 1) and N (k ; –1).



2.1.1 Determine the equation of the circle. (3)

2.1.2 Calculate the value of k, if N lies in the third quadrant. (2)

2.1.3 Hence, determine the equation of the straight line parallel to RN and which passes through the point (2 ; 0). (2)

2.1.4 Calculate the gradient of the tangent to the circle at R. (3)

2.1.5 Hence, determine the equation of the tangent to the circle at R. (3)

2.2 A (2 ; – 4) and B (0 ; 1) are points in a Cartesian plane.
Show that the equation of the locus of point P $(x ; y)$ if $P\hat{A}B = 90^\circ$,
is given by $5y = 2x - 24$ (4)

[17]

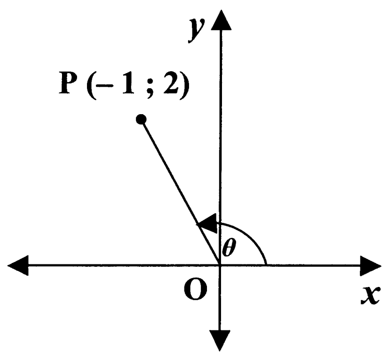
TRIGONOMETRY

QUESTION 3

Answer this question without the use of a calculator.

3.1 In the diagram alongside, P (– 1 ; 2) is a point in a Cartesian plane.

θ is an obtuse angle.



Calculate:

3.1.1 The length of OP (2)

3.1.2 The numerical value of $\sec^2(180^\circ - \theta)$ (3)

3.2 Simplify to a single trigonometric ratio of A:

$$\frac{\sin(180^\circ + A)}{\text{cosec}(90^\circ - A)\,.\cos(360^\circ - A)}$$ (5)

3.3 If $\cos\theta = -\frac{\sqrt{2}}{2}$, $0^\circ \leq \theta \leq 180^\circ$ and $\sin\alpha = \frac{1}{2}$, $0^\circ \leq \alpha \leq 90^\circ$, calculate the value of:

3.3.1 θ (2)

3.3.2 α (1)

3.3.3 $\cot 2(\theta - \alpha)$ (leave the answer in surd form) (3)

[16]

QUESTION 4

Given: $f(x) = -\sin x$ and $g(x) = \cos 2x$, for $x \in [0° ; 180°]$

4.1 Use the system of axes provided on the diagram sheet to sketch the curves of f and g, for $x \in [0° ; 180°]$.
Show clearly the coordinates of all the turning points and intercepts with the axes. (8)

4.2 Use the graphs in QUESTION 4.1, to determine the value(s) of $x \in [0° ; 180°]$ for which:

4.2.1 $f(x) = g(x)$ (1)

4.2.2 $g(x) < 0$ (3)

[12]

QUESTION 5

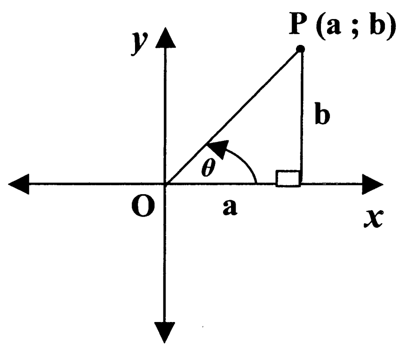
5.1 Solve for x, rounded off to ONE decimal digit:

$2 \tan x = -0{,}924$ and $0° \leq x \leq 270°$ (3)

5.2 In the diagram alongside, P (a ; b) is a point in a Cartesian plane such that θ is an acute angle.

Use the diagram to prove that:

$$\cot \theta = \frac{\cos\theta}{\sin\theta}$$



(5)

5.3 Use fundamental trigonometric identities and **not a diagram** to prove the following identity:

$$\cos \theta \, (\tan \theta + \cot \theta) = \text{cosec}\, \theta$$ (5)

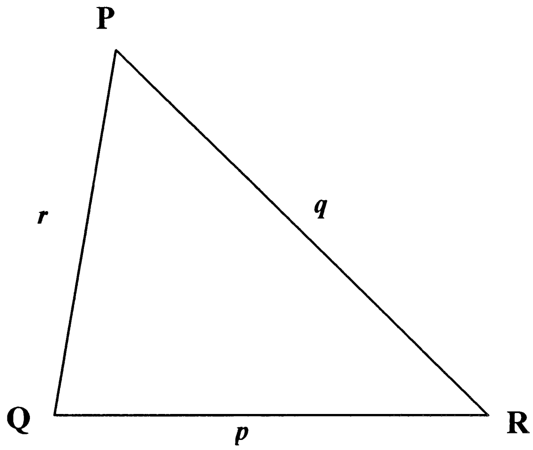
[13]

QUESTION 6

6.1 In the diagram alongside, ΔPQR is an acute-angled triangle.

Use the diagram on the diagram sheet, or redraw the diagram in your answer book to prove that:

$$q^2 = p^2 + r^2 - 2(p)(r)\cos Q$$



(6)

6.2 Farmer Molefe has a piece of farming land in the form of a cyclic quadrilateral ABCD.

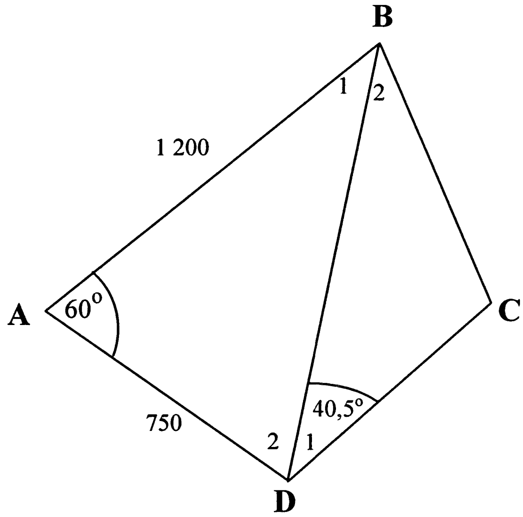
The following measurements are given:

AB = 1 200 m

AD = 750 m

$\hat{A} = 60°$

$\hat{D}_1 = 40{,}5°$



Determine:

6.2.1 The length of BD, rounded off to the nearest metre (4)

6.2.2 The size of $\hat{C}$ (1)

6.2.3 The length of BC, rounded off to the nearest metre (4)

6.2.4 The area of ABD, rounded off to the nearest square metre (3)

6.2.5 The number of bags of fertiliser needed for area ABD if one bag is sufficient for 400 square metres of land (2)

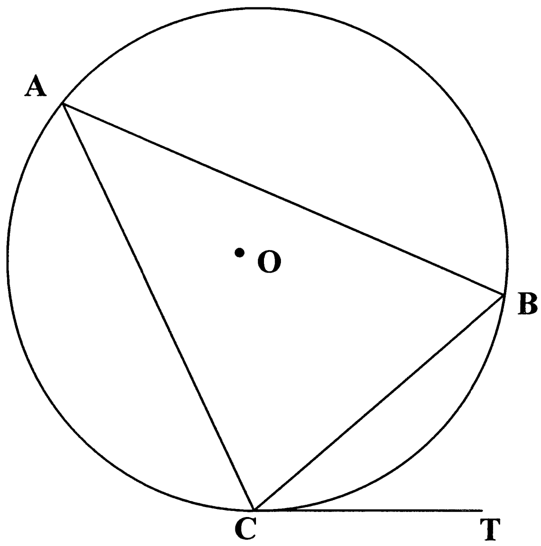
[20]

EUCLIDEAN GEOMETRY

NOTE:
- **DIAGRAMS FOR PROVING THEORY MAY BE USED ON THE DIAGRAM SHEETS OR REDRAWN IN YOUR ANSWER BOOK.**
- **DETACH THE DIAGRAM SHEETS FROM THE QUESTION PAPER AND PLACE THEM IN YOUR ANSWER BOOK.**
- **GIVE A REASON FOR EACH STATEMENT, UNLESS OTHERWISE STATED.**

QUESTION 7

7.1 In the diagram alongside, CT is a tangent to circle ABC at C. BC is a chord and O is the centre. Use the diagram on the diagram sheet or redraw the diagram in your answer book to prove the theorem which states that:



If CT is a tangent then $B\hat{C}T = \hat{A}$ (6)

7.2 In the diagram below, O is the centre of circle TRNM.

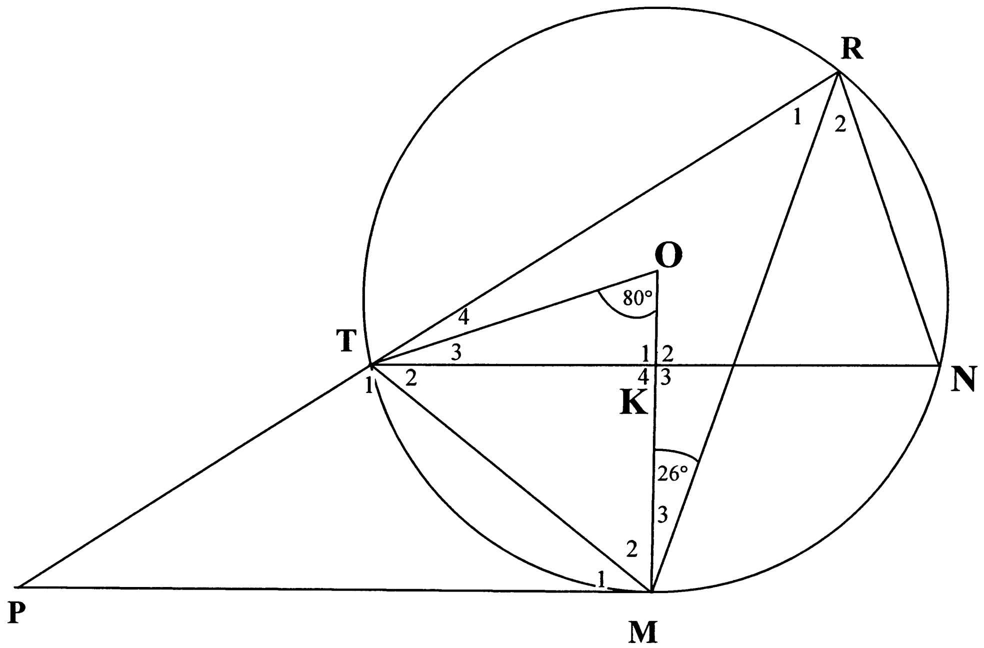
RT produced meets tangent MP at P.

OM intersects TN at K.

K is the midpoint of TN.

$M\hat{O}T = 80°$

$\hat{M}_3 = 26°$



Calculate, stating reasons, the size of:

7.2.1 $\hat{M}_1$ (4)

7.2.2 $\hat{N}$ (3)

7.2.3 $\hat{T}_3$ (3)

[16]

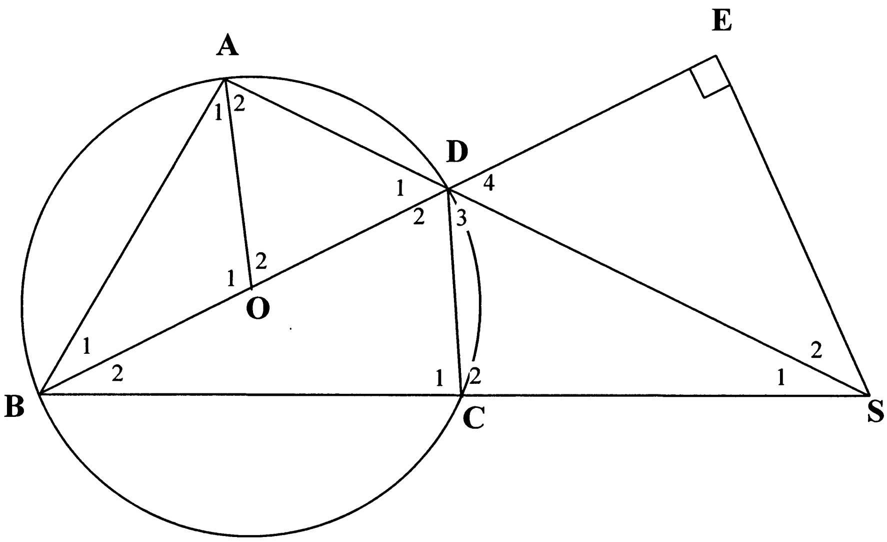
QUESTION 8

In the diagram below, O is the centre of circle ABCD.

AD and BC are produced to meet at S.

Diameter BD is produced to E such that $\hat{E} = 90°$ and

BD = DS



8.1	Prove that DESC is a cyclic quadrilateral.	(4)
8.2	Name, without stating a reason, ONE other cyclic quadrilateral in the diagram.	(1)
8.3	Prove that AB = ES	(5)

[10]

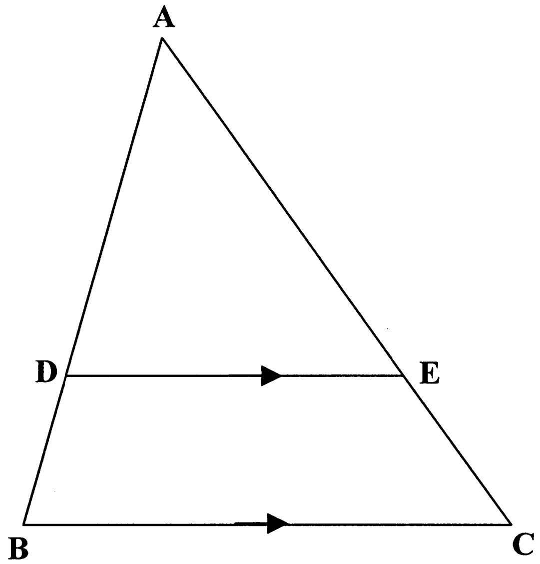
QUESTION 9

9.1 In the diagram alongside, Δ ABC is given.

D is a point on AB.

E is a point on AC.

Use the diagram on the diagram sheet or redraw the diagram in your answer book to prove the theorem which states that:



If DE || BC then $\frac{\mathbf{AD}}{\mathbf{DB}} = \frac{\mathbf{AE}}{\mathbf{EC}}$ (6)

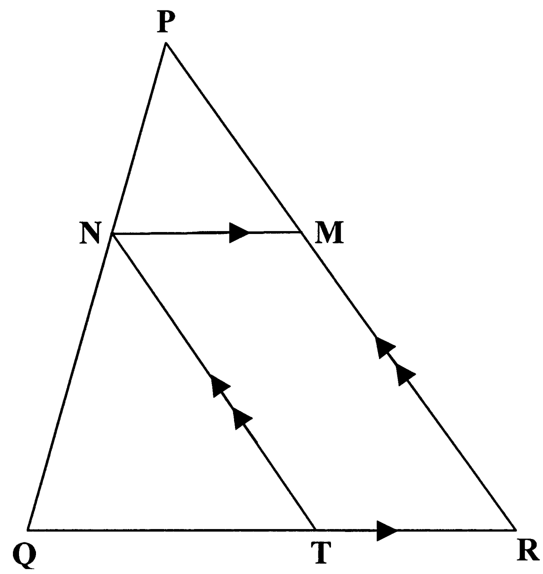
9.2 In the diagram alongside, N is a point on PQ and M is a point on PR of Δ PQR such that NM || QR.

T is a point on QR such that NT || PR.

QT : TR = 3 : 2

PQ = 30 units

PM = 16 units



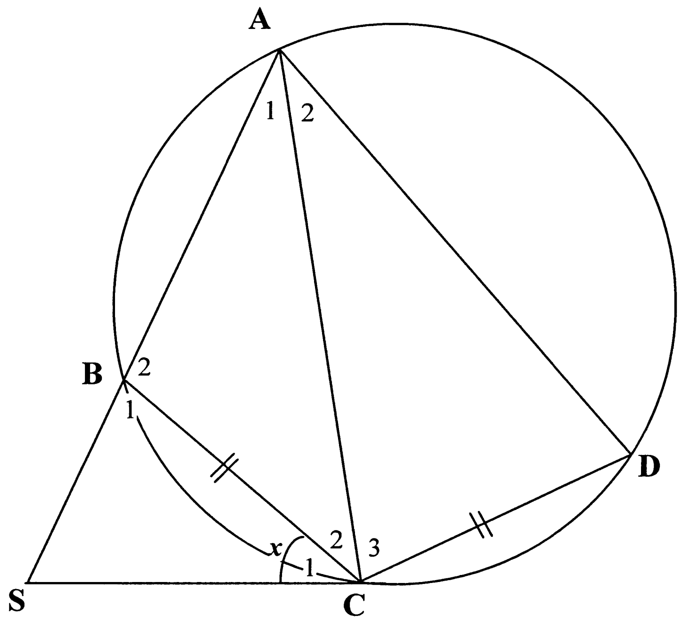
Calculate, stating reasons, the lengths of:

9.2.1 PN (4)

9.2.2 MR (4)

9.3 In the diagram alongside, ABCD is a cyclic quadrilateral with BC = CD. The tangent through C meets AB produced at S.

Let $\hat{C}_1 = x$



Prove that:

9.3.1 $\hat{C}_1 = \hat{A}_2$ (4)

9.3.2 $\Delta BCS \mid\mid\mid \Delta DAC$ (4)

9.3.3 $BC^2 = DA \cdot BS$ (2)

[24]

TOTAL: 150

Mathematics Formula Sheet (HG and SG)
Wiskunde Formuleblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}(a + T_n)$ $\quad S_n = \frac{n}{2}(a + \ell)$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = a.r^{n-1}$ $\quad S_n = \frac{a(1 - r^n)}{1 - r} \quad (r \neq 1)$ $\quad S_n = \frac{a(r^n - 1)}{r - 1} \quad (r \neq 1)$

$$S_\infty = \frac{a}{1 - r} \quad (|r| < 1)$$

$A = P\left(1 + \frac{r}{100}\right)^n$ **OR / OF** $A = P\left(1 - \frac{r}{100}\right)^n$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$(x_3 \, ; \, y_3) = \left(\frac{x_1 + x_2}{2} \, ; \, \frac{y_1 + y_2}{2}\right)$$

$$x^2 + y^2 = r^2$$

$$(x - p)^2 + (y - q)^2 = r^2$$

In Δ ABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta ABC = \tfrac{1}{2} ab.\sin C$$

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SENIOR CERTIFICATE EXAMINATION/SENIORSERTIFIKAAT-EKSAMEN
MATHEMATICS SG/WISKUNDE SG
PAPER II/VRAESTEL II
FEBRUARY/MARCH 2006

DIAGRAM SHEET/DIAGRAMVEL

INSTRUCTION

This diagram sheet must be handed in with your answer book. Please ensure that your details are complete.

INSTRUKSIE

Hierdie diagramvel moet saam met jou antwoordeboek ingelewer word. Maak asseblief seker dat jou besonderhede volledig ingevul is.

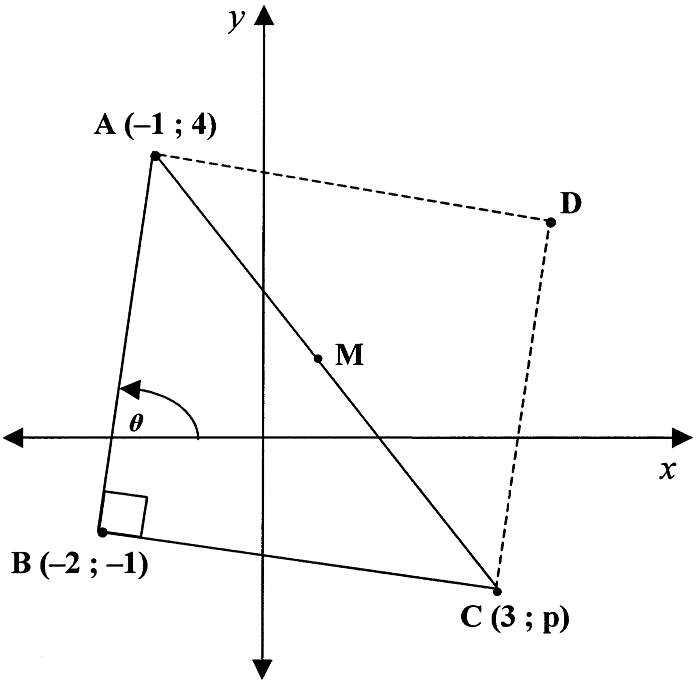
EXAMINATION NUMBER
EKSAMENNOMMER

CENTRE NUMBER
SENTRUMNOMMER

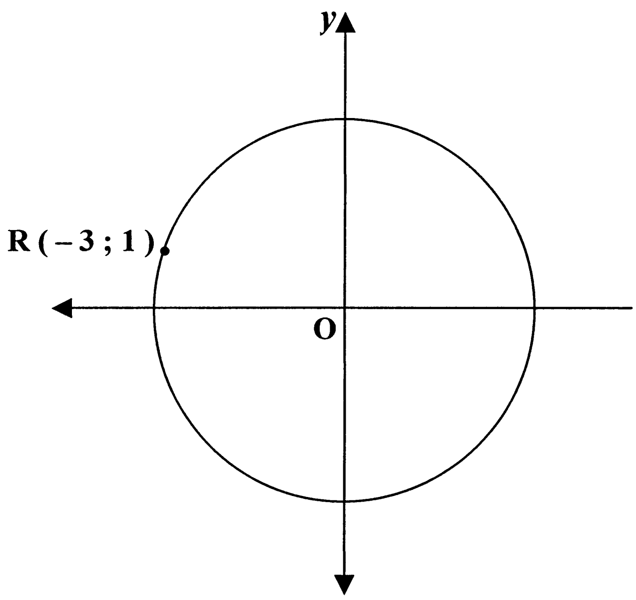
DIAGRAM SHEET/DIAGRAMVEL

EXAMINATION NUMBER
EKSAMENNOMMER

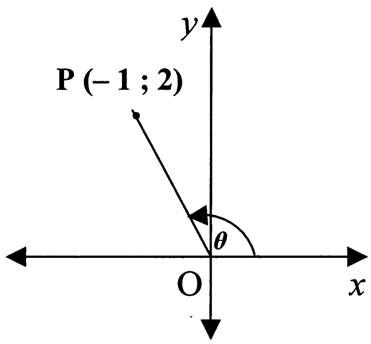
QUESTION 1 / VRAAG 1



QUESTION 2.1 / VRAAG 2.1



QUESTION 3.1 /VRAAG 3.1



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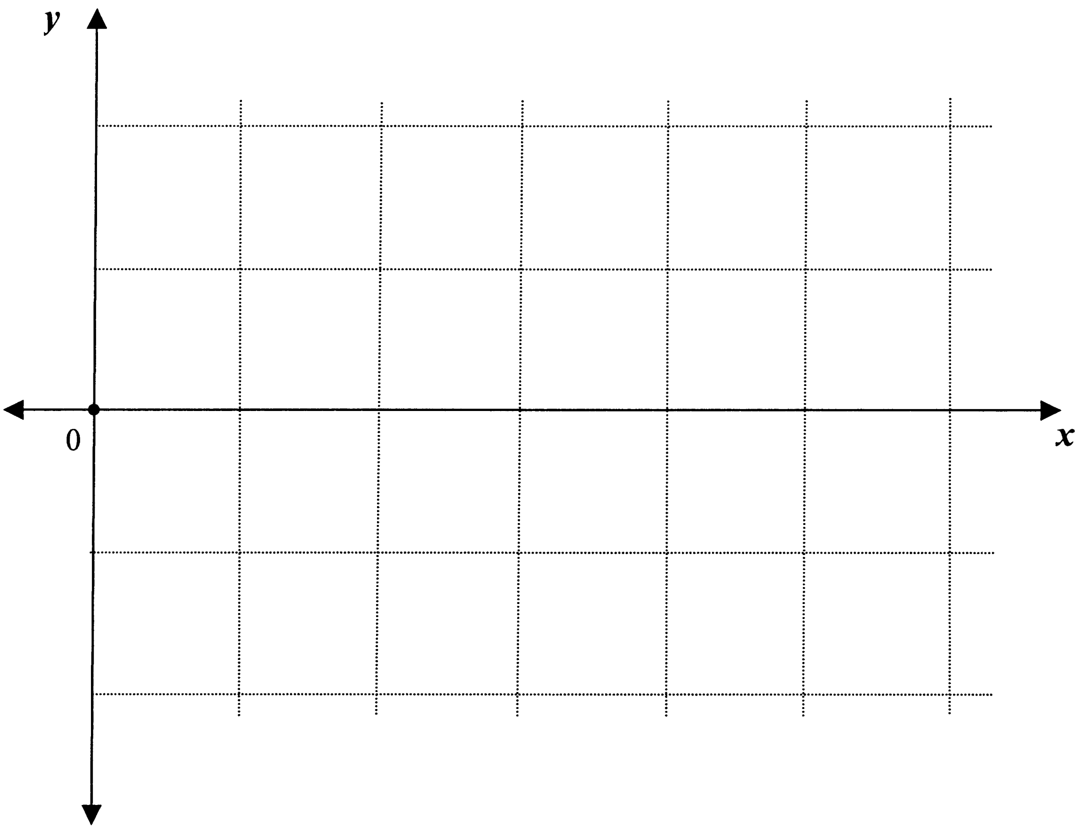
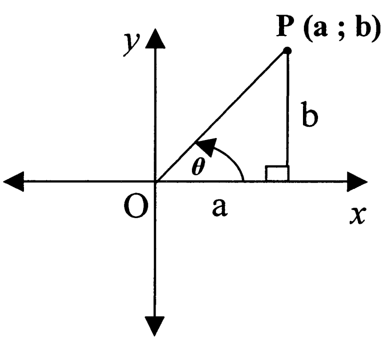
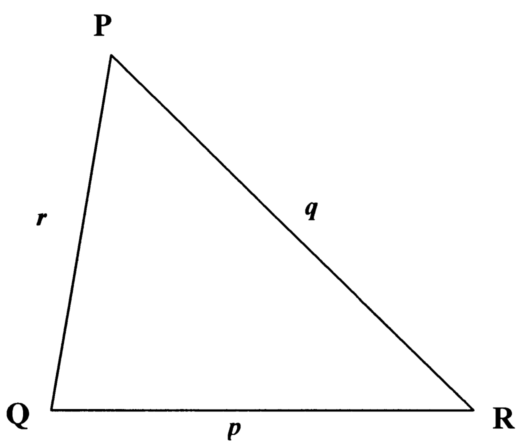
QUESTION 4.1 / VRAAG 4.1



QUESTION 5.2 / VRAAG 5.2



QUESTION 6.1 / VRAAG 6.1



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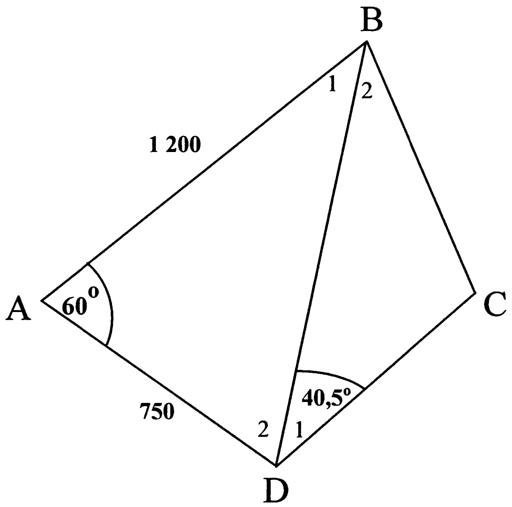
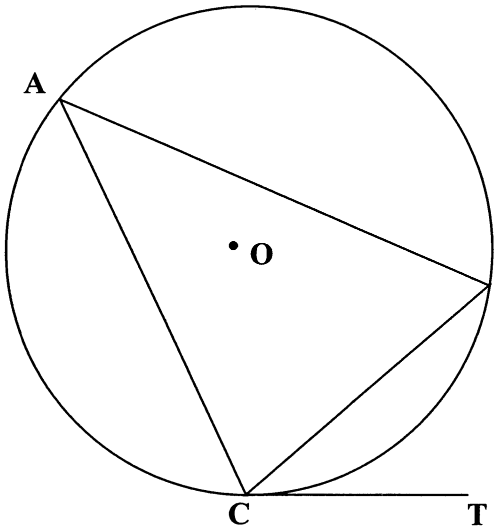
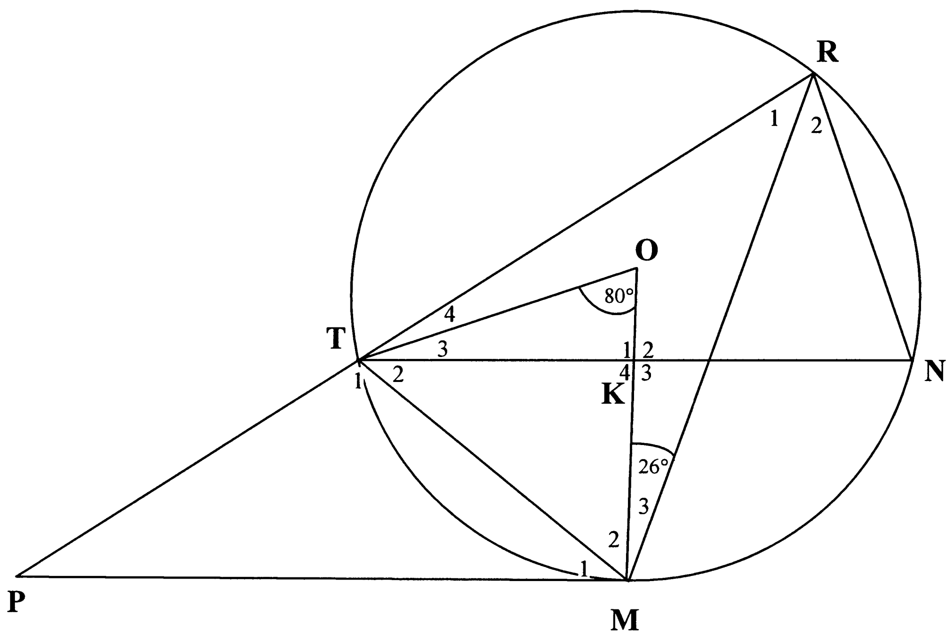
QUESTION 6.2 / VRAAG 6.2



QUESTION 7.1 / VRAAG 7.1



QUESTION 7.2 / VRAAG 7.2



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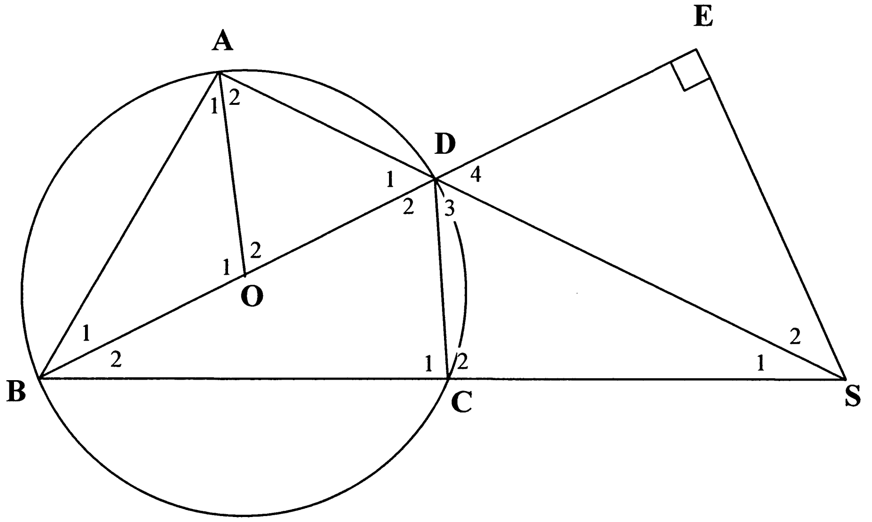
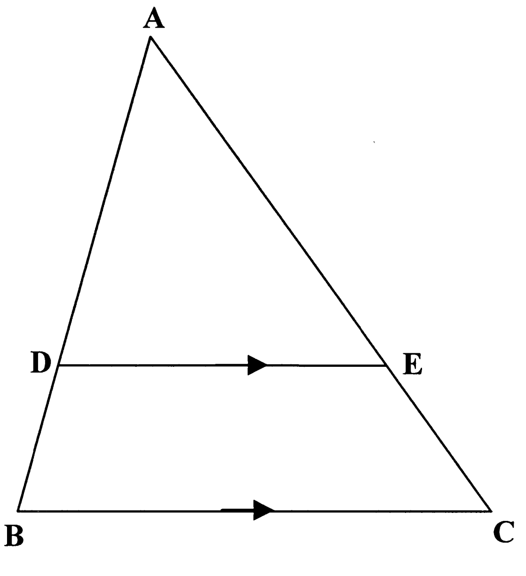
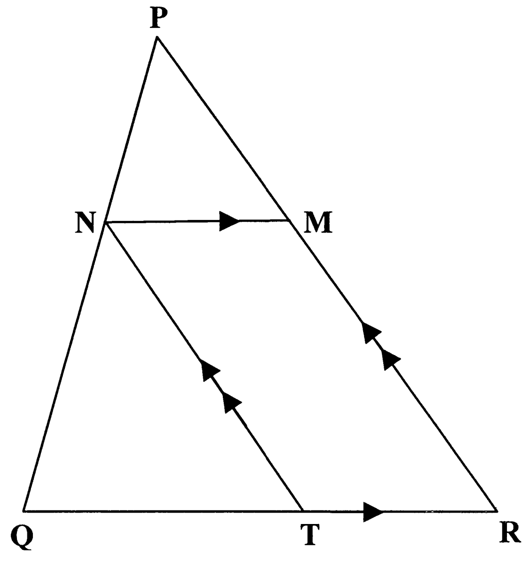
QUESTION 8 / VRAAG 8



QUESTION 9.1 / VRAAG 9.1



QUESTION 9.2 / VRAAG 9.2



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QUESTION 9.3 / VRAAG 9.3