education

Department of Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATION - 2006

MATHEMATICS P2

STANDARD GRADE

OCTOBER/NOVEMBER 2006

MARKS: 150

TIME: 3 hours

This question paper consists of 11 pages, a formula sheet and 5 diagram sheets.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions:

1. This question paper consists of 9 questions, a formula sheet and 5 diagram sheets.
2. Use the formula sheet to answer this question paper.
3. Detach the diagram sheets from the question paper and place them inside your ANSWER BOOK.
4. The diagrams are not drawn to scale.
5. Answer ALL the questions.
6. Number ALL the answers correctly and clearly.
7. ALL the necessary calculations must be shown.
8. Non-programmable calculators may be used, unless otherwise stated.
9. The number of decimal digits to which answers must be rounded off will be stated in the question where necessary.

ANALYTICAL GEOMETRY

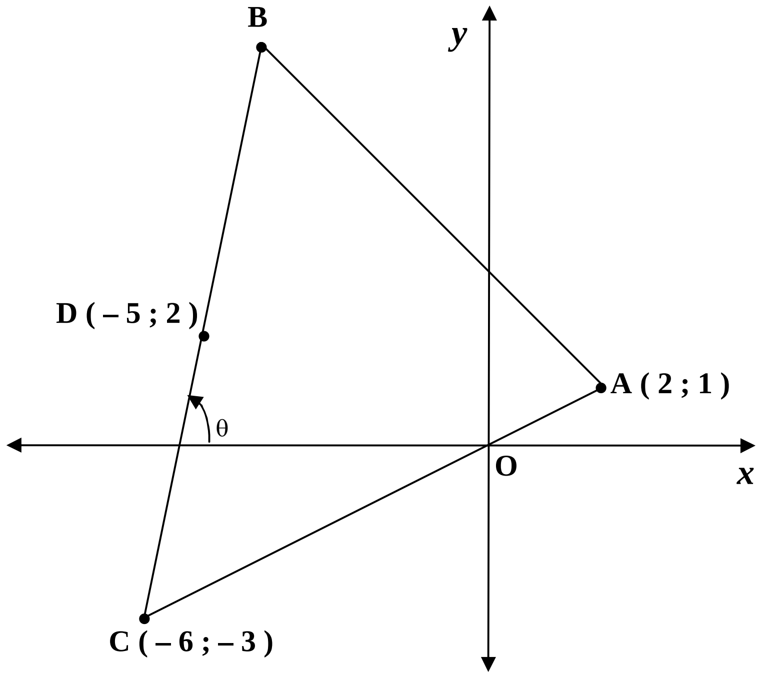
NOTE: – USE ANALYTICAL METHODS IN THIS SECTION.
– CONSTRUCTION AND MEASUREMENT METHODS MAY NOT BE USED.

QUESTION 1

In the diagram alongside, A(2 ; 1), C (– 6 ; – 3) and B are the vertices of ΔABC.

D (– 5 ; 2) is the midpoint of BC.

Straight line BC makes an angle θ with the *x*–axis.



1.1 Calculate the length of AC, **without using a calculator**. (3)

1.2 Determine:

1.2.1 The gradient of DC (3)

1.2.2 The size of θ, rounded off to ONE decimal digit (2)

1.2.3 The value of k if D, C and E (–3 ; k) are collinear (4)

1.2.4 The co–ordinates of B (4)

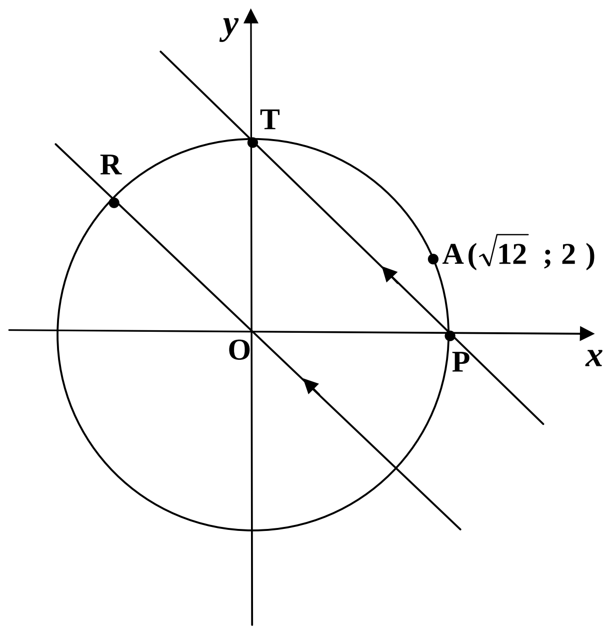
1.3 1.3.1 Prove that the equation of the locus of point P (*x* ; *y*), which is equidistant from points A and C, is given by $2x + y + 5 = 0$ (6)

1.3.2 Determine whether point (1 ; – 3) lies on the straight line $2x + y + 5 = 0$ (3)

[25]

QUESTION 2

In the diagram alongside, $A(\sqrt{12}\ ;\ 2)$ is a point on the circle with centre O (0 ; 0). The circle cuts the *x*–axis and *y*–axis at P and T respectively. Straight line RO intersects the circle at R. RO || TP



2.1 Show that the equation of the circle is given by $x^2 + y^2 = 16$ (2)

2.2 Write down the co–ordinates of P and T. (4)

2.3 Hence, write down the equation of the tangent to the circle at T. (2)

2.4 Determine:

2.4.1 The gradient of RO (3)

2.4.2 The equation of RO (2)

[13]

TRIGONOMETRY

QUESTION 3

3.1 If $\hat{A} = 121°$ and $\hat{B} = 61°$, calculate the values of the following (rounded off to TWO decimal digits):

3.1.1 cosec A – tan B (2)

3.1.2 $\cos^2 (A + 2B)$ (2)

3.2 If $12 \operatorname{cosec} \theta = 13$ and $\theta \in [90° ; 270°]$, by using a sketch and **without using a calculator**, calculate the values of the following:

3.2.1 $\cot \theta$ (5)

3.2.2 $\tan \theta - \sec \theta$ (4)

3.3 Simplify to a single trigonometric ratio of x:

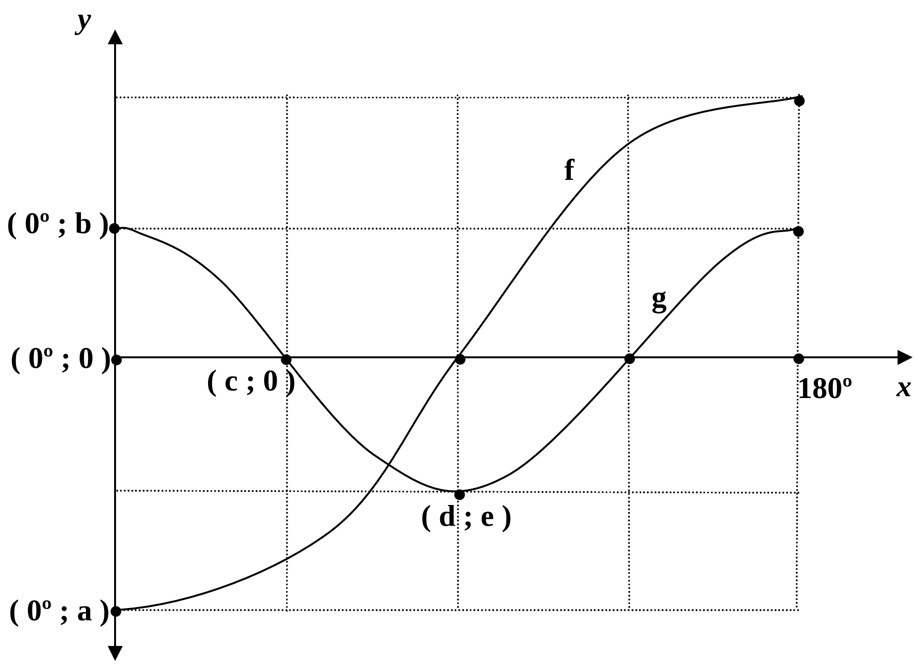
$$\frac{\sin(180° + x) \,.\, \tan 135°}{\operatorname{cosec}(90° - x) \,.\, \cos(360° - x)}$$ (6)

[19]

QUESTION 4

Sketch graphs of the curves of f and g are shown in the diagram below:

Given: $f(x) = -2\cos x$ and $g(x) = \cos 2x$ for $x \in [0°\,;180°]$



4.1 Determine the numerical values of a , b , c , d and e. (5)

4.2 Determine the value(s) of x, for which:

4.2.1 $g(x) - f(x) = 3$ for $x \in [0°\,;180°]$ (1)

4.2.2 $f(x) < 0$ for $x \in [0°\,;180°]$ (3)

4.2.3 $f(x)\,.\,g(x) > 0$ for $x \in [0°\,;135°]$ (3)

[12]

QUESTION 5

5.1 Use fundamental trigonometric identities and NOT a sketch to simplify the following:

$$(\tan^2\theta + 1)(1 - \sin^2\theta)$$ (4)

5.2 Solve for α, rounded off to TWO decimal digits if

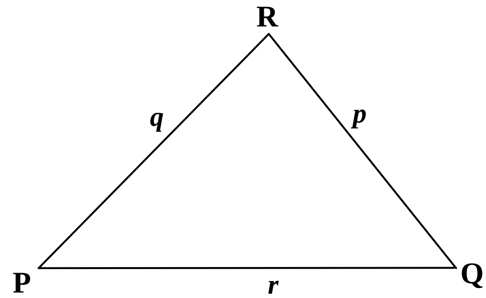
$$\sin 2\alpha = -0{,}4 \quad \text{for} \quad 2\alpha \in [0°\,;270°]$$ (4)

[8]

QUESTION 6

6.1 In the diagram alongside, ΔPQR is an acute–angled triangle.

Use the diagram on the diagram sheet, or redraw the diagram in your answer book to prove that:



$$p^2 = q^2 + r^2 - 2(q)(r)\cos P$$

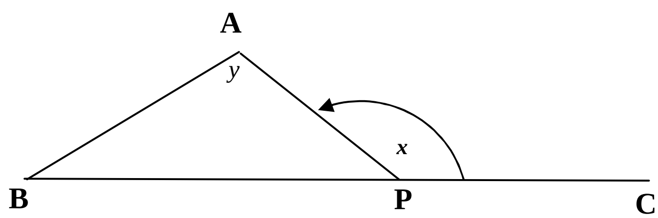
(6)

6.2 In ΔDEF, $d = 5$ units, $e = 8$ units and $f = 12$ units.

6.2.1 Calculate the size of $\hat{F}$, rounded off to TWO decimal digits. (5)

6.2.2 Hence, calculate the area of ΔDEF, rounded off to TWO decimal digits. (3)

6.3 In the diagram alongside, A, B, P and C are the positions of four players on a sports field.



B, P and C are in a straight line.

$A\hat{P}C = x$

$\hat{A} = y$

6.3.1 Express, without reasons, $A\hat{P}B$ in terms of x. (1)

6.3.2 Prove that the distance between players A and B is given by

$$AB = \frac{BP \cdot \sin x}{\sin y}$$ (4)

6.3.3 If $BP = 50$ m, $x = 150°$ and $\hat{B} = 30°$,
calculate, **without using a calculator**, the distance AB.
(Leave the answer in surd form.) (5)

[24]

EUCLIDEAN GEOMETRY

NOTE:
- **DIAGRAMS FOR PROVING THEORY MAY BE USED ON THE DIAGRAM SHEETS OR REDRAWN IN YOUR ANSWER BOOK.**
- **DETACH THE DIAGRAM SHEETS FROM THE QUESTION PAPER AND PLACE THEM IN YOUR ANSWER BOOK.**
- **GIVE A REASON FOR EACH STATEMENT, UNLESS OTHERWISE STATED.**

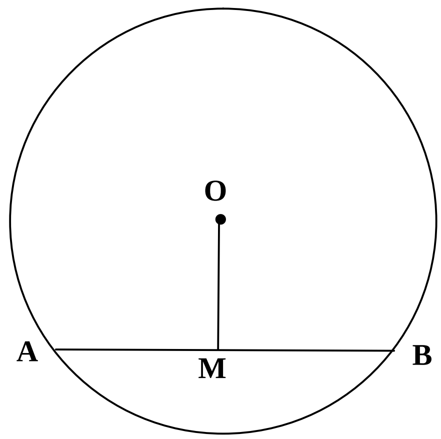
QUESTION 7

7.1 In the diagram alongside, AB is a chord of a circle with centre O.

Use the diagram on the diagram sheet, or redraw the diagram in your answer book to prove the theorem which states that:

If M is the midpoint of AB

then OM ⊥ AB.



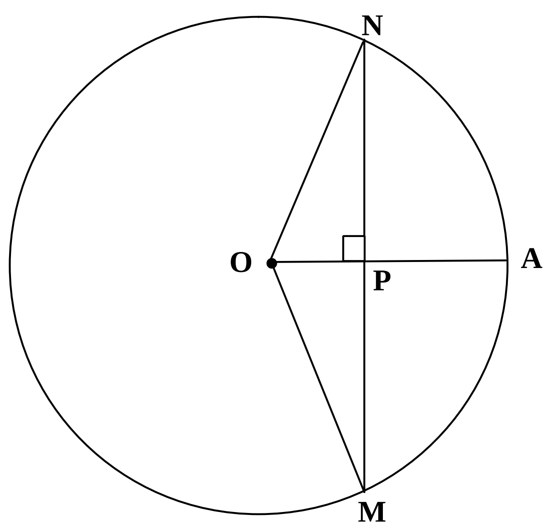
(5)

7.2 In the diagram alongside, O is the centre of circle NAM and

OPA ⊥ MPN.

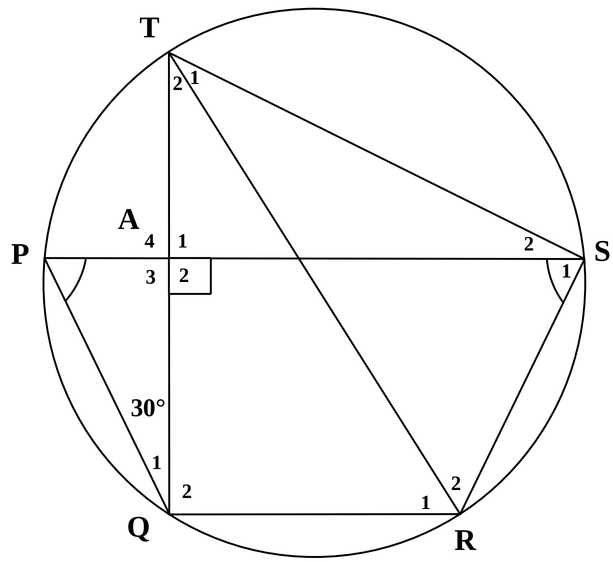
MN = 48 units

OP = 7 units



Calculate, with reasons, the length of PA. (5)

7.3 In the diagram alongside,
TR is a chord of circle PQRST.



QAT $\perp$ PAS

$\hat{Q}_1 = 30°$

$\hat{P} = \hat{S}_1$

In the following questions, give a reason for each statement:

7.3.1 Name THREE angles each equal to 60°. (4)

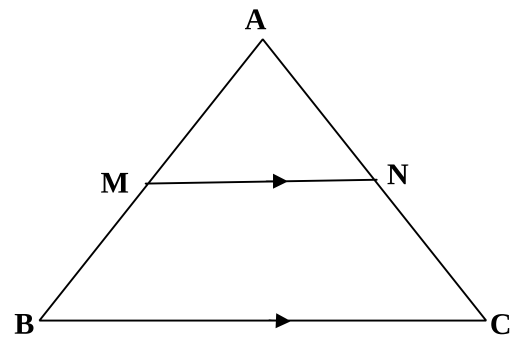
7.3.2 Calculate the size of $Q\hat{R}S$. (2)

7.3.3 Prove that PS || QR. (2)

7.3.4 Prove that TR is a diameter of the circle. (3)
[21]

QUESTION 8

8.1 In the diagram alongside, M and N are points on sides AB and AC respectively of ΔABC.



Use the diagram on the diagram sheet or redraw the diagram in your answer book to prove the theorem which states that:

If MN || BC, then $\mathbf{\frac{AM}{MB} = \frac{AN}{NC}}$

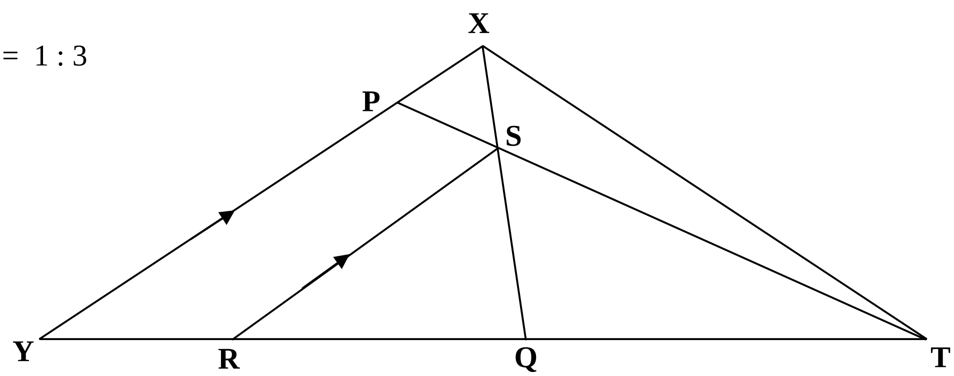
(7)

8.2 In the diagram below, XYT is a triangle with Q the midpoint of YT.

P is a point on XY such that XQ and PT intersect at S.

SR || XY with R on YT.

XS : XQ = 1 : 3



Determine, with reasons, the numerical value of:

8.2.1 $\frac{YR}{RQ}$ (3)

8.2.2 $\frac{TS}{TP}$ (4)

[14]

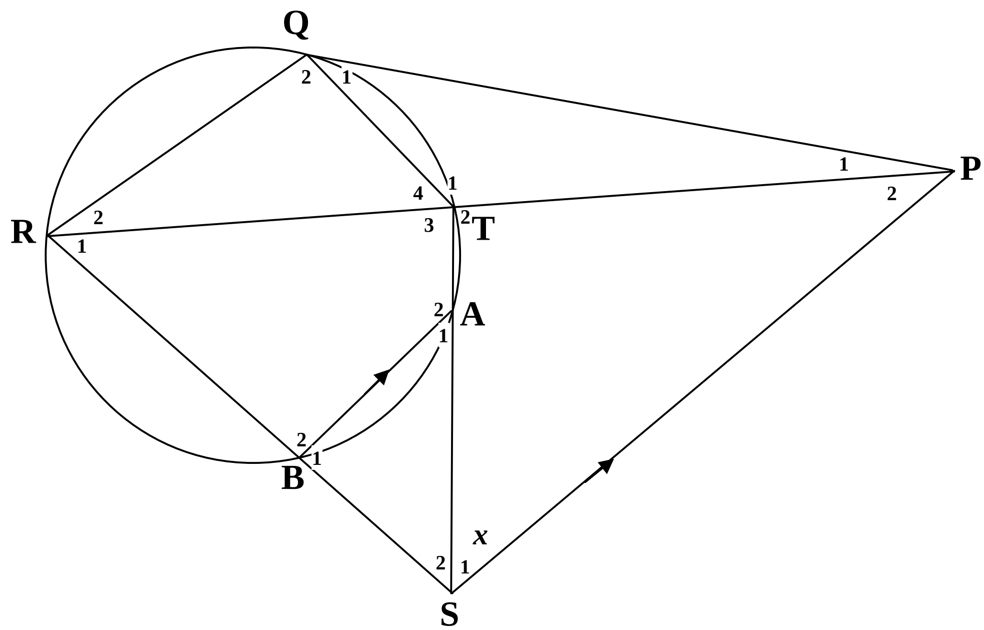
QUESTION 9

In the diagram below, PQ is a tangent to circle QRBAT .

RT is produced to meet tangent QP at P.

TA and RB are produced to meet at S.

SP || BA.



Let $\hat{S}_1 = x$

9.1 Name, with reasons, TWO other angles each equal to x. (3)

9.2 Prove that $\Delta PTS\ |||\ \Delta PSR$. (3)

9.3 9.3.1 Prove that $\Delta PQT\ |||\ \Delta PRQ$. (4)

9.3.2 Hence, show that $PQ^2 = PR \cdot PT$ (1)

9.4 Hence, show that $PQ = PS$ (3)

[14]

TOTAL: 150

Mathematics Formula Sheet (HG and SG)
Wiskundeformuleblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}(a + T_n)$ $S_n = \frac{n}{2}(a + \ell)$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = a.r^{n-1}$ $S_n = \frac{a(1-r^n)}{1-r}$ $(r \neq 1)$ $S_n = \frac{a(r^n - 1)}{r-1}$ $(r \neq 1)$

$S_\infty = \frac{a}{1-r}$ $(|r| < 1)$

$A = P\left(1 + \frac{r}{100}\right)^n$ $A = P\left(1 - \frac{r}{100}\right)^n$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$y = mx + c$

$y - y_1 = m(x - x_1)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \tan\theta$

$(x_3 ; y_3) = \left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$x^2 + y^2 = r^2$

$(x-p)^2 + (y-q)^2 = r^2$

In Δ ABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

area $\Delta ABC = \frac{1}{2} ab.\sin C$