education

Department of Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATION - 2006

MATHEMATICS P2

HIGHER GRADE

OCTOBER/NOVEMBER 2006

MARKS: 200

TIME: 3 hours

This question paper consists of 11 pages, a formula sheet and 5 diagram sheets.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions:

1. This question paper consists of 9 questions, a formula sheet and 5 diagram sheets.
2. Use the formula sheet to answer this question paper.
3. Detach the diagram sheets from the question paper and place them inside your ANSWER BOOK.
4. The diagrams are not drawn to scale.
5. Answer ALL the questions.
6. Number ALL the answers correctly and clearly.
7. ALL the necessary calculations must be shown.
8. Non–programmable calculators may be used, unless otherwise stated.
9. The number of decimal digits to which answers must be rounded off will be stated in the question where necessary.

ANALYTICAL GEOMETRY

NOTE:
- **USE ANALYTICAL METHODS IN THIS SECTION.**
- **CONSTRUCTION AND MEASUREMENT METHODS MAY NOT BE USED.**

QUESTION 1

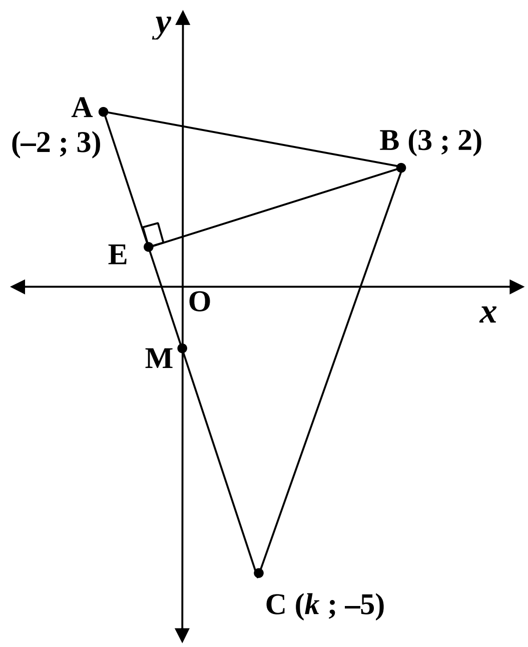
In the diagram alongside,

A (– 2 ; 3), B (3 ; 2) and C (k ; –5)

are three points in a Cartesian plane.

M, the midpoint of AC, lies on the y–axis.

BE ⊥ AC, with E a point on AC.



1.1 Write down the value of k. (1)

1.2 Calculate the size of $\hat{A}$, rounded off to ONE decimal digit. (7)

1.3 Determine the equation of altitude BE. (4)

1.4 Determine the co-ordinates of E. (6)

1.5 Calculate the area of Δ ABM. (6)
[24]

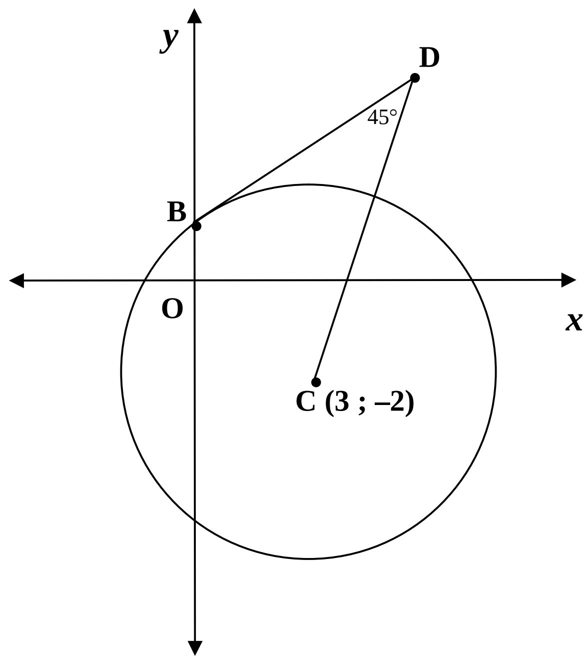
QUESTION 2

2.1 In the diagram alongside, BD is a tangent to the circle at point B, which lies on the y–axis.

The centre of the circle is C (3 ; –2).

The equation of tangent BD is given by $3x - 4y + 8 = 0$

$B\hat{D}C = 45°$



2.1.1 Determine the co-ordinates of B. (2)

2.1.2 Show that $x^2 - 6x + y^2 + 4y - 12 = 0$ is the equation of the circle . (4)

2.1.3 Determine the value(s) of q if $x + q = 0$ is the equation of a tangent to the circle. (4)

2.1.4 (a) Write down the length of BD. (1)

(b) Hence, determine the co-ordinates of D. (6)

2.1.5 Determine whether points E (2 ; – 9), C and D are collinear. (3)

2.2 Determine the equation of the locus of point P (x ; y), if the distance from P to R (1 ; – 4) is equal to two times the distance from P to T (–2 ; –1). (7)

[27]

TRIGONOMETRY

QUESTION 3

Answer this question without the use of a calculator.

3.1 If $\sec 751° = k$, express each of the following in terms of k:

3.1.1 Cos 31° (2)

3.1.2 2 cosec (-121°) (3)

3.1.3 tan 329° (3)

3.2 Simplify:

$$\sqrt{\tan(-207°)\,.\,\cot 333° - \frac{\sin^2(x-360°)\,.\,\operatorname{cosec}(x-90°)}{\cos x}}$$ (10)

[18]

QUESTION 4

Given: $f(x) = \cos\frac{1}{2}x$ and $g(x) = \sin(x + 60°)$

4.1 Solve for x if $\sin(x + 60°) = \cos\frac{1}{2}x$ and $x \in [-60° ; 300°]$ (7)

4.2 Use the set of axes provided on the diagram sheet to draw sketch graphs of the curves of f and g for $x \in [-60° ; 300°]$. Show clearly the co-ordinates of all turning points and end points and the intercepts with the axes. (10)

4.3 Use the solution obtained in QUESTION 4.1 as well as the graph drawn in QUESTION 4.2 to determine the value(s) of $x \in [-60° ; 300°]$ for which:

4.3.1 $f(x) < g(x)$ (3)

4.3.2 $f(x)\,.\,g(x) \leq 0$ (3)

[23]

QUESTION 5

5.1 Determine the general solution of the equation:

$$2\sin x + \operatorname{cosec} x - 3 = 0$$ (9)

5.2 5.2.1 Write down an expression for $\cos 2\theta$ in terms of $\cos\theta$. (1)

5.2.2 Prove the identity:

$$2\cos\theta \cdot \cos 2\theta + \sec\theta \cdot \sin^2 2\theta = 2\cos\theta$$ (7)

5.3 5.3.1 Write down an expression for $\tan(A + B)$ in terms of $\tan A$ and $\tan B$ (1)

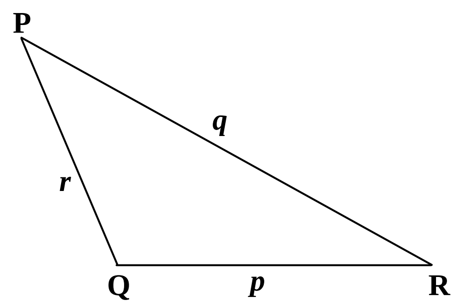
5.3.2 Hence, if $\tan(A + B) = 4$ and $\tan A = 1$, calculate, without the use of a calculator, the numerical value of $\tan B$. (4)

[22]

QUESTION 6

6.1 In the diagram alongside Δ PQR is shown.

Use the diagram on the diagram sheet or redraw the diagram in your answer book, to prove that:



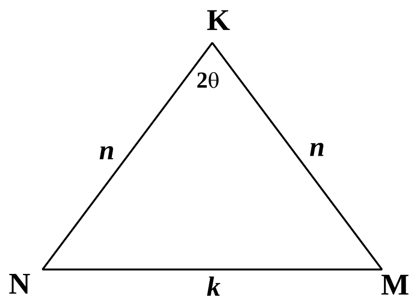
$$\frac{\sin Q}{q} = \frac{\sin P}{p}$$ (3)

6.2 In the diagram alongside, Δ KMN is given with $\hat{K} = 2\theta$, and

KM = KN = n units

NM = k units

Prove that $k = 2n.\sin\theta$ (5)



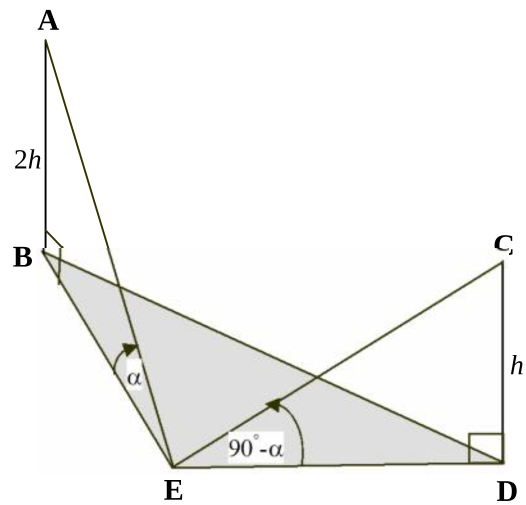
6.3 In the diagram alongside, points B, D and E lie in the same horizontal plane with $B\hat{E}D = 120°$.

AB and CD are two vertical towers.

AB = 2CD = $2h$ metres.

The angle of elevation of A from point E is α.

The angle of elevation of C from point E is $(90° - \alpha)$.



6.3.1 Determine the length of BE in terms of h and α. (2)

6.3.2 Prove that the distance between the bases of the two towers is given by:

$$BD = \frac{h\sqrt{\tan^4\alpha + 2\tan^2\alpha + 4}}{\tan\alpha}$$ (8)

6.3.3 Hence, determine the height of tower CD, rounded off to the nearest metre, if $\alpha = 48°$ and BD = 509 m. (4)

[22]

EUCLIDEAN GEOMETRY

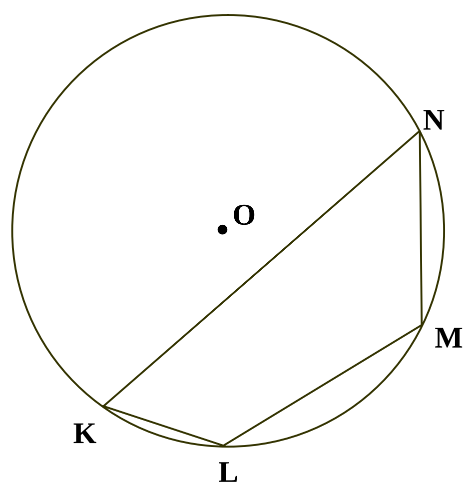
NOTE:
- **DIAGRAMS FOR PROVING THEORY MAY BE USED ON THE DIAGRAM SHEETS OR REDRAWN IN YOUR ANSWER BOOK.**
- **DETACH THE DIAGRAM SHEETS FROM THE QUESTION PAPER AND PLACE THEM IN YOUR ANSWER BOOK.**
- **GIVE A REASON FOR EACH STATEMENT, UNLESS OTHERWISE STATED.**

QUESTION 7

7.1 In the diagram below, circle KLNM is drawn.

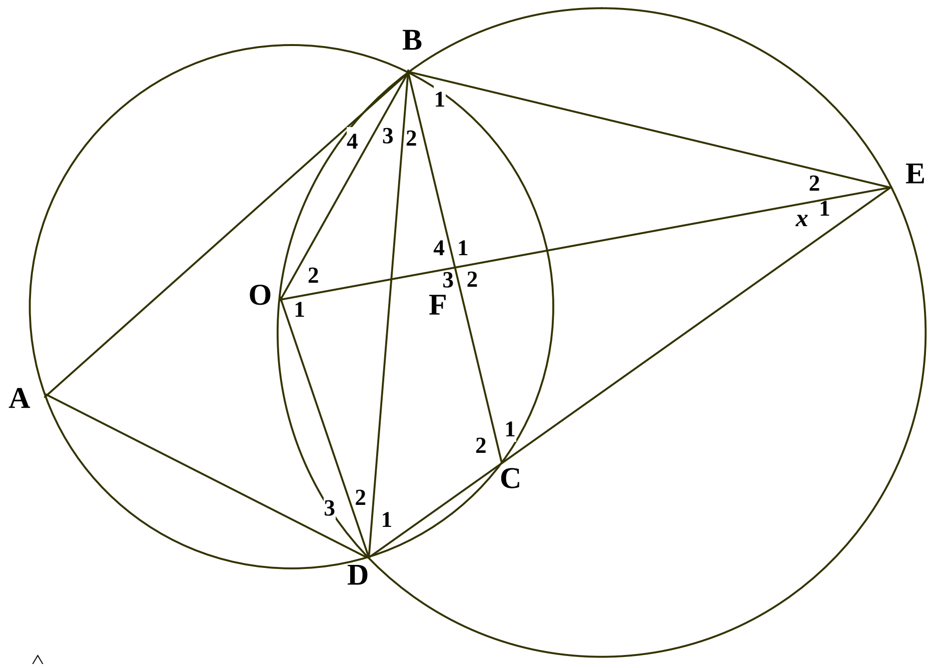
Use the diagram on the diagram sheet or redraw the diagram in your answer book to prove the theorem which states that:

If O is the centre of the circle, then $\hat{L} + \hat{N} = 180°$



(6)

7.2 In the diagram below, O is the centre of circle ABCD.
DC is produced to meet circle BODE at E.
OE intersects BC at F.



Let $\hat{E}_1 = x$

7.2.1 Determine the size of $\hat{A}$ in terms of x. (6)

7.2.2 Prove that:

(a) BE = EC (7)

(b) BE is NOT a tangent to circle ABCD (3)

[22]

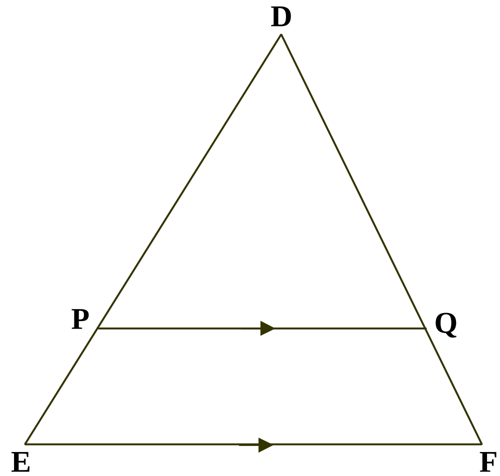
QUESTION 8

8.1 In the diagram alongside,

P is a point on DE and Q is a point on DF of Δ DEF.

Use the diagram on the diagram sheet or redraw the diagram in your answer book to prove the theorem which states that:

If PQ || EF, then $\frac{\mathbf{DP}}{\mathbf{DE}} = \frac{\mathbf{DQ}}{\mathbf{DF}}$



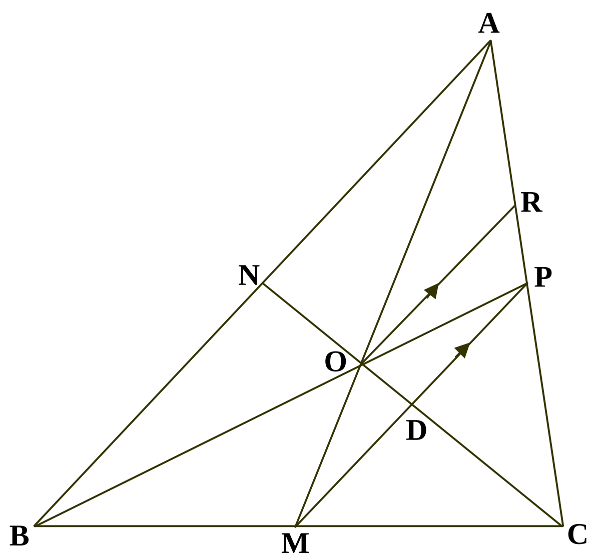
(7)

8.2 In the diagram alongside, medians AM and CN of Δ ABC, intersect at O.

BO is produced to cut AC at P.

MP and CN intersect at D.

OR || MP with R on AC.



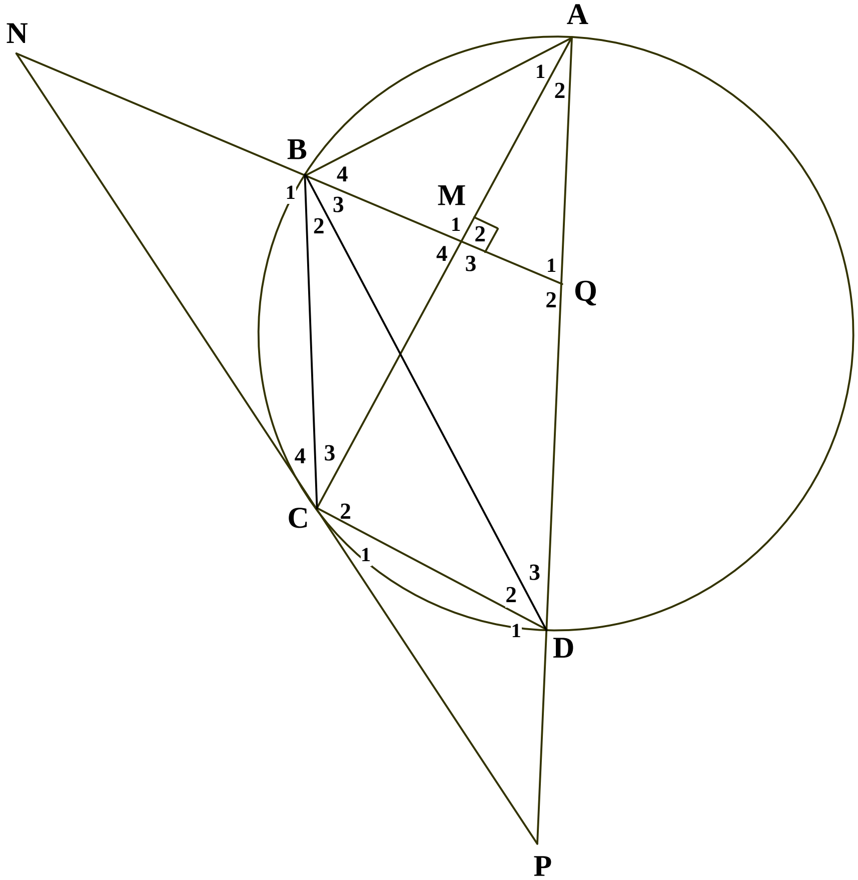
8.2.1 Calculate, with reasons, the numerical value of $\frac{ND}{NC}$ (6)

8.2.2 Use AO : AM = 2 : 3 , to calculate the numerical value of $\frac{RP}{PC}$ (5)

[18]

QUESTION 9

In the diagram below AD is the diameter of circle ABCD.
AD is produced to meet tangent NCP at P.
Straight line NB is produced to Q and intersects
AC at M with Q on ADP.
AC $\perp$ NQ at M.



9.1 Prove that NQ $\parallel$ CD. (3)

9.2 Prove that ANCQ is a cyclic quadrilateral. (4)

9.3 9.3.1 Prove that Δ PCD $|||$ Δ PAC (3)

9.3.2 Hence, complete the statement: $PC^2 = \ldots$ (2)

9.4 Prove that $BC^2 = CD \,.\, NB$ (7)

9.5 If it is further given that PC = MC, prove that

$$1 - \frac{BM^2}{BC^2} = \frac{AP \,.\, DP}{CD \,.\, NB}$$ (5)

[24]

TOTAL: 200

Mathematics Formula Sheet (HG and SG)
Wiskundeformuleblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}(a + T_n)$ $\quad S_n = \frac{n}{2}(a + \ell)$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = a.r^{n-1}$ $\quad S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$ $\quad S_n = \frac{a(r^n - 1)}{r-1} \quad (r \neq 1)$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

$A = P\left(1 + \frac{r}{100}\right)^n$ $\quad A = P\left(1 - \frac{r}{100}\right)^n$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$(x_3 ; y_3) = \left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

$$x^2 + y^2 = r^2$$

$$(x-p)^2 + (y-q)^2 = r^2$$

In Δ ABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta ABC = \tfrac{1}{2} ab.\sin C$$