education

Department:
Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATION - 2006

MATHEMATICS P1

HIGHER GRADE

OCTOBER/NOVEMBER 2006

MARKS: 200

TIME: 3 hours

This question paper consists of 10 pages, 1 sheet of graph paper and 1 formula sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions:

1. This question paper consists of 8 questions. Answer ALL the questions.
2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining the answers.
3. An approved calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
5. The attached graph paper must be used only for QUESTION 8.
6. Number the answers EXACTLY as the questions are numbered.
7. Diagrams are not necessarily drawn to scale.
8. It is in your own interest to write legibly and present the work neatly.
9. A formula sheet is included at the end of the question paper.

QUESTION 1

1.1 Given: $5x(x-2)=2$

1.1.1 Prove that the equation does not have rational roots. (4)

1.1.2 Solve the equation for x, correct to TWO decimal places. (4)

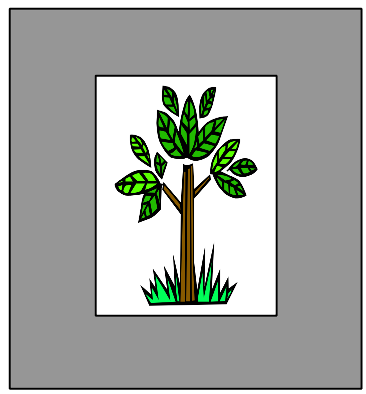
1.2 Solve for x:

1.2.1 $(2x-5)^2-49x^2=0$ (4)

1.2.2 $|x-5|>|4-8|$ (5)

1.2.3 $\frac{3x}{x-3}\geq 4$ (6)

1.3 A rectangular photograph has a breadth of x cm and a length of y cm. There is a border with a constant width of 2 cm around the photograph. The area of the photograph is 540 cm^2 and the area of the border is 208 cm^2.



1.3.1 Prove that $x+y=48$. (3)

1.3.2 If $x<y$, calculate the values of x and y. (6)

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QUESTION 2

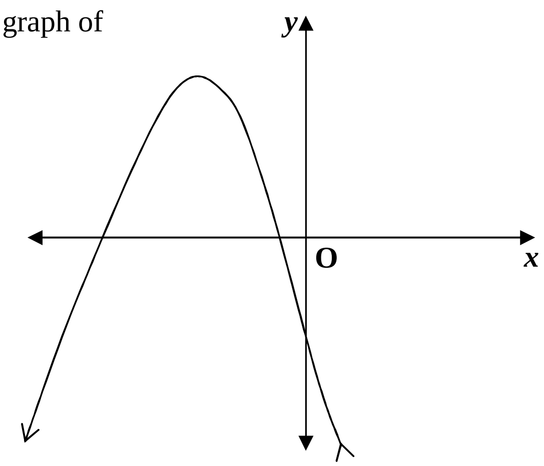
2.1 Given: $f(x) = |x - 2|$ and $g(x) = x - 2$

2.1.1 Draw sketch graphs of f and g on the same set of axes. Clearly indicate the co-ordinates of all intercepts with the axes. (6)

2.1.2 Use the graphs to determine the values of x for which $f(x) - g(x) = 0$. (2)

2.1.3 Give the equation of the graph which is symmetrical to f with respect to the line $y = 0$. (2)

2.2 The accompanying figure shows the graph of $f(x) = -x^2 - 6x - 4$.



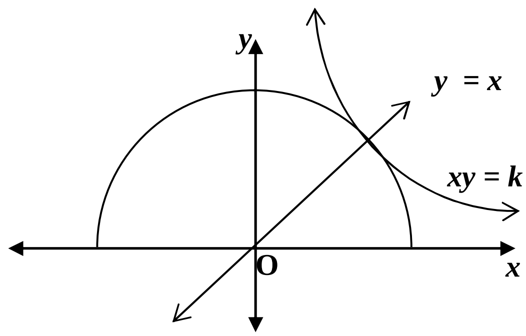
2.2.1 Change the equation to the form $f(x) = -(x - p)^2 + q$. (4)

2.2.2 Hence, prove that $f(x) \leq 5$ for all values of x. (2)

2.2.3 Determine, with the aid of the graph, the values of k for which $-x^2 - 6x - 4 = k$ has roots which are unequal, negative and real. (5)

2.2.4 Determine, using QUESTION 2.2.1 or otherwise, three positive integral values of t for which $-x^2 - 6x - 4 = t$ has rational roots. (5)

2.3 The semi-circle $y = \sqrt{9 - x^2}$ and the hyperbola $xy = k$ touch only at one point. This point lies on the line $y = x$.



Calculate the value of k. (5)

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QUESTION 3

Given: $p(x) = 2x^3 + x^2 - 2m^2 x - 3m$

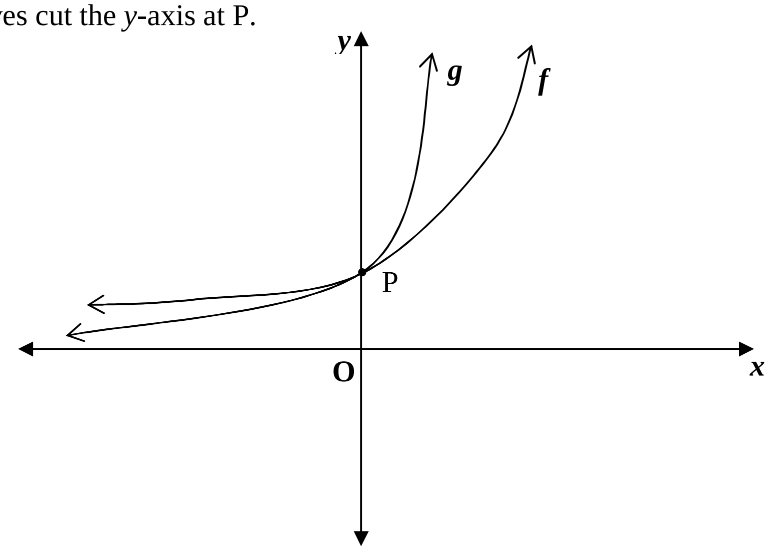
3.1 Calculate the value of m for which both $(x + 3)$ and $(x - 3)$ are factors of $p(x)$. (9)

3.2 If m has the value found in QUESTION 3.1, factorise $p(x)$ completely. (3)

[12]

QUESTION 4

4.1 The sketch graph below shows the curves of $f(x) = a^x$ and $g(x) = 5^x$.
The curves cut the y-axis at P.



4.1.1 Write down the co-ordinates of P. (1)

4.1.2 Determine ALL possible values of a. (3)

4.1.3 Draw a sketch graph of g^{-1}, the inverse of g. Indicate the co-ordinates of any intercepts with the axes. (3)

4.1.4 Write down the values of x for which $\log_5 x < 0$. (2)

4.2 Given: $\log M = p$

Prove that:

4.2.1 $\dfrac{15.5^{p-1} + 5^{p+1}}{2^{-p}} = 8M$ (5)

4.2.2 $\log 2.\log_2 5.\log_{25} M = \frac{1}{2} p$ (4)

4.3 Solve for x:

4.3.1 $\sqrt{3}\, x^{\frac{3}{4}} - \sqrt{24} = 0$ (Without the use of a calculator.) (4)

4.3.2 $2^{2x+1} - 2^x = 3$ (Round off answers to TWO decimal places.) (7)

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QUESTION 5

5.1 Prove that the sum to n terms of an arithmetic series with first term a and common difference d is given by $S_n = \frac{n}{2}[2a + (n-1)d]$. (4)

5.2 A fitness test requires athletes to repeatedly run a distance of 20 metres. They complete the distance 5 times in the first minute, 6 times in the second minute and 7 times in the third minute. They continue in this manner, increasing the number of repetitions by 1 in each successive minute. Calculate after how many minutes the athletes will have run a total of 2 200 metres. (6)

5.3 The first three terms of a geometric series are $m+2$, m and $2m-3$.

5.3.1 Calculate the values of m. (5)

5.3.2 Determine the value of m for which the series converges. (3)

5.3.3 Write down the first THREE terms of the convergent series. (2)

5.3.4 Calculate the sum to infinity of the convergent series. (2)

5.4 The first term of a geometric series is 1 and the common ratio is 3. Calculate the smallest value of n for which the sum of the first n terms is greater than 100 000. Show the necessary calculations. (6)

5.5 Given: $\sum_{k=1}^{n} T_k = n^3$ where T_k is the k^{th} term of the series.

Calculate the 4^{th} term of the series. (4)

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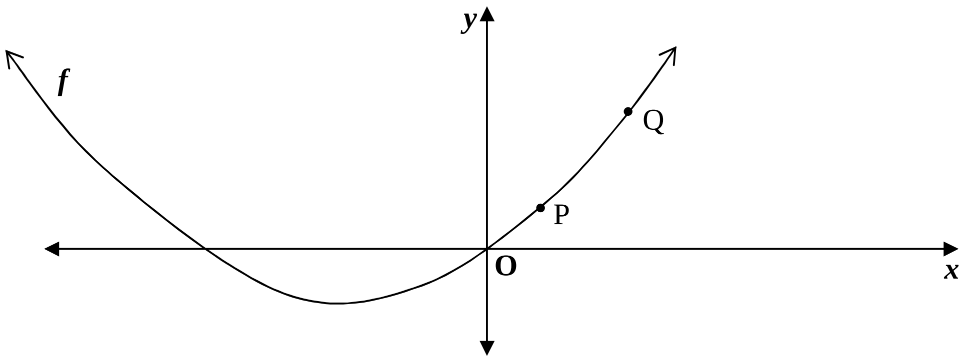
QUESTION 6

6.1 The diagram below shows the graph of $y = f(x)$.
$P(x; f(x))$ and $Q(x+h; f(x+h))$ are points on the graph.
The gradient of the straight line through P and Q
is given by $m = \dfrac{f(x+h) - f(x)}{(x+h) - x}$.



6.1.1 Which line has a gradient given by $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$? (2)

6.1.2 Calculate the gradient of PQ in terms of h and x if $f(x) = \frac{1}{2}x^2 + x$. (4)

6.1.3 Hence, determine the value of $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$. (2)

6.1.4 Determine whether $f'(1+2) = f'(1) + f'(2)$ for the function in QUESTION 6.1.2. Show ALL calculations. (4)

6.2 Given: $g(x) = \dfrac{-2x}{\sqrt{x}} - x^{10}$ and $h(x) = (x^5 + 5x^{-1})(x^5 - 5x^{-1})$

Determine:

6.2.1 $g'(x)$ (3)

6.2.2 $h'(x)$ (3)

6.2.3 $\dfrac{d}{dx}\left[2g(x) + h(x)\right]$ (4)

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QUESTION 7

7.1 Given: $f(x) = x^3 - 9x^2 + 24x$

7.1.1 Show by calculation that P(5 ; 20) is a point on the graph of $y = f(x)$. (2)

7.1.2 Calculate the co-ordinates of the turning points of the graph of *f*. (6)

7.1.3 Draw a neat sketch graph of *f*. Indicate the co-ordinates of any intercepts with the axes and of the turning points. (5)

7.1.4 If $x \in [0 ; 5]$, state:

(a) The maximum value of $f(x)$ on the interval (1)

(b) The values of x for which this maximum is attained (3)

(c) The minimum value of $f(x)$ for the interval (1)

7.2 A clothing manufacturer estimates that the cost (in rands) of producing *x* shirts is given by the function:

$$C(x) = 10 + 5x + 0{,}001x^3$$

Calculate the rate at which the cost is changing when the 100th shirt is being produced. (4)

7.3 There are 40 fruit trees in an orchard. The average yield per tree in a season is 580 fruit. The farmer calculates that for each additional tree planted in the orchard, the yield per tree will drop by 10 fruit.

If the number of additional trees planted in the orchard is x and the total yield of the orchard in a season is N, then:

$$N = (40 + x)(580 - 10x)$$

Calculate how many additional trees must be planted to maximise the total yield of the orchard. (4)

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QUESTION 8

A small business enterprise uses x landline phones and y cellphones. The following constraints apply and are shown on the attached graph paper. The feasible region is shaded.

- The small business enterprise needs at least n phones.
- At least j of these phones must be landline phones.
- At least m must be cellphones.
- The monthly contract charge for a landline phone is p rands per month and for a cell-phone q rands per month. The company has a budget of at most R960 per month available to pay such contract charges.

8.1 Make use of the graph to determine the values of n, j, m, p and q. (8)

8.2 The monthly cost for calls is R500 per landline phone and R700 per cellphone. Write down an equation which expresses the total monthly cost for calls (C) in terms of x and y. (1)

8.3 If the objective is to minimise the monthly call costs, determine the values of x and y for which C is a minimum. (4)

8.4 The objective changes and the company now wants to maximise the use of cell-phones. Determine how many phones of each type should be used to achieve this objective. (3)

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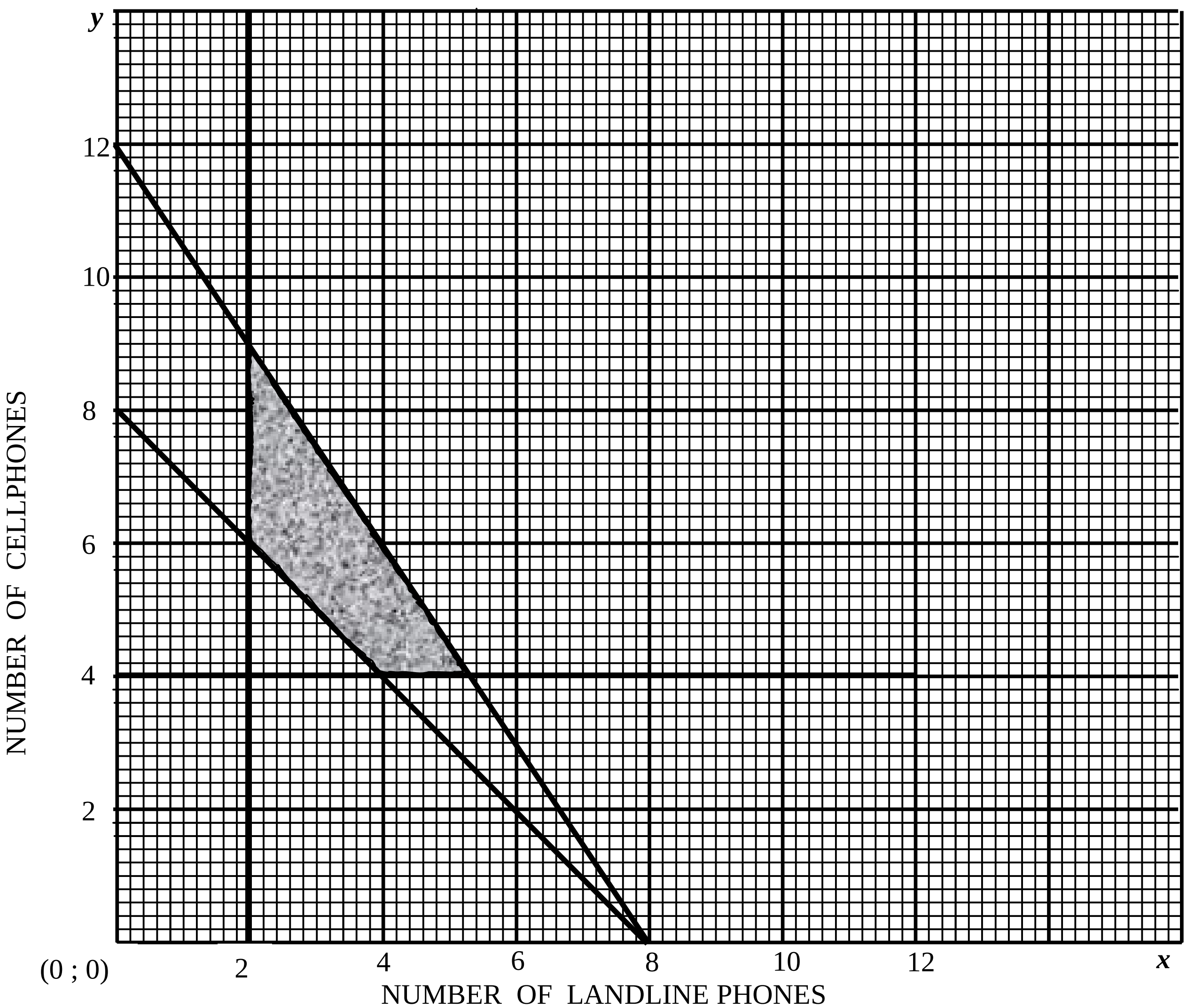
TOTAL: 200

GRAPH PAPER FOR QUESTION 8



Mathematics Formula Sheet (HG and SG)
Wiskunde Formuleblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}(a + T_n) \quad \text{or / of} \quad S_n = \frac{n}{2}(a + l)$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(1-r^n)}{1-r} \ (r \neq 1) \quad S_n = \frac{a(r^n - 1)}{r-1} \ (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

$$A = P\left(1 + \frac{r}{100}\right)^n \quad \text{or/of} \quad A = P\left(1 - \frac{r}{100}\right)^n$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$(x_3; y_3) = \left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$x^2 + y^2 = r^2$$

$$(x-p)^2 + (y-q)^2 = r^2$$

In ΔABC: $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta ABC = \frac{1}{2}ab.\sin C$$