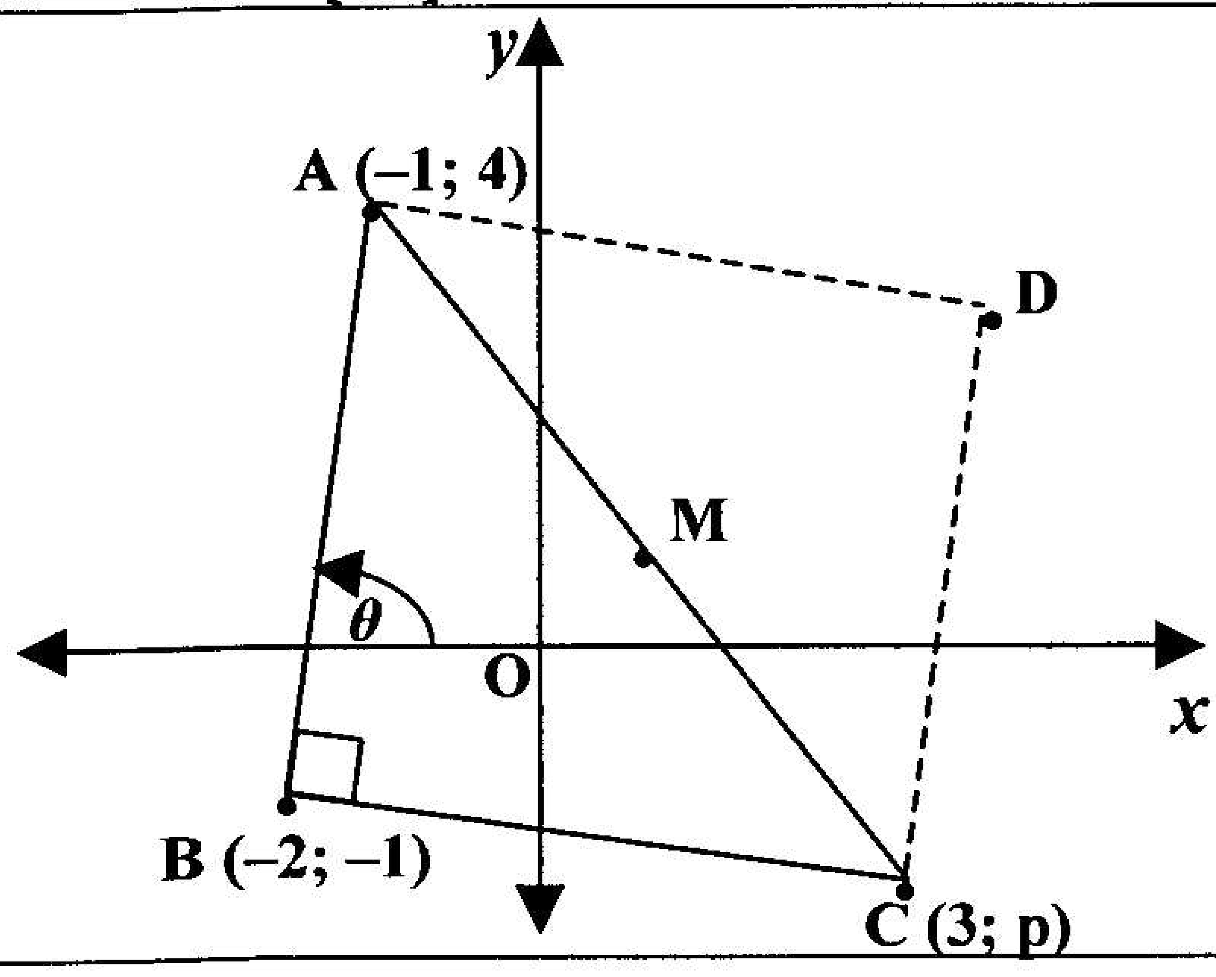
Possible Answers March 2006

MATHEMATICS STANDARD GRADE SECOND PAPER

QUESTION 1 [22]



1.1	$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$ ✓M	use of gradient formula
	$= \frac{4 - (-1)}{-1 - (-2)}$	
	$= 5$ ✓A	calculation of correct gradient
	$\tan\theta = 5$ ✓M	use of inclination formula
	$\theta = 78{,}7°$ ✓CA (4)	answer consistent with m_{AB}

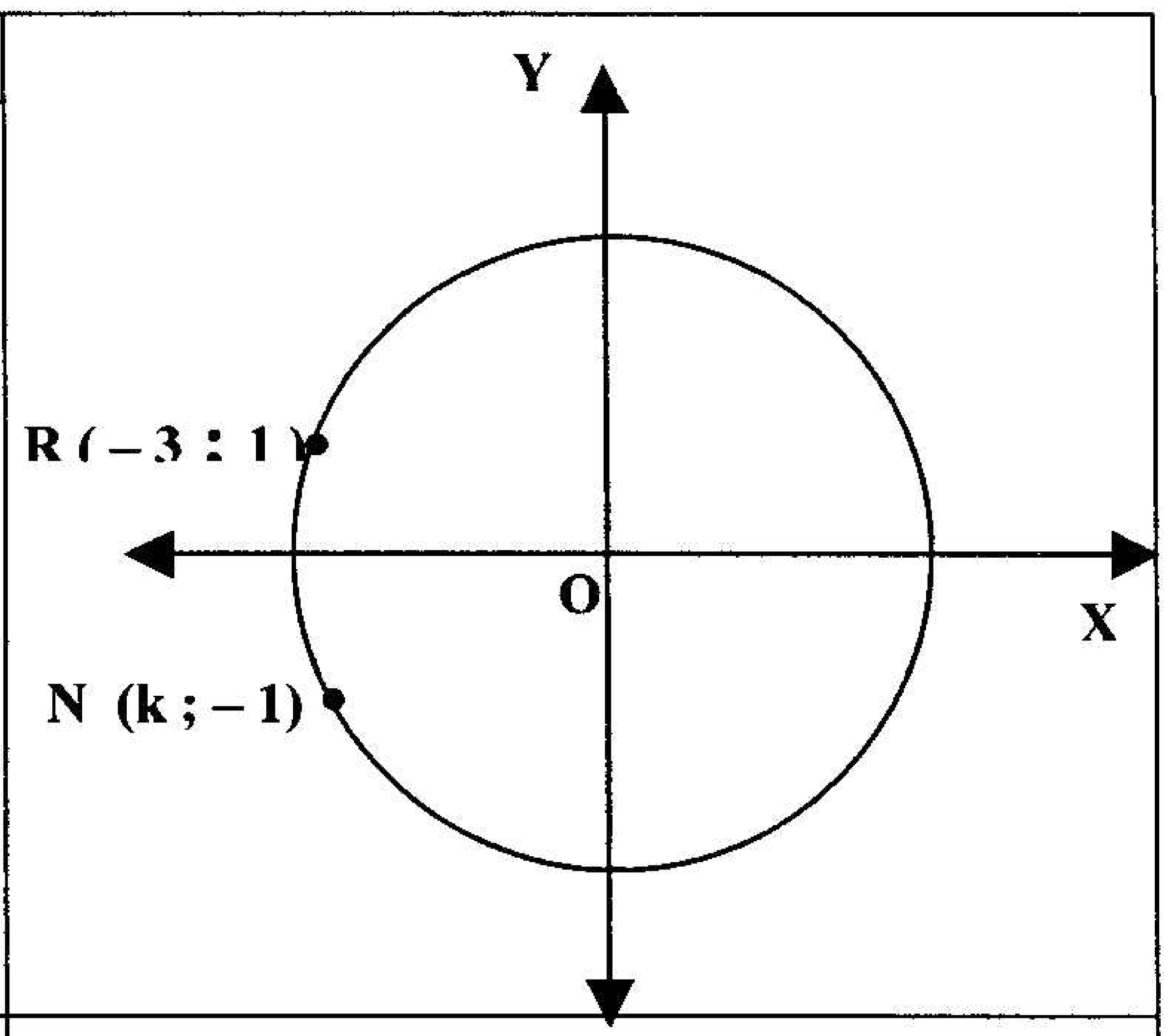
1.2	$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$	
	$\frac{-1}{5} = \frac{p-(-1)}{3-(-2)}$ ✓M ✓M	calculating m_{BC} in terms of p gradient of ⊥
	$\frac{-1}{5} = \frac{p+1}{5}$ ✓A	equating
	$5(p+1) = -5$ ✓CA	simplification
	$p = -2$	
	OR	
	$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$	
	$= \frac{p-(-1)}{3-(-2)}$ ✓M	calculating m_{BC} in terms of p
	$= \frac{p+1}{5}$ ✓A	gradient of ⊥
	$m_{AB} \times m_{BC} = -1$ ✓M	equating
	$5 \times \frac{p+1}{5} = -1$	
	$p + 1 = -1$ ✓A	simplification
	$p = -2$	
	OR	

	$AC^2 = AB^2 + BC^2$ ✓M ✓A $(3+1)^2 + (p-4)^2 = (-2+1)^2 + (-1-4)^2 + (3+2)^2 + (p+1)^2$ $16 + p^2 - 8p + 16 = 1 + 25 + 25 + p^2 + 2p + 1$ ✓M $-10p = 20$ ✓CA $p = -2$ (4)	use of Pythagoras correct substitution multiplying simplification
1.3.1	$M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$ ✓M $M\left(\frac{-1+3}{2} ; \frac{4+(-2)}{2}\right)$ ✓A $M(1 ; 1)$ ✓CA (3)	use of midpoint formula correct substitution answer
1.3.2	$x_M = \frac{x_D + x_B}{2}$ $y_M = \frac{y_D + y_B}{2}$ $1 = \frac{x_D + (-2)}{2}$ ✓M $1 = \frac{y_D + (-1)}{2}$ ✓M $2 = x_D - 2$ $2 = y_D - 1$ $x_D = 4$ ✓A $y_D = 3$ ✓A D(4 ; 3) **OR** $D(r ; t): \frac{t-4}{r+1} \cdot \frac{t+2}{r-3} = -1$ $t^2 - 2t - 8 = -1(r^2 - 2r - 3)$……(1) ✓M $\frac{t-4}{r+1} = -\frac{1}{5}$ $5t - 20 = -r - 1$ ✓M $r = -5t + 19$ …………(2) $t^2 - 2t - 8 = -1[(-5t+19)^2 - 2(-5t+19) - 3]$ $t^2 - 2t - 8 + 25t^2 - 190t + 361 + 10t - 38 - 3 = 0$ $26t^2 - 182t + 312 = 0$ $(\div 26)$ ….. $t^2 - 7t + 12 = 0$ $(t-3)(t-4) = 0$ $y = 3$ or 4 ✓A $t = 3 ; r = -5(3) + 19 = 4$ ✓A $t = 4 ; r = -5(4) + 19 = -1$ D (4 ; 3)	correct use of midpt formula for x_D correct use of midpt. formula for y_D answer for x_D answer for y_D **Answer only – full marks** $m_{AD} \times m_{DC} = -1$ (det. eq. of line using ⊥ lines) $m_{AD} = m_{BC} = -\frac{1}{5}$ (det. eq. of line using // lines) answer for y_D answer for x_D

	OR D (r ;t) : $m_{AD} = m_{BC}$ $\frac{t-4}{r+1} = -\frac{1}{5}$ ✓M $5(t-4) = -1(r+1)$ $5t - 20 = -r - 1$ $r = -5t + 19$ .... (Eq.1) $m_{CD} = m_{AB}$ $\frac{t+2}{r-3} = 5$ ✓A $1(t+2) = 5(r-3)$ $t + 2 = 5r - 15$ $5r - t - 17 = 0$ .... (Eq. 2) subst. (1) into (2) $5(-5t + 19) - t - 17 = 0$ $-25t + 95 - t - 17 = 0$ $-26t = -78$ $t = 3$ ✓A $r = -5(3) + 19 = 4$ ✓A D(4 ; 3) (4)	$m_{AD} = m_{BC} = -\frac{1}{5}$ det. eq. of line using // lines answer for y_D answer for x_D
1.4.1	$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ ✓M $= \sqrt{(-2-(-1))^2 + (-1-4)^2}$ ✓A $= \sqrt{1 + 25}$ $= \sqrt{26}$ ✓CA (3)	use of distance formula correct substitution answer

1.4.2	$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ ✓M	correct use of distance formula
	$= \sqrt{(-2-3)^2 + (-1-(-2))^2}$ ✓A	correct substitution
	$= \sqrt{25 + 1}$	
	$= \sqrt{26}$ ✓CA	answer
	$\therefore$ AB = BC ✓Conclusion $\therefore$ ABCD is a square (adjacent sides =, angles 90°)	conclusion
	OR	
	$mAC = \dfrac{4+2}{-1-3}$ ✓M	correct use of gradient formula
	$= \dfrac{6}{4} = -\dfrac{3}{2}$ ✓A	gradient of AC
	$mBD = \dfrac{3+1}{4+2}$	
	$= \dfrac{4}{6} = \dfrac{2}{3}$ ✓CA	gradient of BD
	mAC x mBD = – 1 ✓Conclusion $\therefore$ ABCD is a square (diag bisect at 90°, angle's 90°,)	conclusion
	(4)	

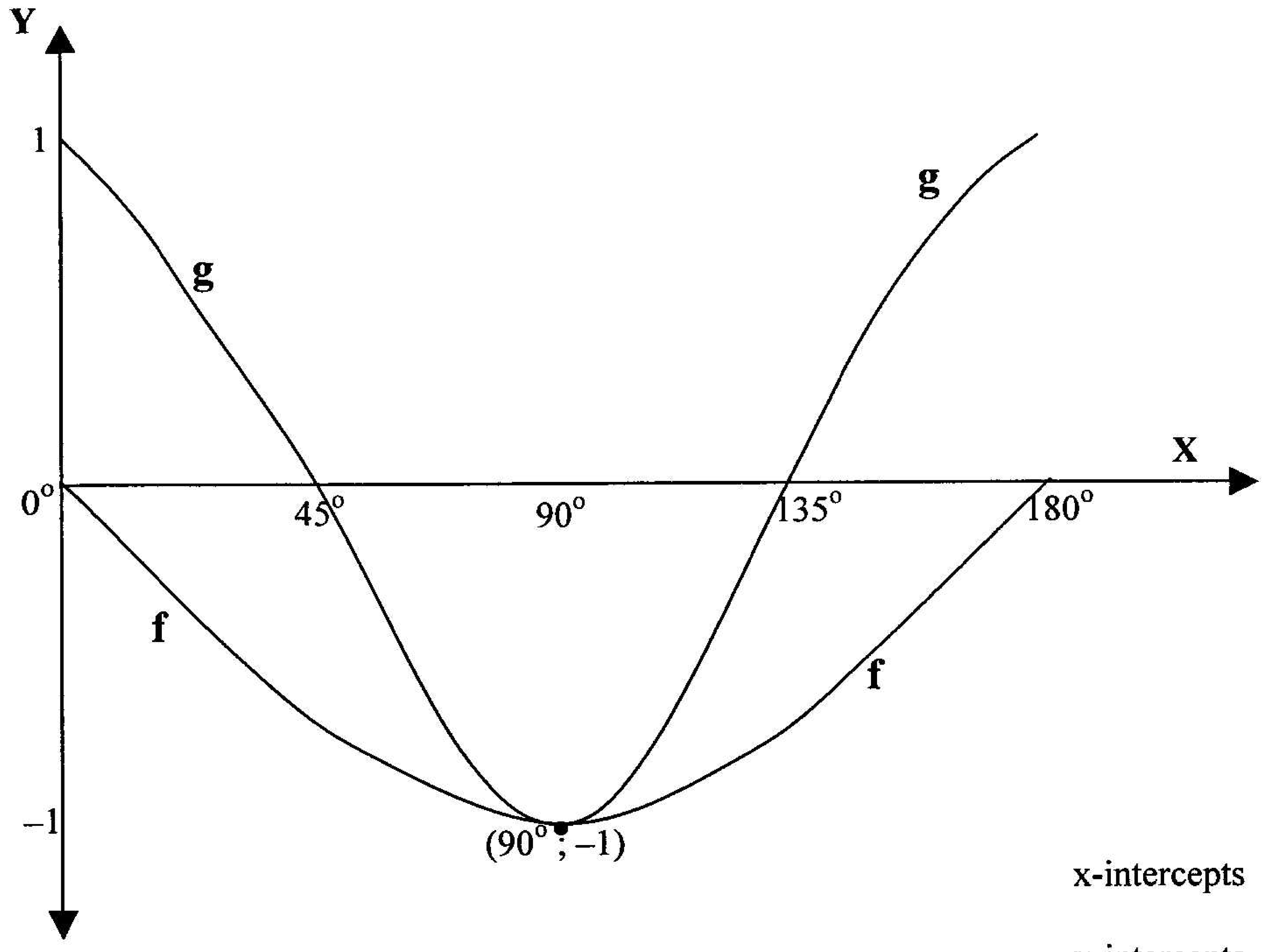
QUESTION 2 [17]

2.1.1	$r^2 = x^2 + y^2$ ✓M $= (1)^2 + (-3)^2$ ✓A $= 1 + 9$ $= 10$ $\therefore x^2 + y^2 = 10$ ✓CA (3)	use of pythagoras correct substitution equation of circle	
2.1.2	$d(NO) = \sqrt{k^2 + 1}$ $k^2 + 1 = 10$ ✓CA $k^2 = 9$ $k = -3$ ✓CA $N(-3\,;-1)$ (2)	sub. into eq. of circle answer **(Correct answer only – full marks)**	
2.1.3	$x = 2$ ✓M ✓CA (2)	correct form of equation of line answer **(Correct answer only – full marks)**	
2.1.4	$m_{OR} = \frac{y_2 - y_1}{x_2 - x_1}$ ✓M $= -\frac{1}{3}$ ✓A $m_{tangent} = 3$ ✓CA (3)	correct sub. into gradient form. answer answer consistent from m_{OP} **(Correct answer only – full marks)**	
2.1.5	$y - y_1 = m(x - x_1)$ ✓M $y - (1) = 3(x + 3)$ ✓A $y - 1 = 3x + 9$ $y = 3x + 10$ ✓CA **OR** $y = mx + c$ ✓M $y = 3x + c$ $1 = 3(-3) + c$ ✓A $c = 10$ $y = 3x + 10$ ✓CA (3)	Use of any eq. of str. line correct subst. answer	
2.2	$m_{AP} \times m_{AB} = -1$ ✓M ✓A $\left(\frac{y+4}{x-2}\right) \times \left(\frac{-4-1}{2-0}\right) = -1$ ✓A $(y + 4)(-5) = -1\,(x - 2)(2)$ $-5y - 20 = -2x + 4$ ✓CA $5y = 2x - 24$ (4)	correct method product of gradients = – 1 correct substitution m_{AP} correct substitution m_{BP} simplification	

QUESTION 3 [16]		
3.1.1	$OP^2 = (-1)^2 + 2^2$ ✓M $= 1 + 4$ $= 5$ $OP = \sqrt{5}$ ✓CA (2)	correct use of Pythagoras answer **(Correct answer only – full marks)**
3.1.2	$\sec^2(180° - \theta)$ $= \sec^2\theta$ ✓A $= \left(-\frac{\sqrt{5}}{1}\right)^2$ ✓CA $= 5$ ✓CA (3)	reduction substitution sec θ simplification
3.2	$\frac{\sin(180° + A).}{\text{cosec}(90° - A).\cos(360° - A)}$ $= \frac{-\sin A}{\sec A \times \cos A}$ ✓A ✓A ✓A $= \frac{-\sin A}{\frac{1}{\cos A} \times \cos A}$ ✓A $= -\sin A$ ✓CA (5)	correct reduction with correct sign application of identities simplification
3.3.1	$\theta = 180° - 45°$ ✓A $= 135°$ ✓CA (2)	special angle answer **(Correct answer only – full marks)**
3.3.2	$\alpha = 30°$ ✓A (1)	answer
3.3.3	$\cot 2(135° - 30°)$ $= \cot(210°)$ ✓CA $= \cot 30°$ ✓CA $= \sqrt{3}$ ✓CA (3)	sub. of θ and α simplification answer

QUESTION 4 [12]

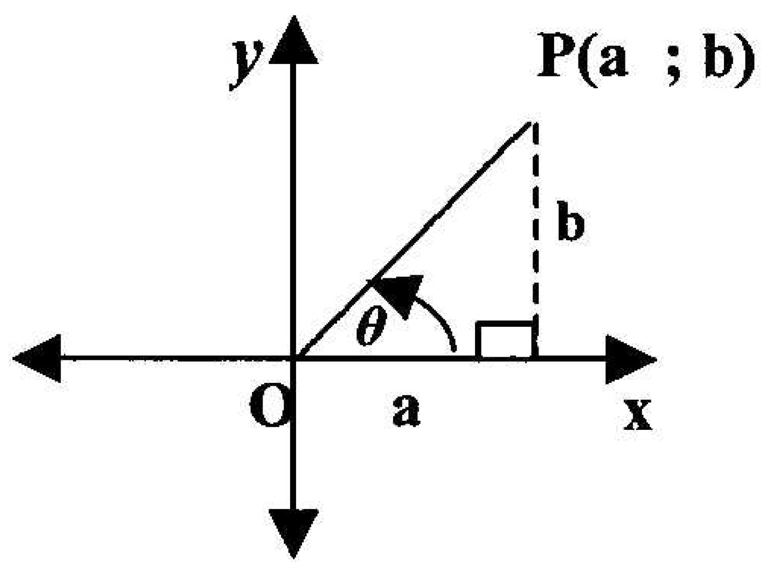
4.1



	f	g
x-intercepts	✓A	✓A
y-intercepts	✓A	✓A
shape	✓A	✓A
turning point	✓A	✓A

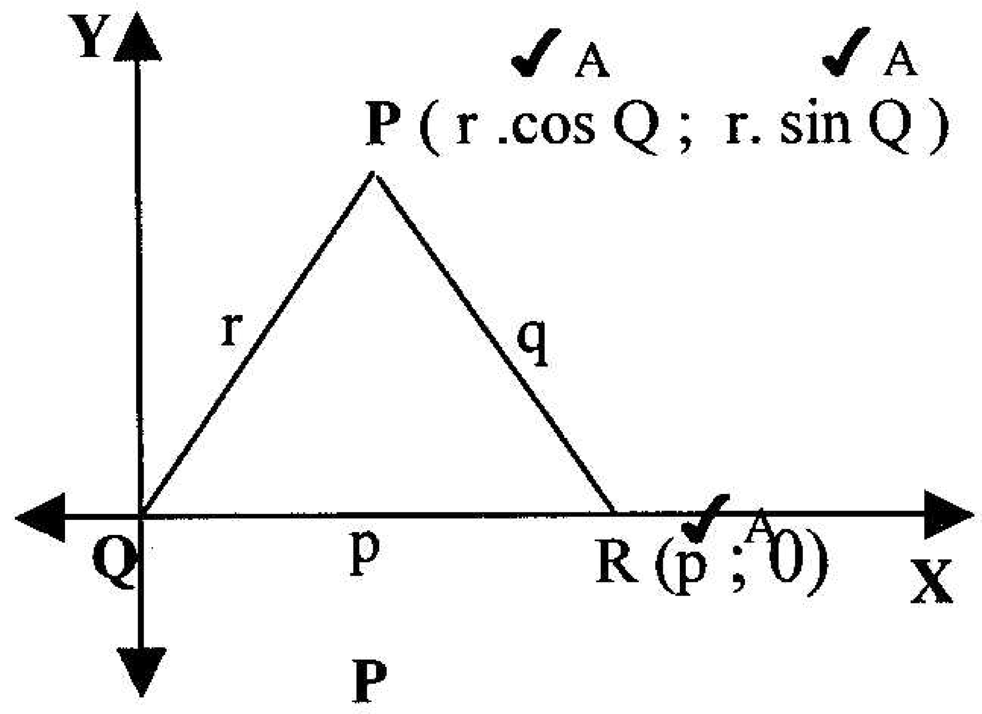
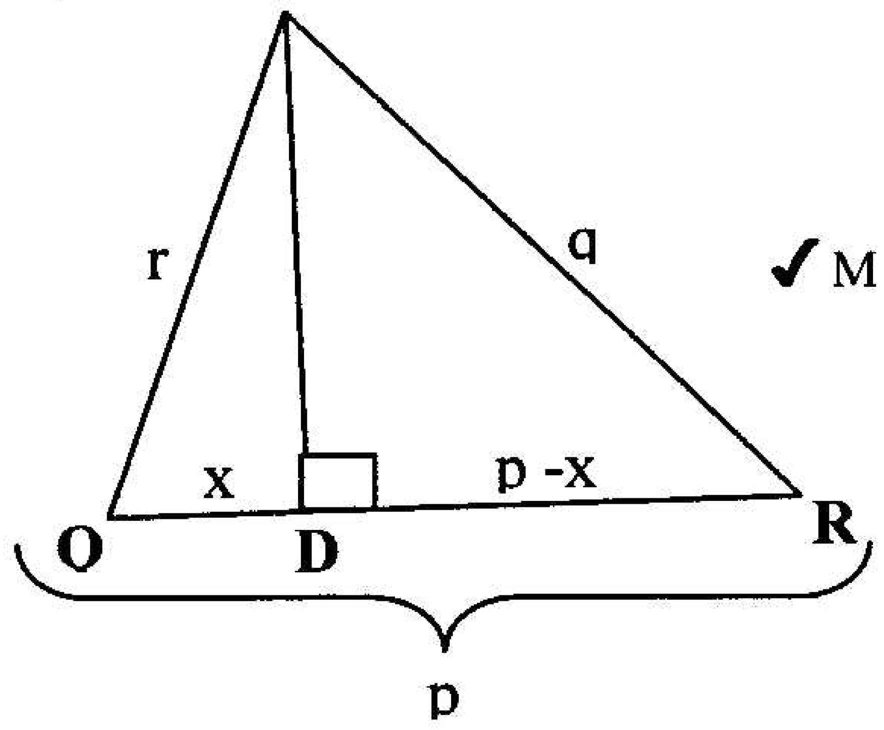
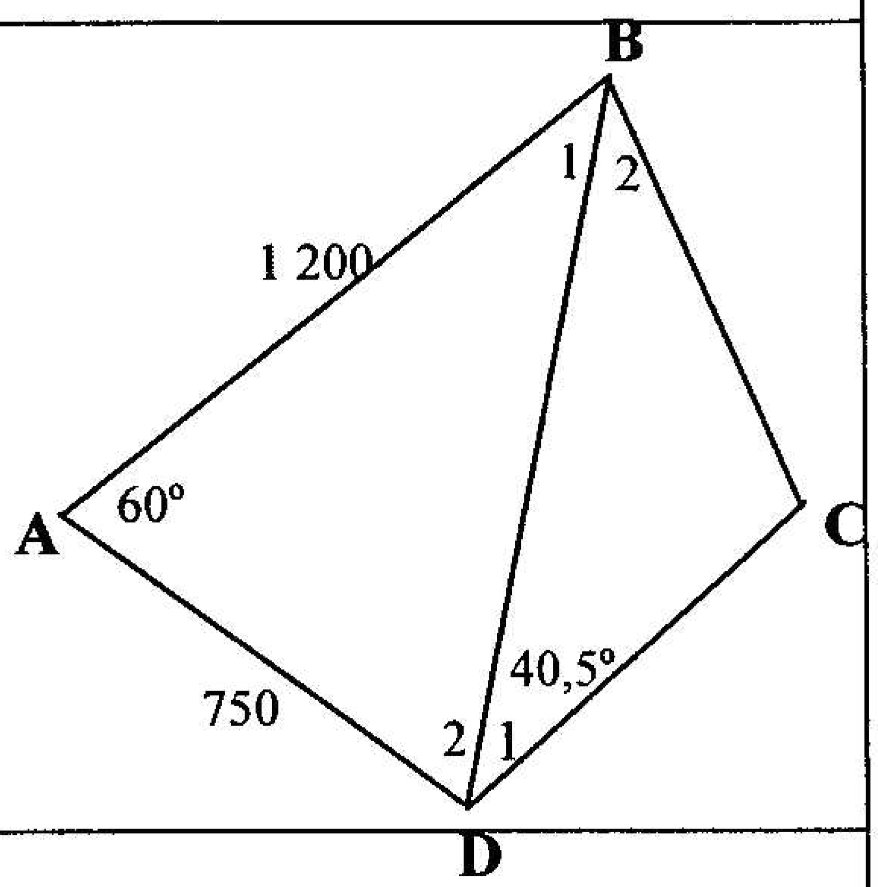
(8)

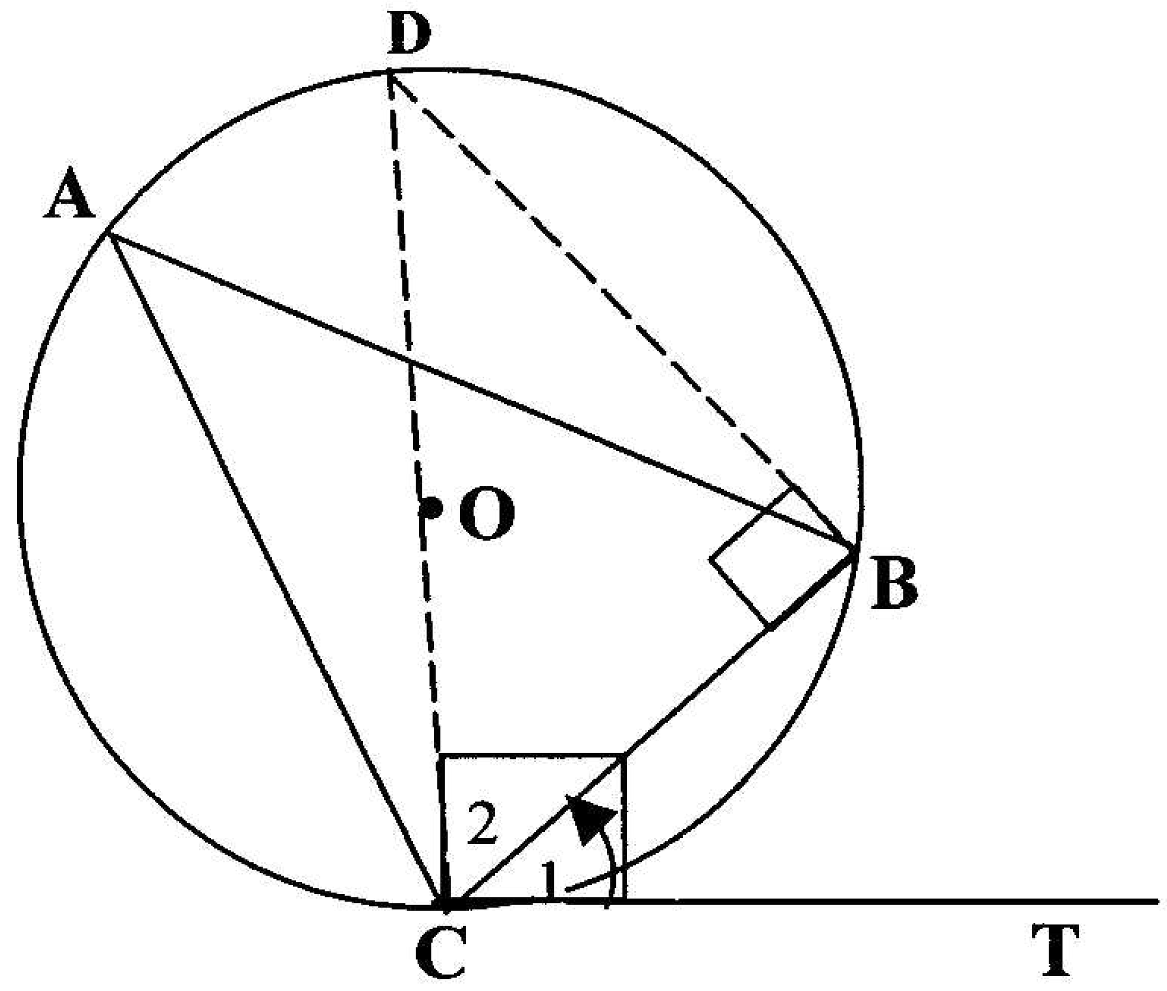
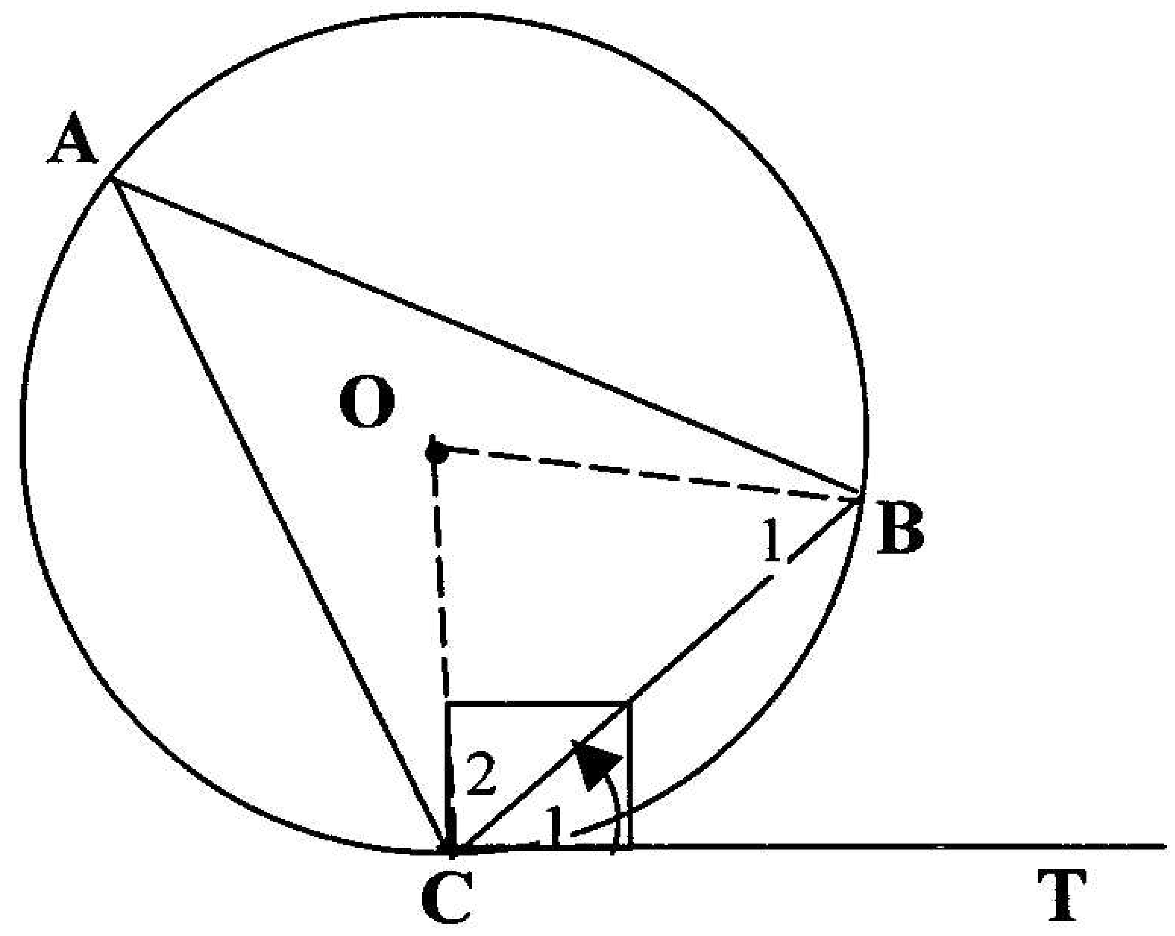
4.2.1	$x = 90°$ ✓CA (1)	answer from graph
4.2.2	✓CA ✓CA $x \in (45°; 135°)$ ✓N **OR** ✓CA ✓CA $45° < x < 135°$ ✓N (3)	(answer consistent with graph) 1 mark for each end point correct notation N (answer consistent with graph) 1 mark for each end point correct notation N

QUESTION 5	**[13]**	
5.1	$2\tan x = -0{,}924$ $\tan x = -0{,}462$ ✓M $x = 180° - 24{,}8°$ ✓A $= 155{,}2°$ ✓CA (3)	 dividing by 2 correct key/reference angle answer
5.2	$OP^2 = a^2 + b^2$ ✓M $OP = \sqrt{a^2 + b^2}$  LHS: $\cot\theta$ $= \frac{a}{b}$ ✓A RHS: $\frac{\cos\theta}{\sin\theta} = \frac{\frac{a}{\sqrt{a^2+b^2}}}{\frac{b}{\sqrt{a^2+b^2}}}$ ✓A ✓A $= \frac{a}{b} = \text{LHS}$ ✓A (5)	correct use of Pythagoras value of cot θ substitution of sin θ substitution of cos θ conclusion / simplification **No conclusion penalty 1**
5.3	LHS: $\cos\theta(\tan\theta + \cot\theta)$ $= \cos\theta\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$ ✓A ✓A $= \cos\theta \times \frac{(\sin^2\theta + \cos^2\theta)}{\sin\theta.\cos\theta}$ ✓CA ✓M $= \frac{1}{\sin\theta}$ ✓CA $= \text{cosec}\,\theta = \text{RHS}$ **OR** LHS: $\cos\theta(\tan\theta + \cot\theta)$ $= \cos\theta.\tan\theta + \cos\theta.\cot\theta$ $= \frac{\cos\theta}{1} \times \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{1} \times \frac{\cos\theta}{\sin\theta}$ ✓A ✓A $= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta}$ ✓CA ✓M $= \frac{1}{\sin\theta}$ ✓CA	 cot θ in terms of sin θ and cos θ finding common denominator calculating numerator use of identity $\sin^2\theta + \cos^2\theta = 1$ simplification/ conclusion **No conclusion penalty 1** cot θ in terms of sin θ and cos θ tan θ in terms of sin θ and cos θ finding common denominator calculating numerator use of identity $\sin^2\theta + \cos^2\theta = 1$ simplification/ conclusion

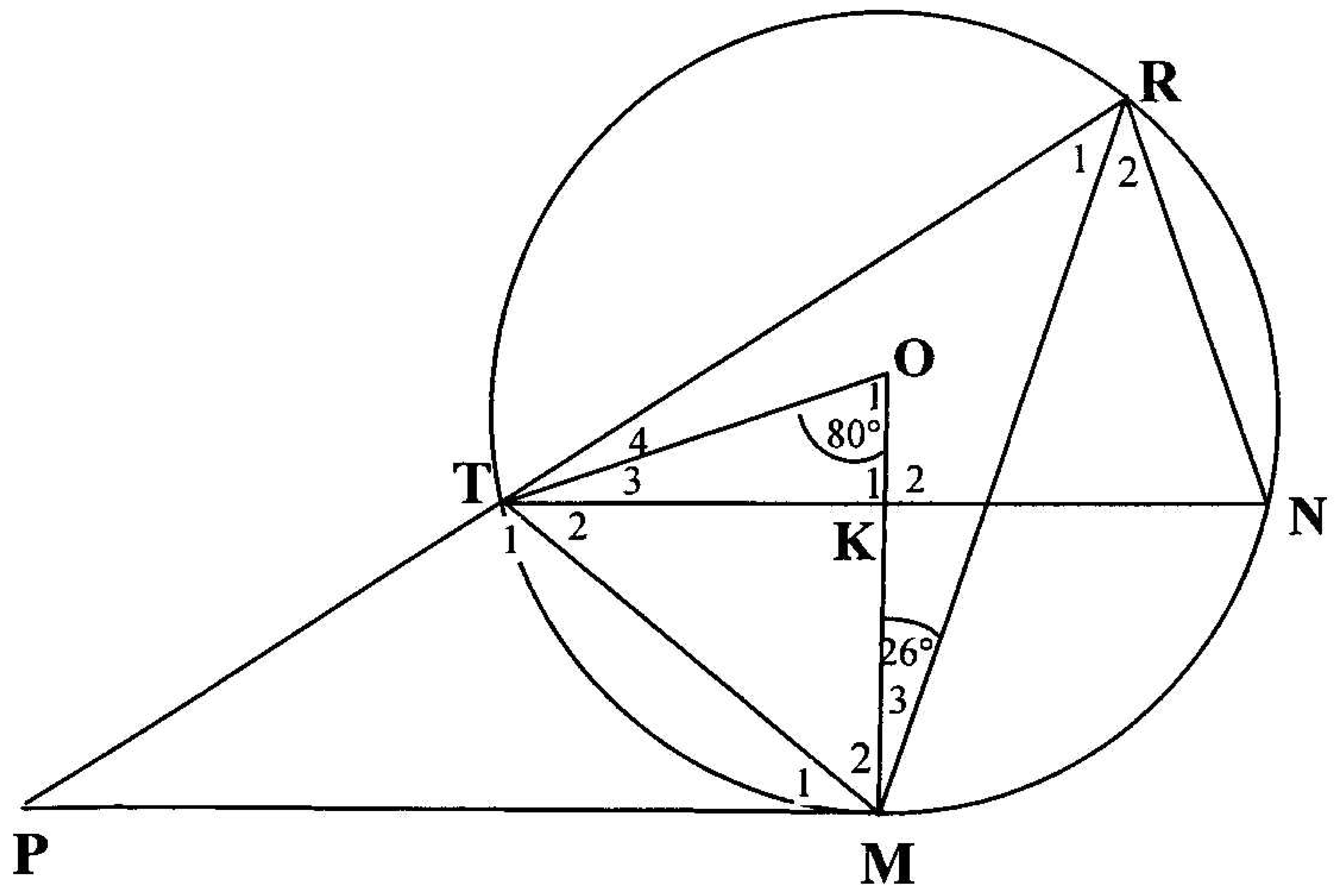
	$= \text{cosec } \theta = \text{RHS}$ (5)	**No conclusion penalty**

QUESTION 6 [20]

6.1	$q^2 = PR^2$ $= (r.\cos Q - p)^2 + (r.\sin Q - 0)^2$ ✓M $= r^2\cos^2 Q - 2pr\cos Q + p^2 + r^2\sin^2 Q$ ✓A $= p^2 + r^2(\sin^2 Q + \cos^2 Q) - 2pr.\cos Q$ ✓M $= p^2 + r^2 - 2pr\cos Q$ **OR** In ΔPQD : $r\cos Q = x$ ✓A ; $PD^2 = r^2 - x^2$ ✓A In ΔPRD : $q^2 = PD^2 + (p-x)^2$ ✓M $= r^2 - x^2 + p^2 - 2px + x^2$ ✓A ✓A $= p^2 + r^2 - 2pr.\cos Q$ (6)	
6.2.1	$BD^2 = AB^2 + AD^2 - 2AB.AD.\cos A$ ✓M $= (1\,200)^2 + (750)^2 - 2(1\,200)(750)\cos 60°$ ✓CA $= 1\,102\,500$ ✓CA $BD = 1\,050\text{m}$ ✓CA (4)	use of cos rule subst. into cos rule simplification answer
6.2.2	$\hat{C} = 120°$ ✓A (1)	correct answer
6.2.3	$\dfrac{BC}{\sin D_1} = \dfrac{BD}{\sin C}$ ✓M $\dfrac{BC}{\sin 40{,}5°} = \dfrac{1050}{\sin 120°}$ ✓CA $BC = \dfrac{1050 \times \sin 40{,}5°}{\sin 120°}$ ✓CA $BC = 787 \text{ m}$ ✓CA (4)	use of sine rule subst. into sine rule simplication answer
6.2.4	Area ΔABD $= \frac{1}{2}AB.AD.\sin A$ ✓M $= \frac{1}{2}(1200)(750)\sin 60°$ ✓CA $= 389\,711 \text{ m}^2$ ✓CA (3)	use of area rule substitution rounding off to nearest metre
6.2.5	Number of bags $= \dfrac{389\,711}{400}$ ✓M $= 975$ ✓CA (2)	dividing by 400 answer must be rounded up

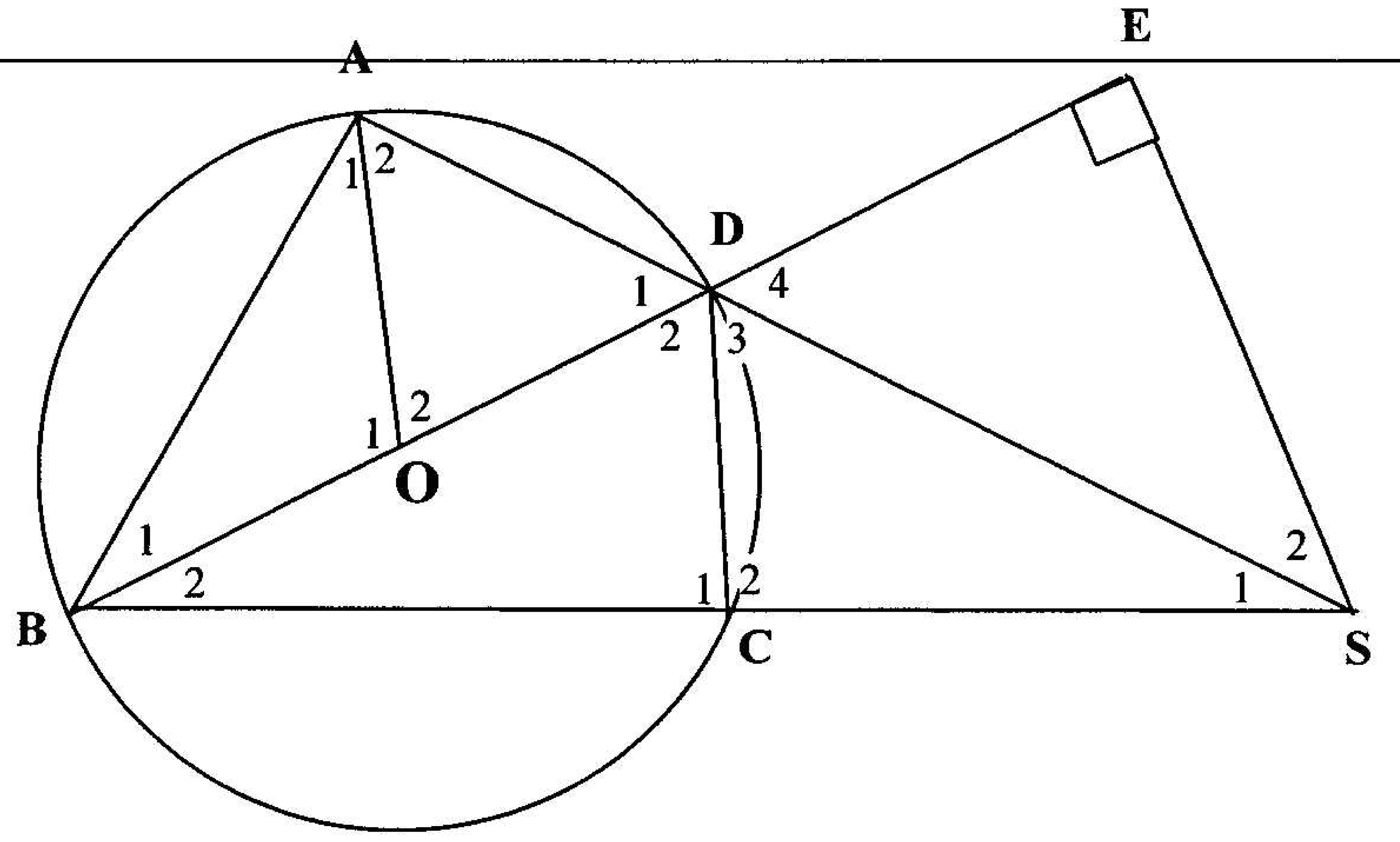
QUESTION 7 **[16]**		
7.1	Const: Draw diameter COD. ✓ **Construction** Join DB Proof: ✓ **S/R** $\hat{C}_1 + \hat{C}_2 = 90°$ (tan. ⊥ radius) $D\hat{B}C = 90°$ (∠ in semi-circle) ✓ **S/R** $\hat{D} = 90° - \hat{C}_2$ (sum ∠'s Δ) ✓ **S/R** $= \hat{C}_1$ $= \hat{A}$ ✓ **S** (∠'s in same segm.) ✓ **R** $B\hat{C}T = \hat{A}$	
	OR Const. Join radii OC and OB ✓ **Construction** Proof: $\hat{C}_1 + \hat{C}_2 = 90°$ (tan. ⊥ radius) ✓ **S/R** $\hat{C}_2 = \hat{B}_1$ (∠'s opp = s's) ✓ **S/R** $C\hat{O}B = 180° - 2\hat{C}_2$ (sum ∠'s Δ) ✓ **S/R** $\hat{A} = \frac{1}{2} C\hat{O}B$ (∠ at centre = 2 ∠ at circ) ✓ **R** $= 90° - \hat{C}_2$ $= B\hat{C}T$ ✓ **S** (6)	

7.2



7.2.1	$\hat{R}_1 = 40°$ ✓S (∠ at centre = 2 ∠ at circ.) ✓R $\hat{M}_1 = 40°$ ✓S (tan – chord) ✓R **OR** $\hat{M}_1 + \hat{M}_2 = 90°$ ✓S (tan ⊥ radius) ✓R $\hat{M}_2 = O\hat{T}M$ (∠'s opp = s's) ✓S/R $= 50°$ $\hat{M}_1 = 40°$ ✓S (4)	
7.2.2	$\hat{M}_2 = \hat{T}_2 + \hat{T}_3$ (∠'s opp = s's) or **(from 7.2.1)** $= 50°$ ✓S/R (sum ∠'s Δ) $\therefore \hat{N} = \hat{M}_2 + \hat{M}_3$ (∠'s in same seg.) ✓R $= 76°$ ✓S **OR** $\therefore \hat{M}_2 = 90° - \hat{M}_1$ (tan ⊥ radius) ✓S/R $= 50°$ $\hat{M}_2 + \hat{M}_3 = 50° + 26°$ $= 76°$ $\therefore \hat{N} = \hat{M}_2 + \hat{M}_3$ $= 76°$ ✓S ✓R (∠'s in same segm.) (3)	
7.2.3	$\hat{K}_1 = 90°$ ✓S ✓R (line from cen. to midpt of chord) $\hat{T}_3 = 10°$ (sum ∠ s of Δ) ✓S/R (3)	

QUESTION 8 [10]



8.1	$\hat{C}_1 = 90°$ ✓S (∠ in semi-circle) ✓R $\hat{E} = 90°$ ✓S $\therefore \hat{C}_1 = \hat{E}$ DESC is a cyclic quad. (ext. ∠ = int. opp. ∠) ✓R (4)	
8.2	BAES is a cyclic quad. ✓S (1)	
8.3	In Δ ABD and Δ ESD BD = DS ✓S $\hat{B}_1 = \hat{S}_2$ ✓S (∠'s in same seg) ✓R $\hat{D}_1 = \hat{D}_4$ (vert. opp. ∠'s) ✓S/R Δ ABD ≡ Δ ESD (S, ∠, ∠) ✓S/R ∴ AB = ES (≡ Δ's) (5)	

QUESTION 9 [24]

9.1

Const: Join DC and BE. ✓ Construction

Proof:

$$\frac{\text{area}\ \Delta\ ADE}{\text{area}\ \Delta\ BED} = \frac{\frac{1}{2}AD \times h}{\frac{1}{2}DB \times h}$$ ✓S

$$= \frac{AD}{DB}$$ ✓S

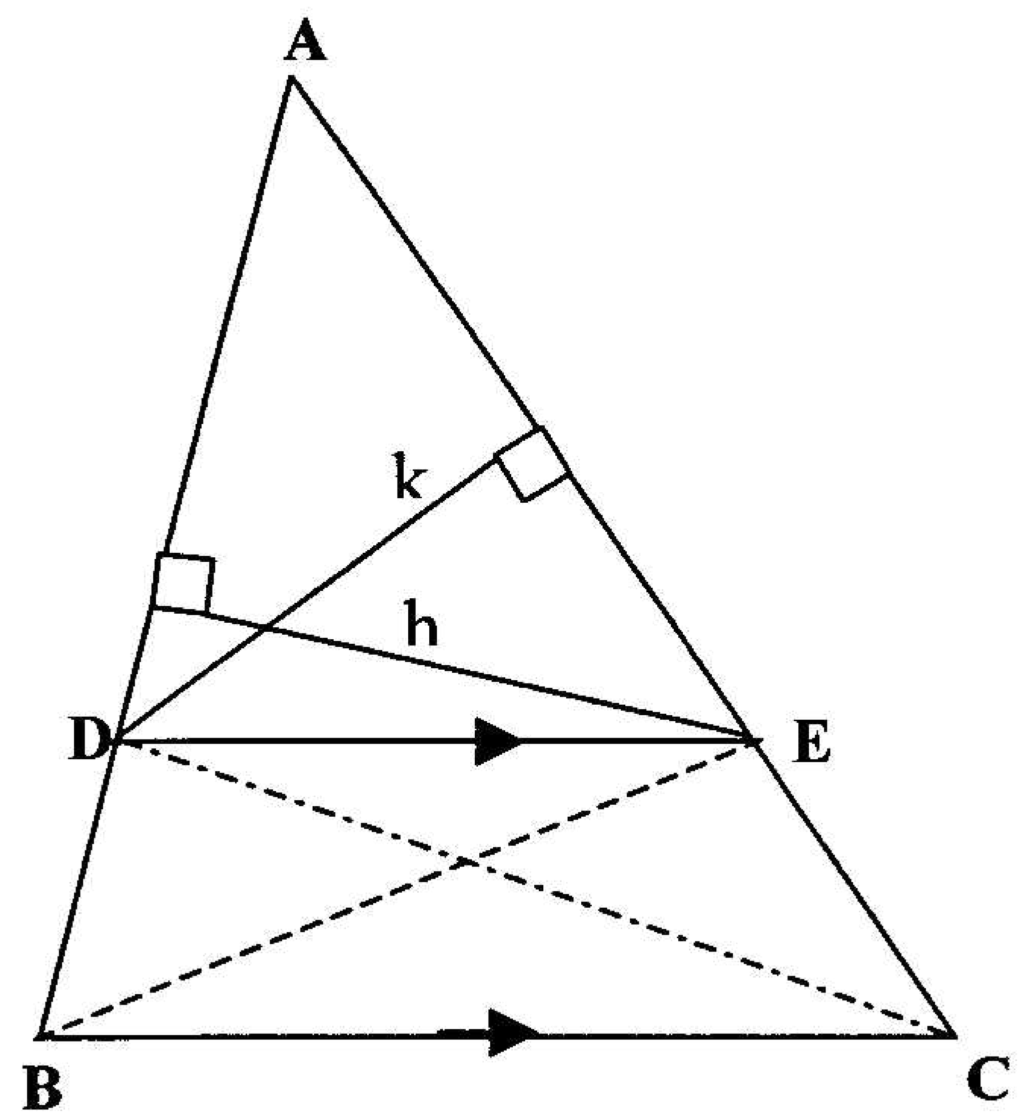
$$\frac{\text{area}\ \Delta\ AED}{\text{area}\ \Delta\ EDC} = \frac{\frac{1}{2}AE \times k}{\frac{1}{2}EC \times k}$$

$$= \frac{AE}{EC}$$ ✓S

but area Δ BDE = area Δ EDC (same base, same height) ✓R

$$\frac{\text{area}\ \Delta\ ADE}{\text{area}\ \Delta\ BED} = \frac{\text{area}\ \Delta\ AED}{\text{area}\ \Delta\ EDC}$$ ✓S

$$\frac{AD}{DB} = \frac{AE}{EC}$$ (6)

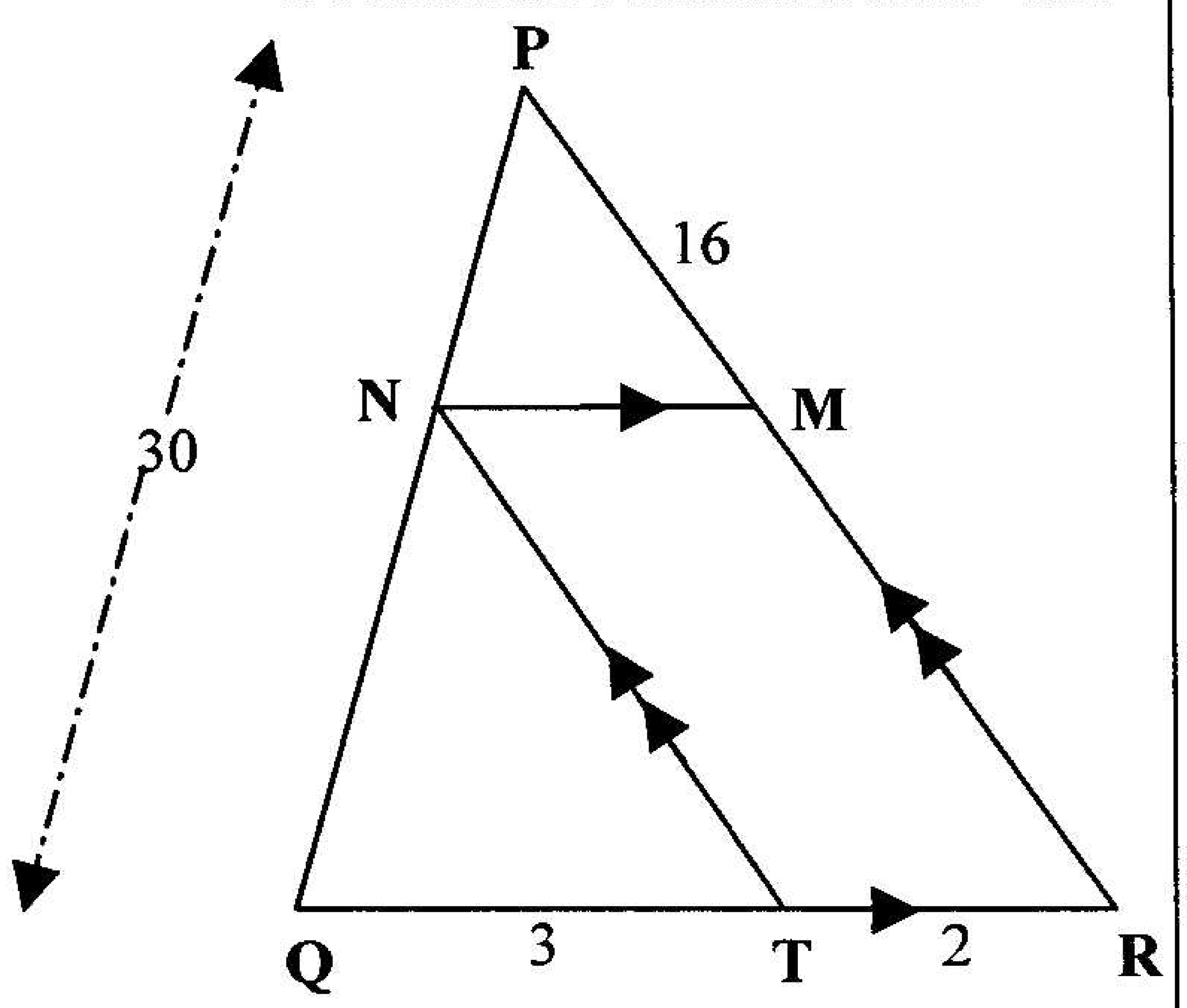


9.2

$$\frac{PN}{PQ} = \frac{RT}{RQ}$$ ✓S (line // one sideΔ) ✓R

$$\frac{PN}{30} = \frac{2}{5}$$ ✓A

PN = 12 ✓CA (4)



9.2.2

$$\frac{MR}{MP} = \frac{QN}{NP}$$ ✓S (line // one sideΔ) ✓R

$$\frac{MR}{16} = \frac{18}{12}$$ ✓CA

MR = 24 ✓CA (4)

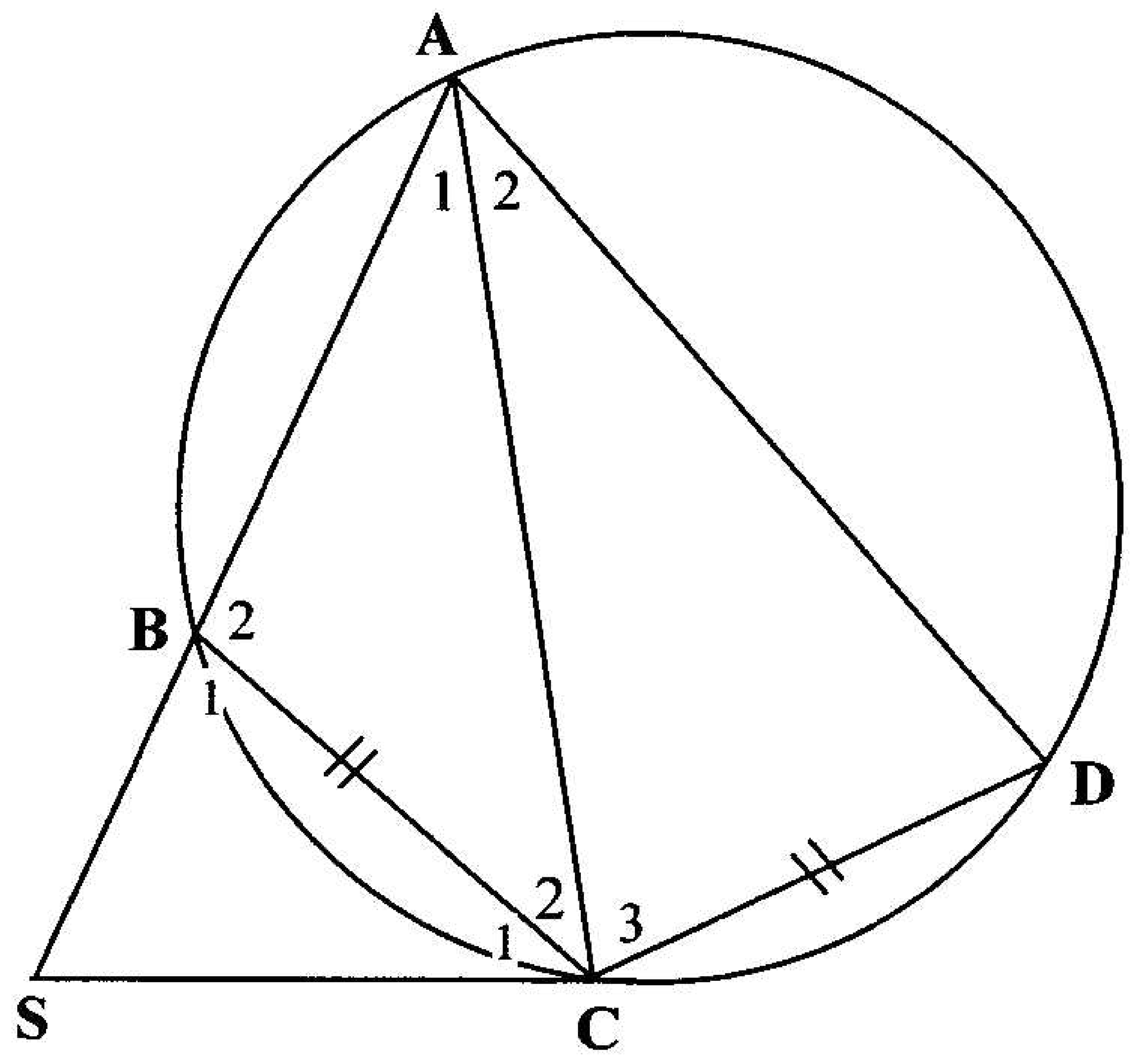
OR

$$\frac{NT}{PR} = \frac{3}{5}$$ (line // one sideΔ) ✓S/R

$$\frac{MR}{16 + MR} = \frac{3}{5}$$ ✓S ✓CA (opp s's parm =)

5 MR = 48 + 3 MR
2 MR = 48
MR = 24 ✓CA

9.3



9.3.1	$\hat{C}_1 = \hat{A}_1$ ✓S (tan – chord) ✓R $\hat{A}_1 = \hat{A}_2$ ✓S (= chords subt. = ∠'s) ✓R $\hat{C}_1 = \hat{A}_2$ (4)	
9.3.2	In Δ BCS and Δ DAC $\hat{C}_1 = \hat{A}_2$ (proved) ✓S/R $\hat{B}_1 = \hat{D}$ ✓S (ext. ∠ of cy. quad.) ✓R $\hat{S} = \hat{C}_3$ (3^{rd} ∠) ∴ Δ BCS ||| Δ DAC (∠, ∠, ∠) ✓R (4)	
9.3.3	$\therefore \frac{BC}{DA} = \frac{BS}{CD}$ ✓S BC. CD = BS . DA BC^2 = DA. BS (given BC = CD) ✓R (2)	
		TOTAL: 150 MARKS