education

Department:
Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATION - 2006

MATHEMATICS P1 : ALGEBRA

STANDARD GRADE

FEBRUARY/MARCH 2006

301-2/1 E

Marks: 150

3 Hours

This question paper consists of 9 pages and 1 information sheet.

MATHEMATICS SG: Paper 1

301 2 1E SG

X05

INSTRUCTIONS TO CANDIDATES

Read the following instructions carefully before answering the questions:

1. This paper consists of **8** questions. Answer **ALL** the questions.
2. Clearly show **ALL** calculations, diagrams, graphs, et cetera you have used in determining the answers.
3. An approved calculator (non-programmable and non-graphical) may be used unless stated otherwise.
4. If necessary, answers should be rounded off to **TWO** decimal places, unless stated otherwise.
5. Graph paper is **NOT** required in this question paper.
6. Number the answers **EXACTLY** as the questions are numbered.
7. Diagrams are not necessarily drawn to scale.
8. It is in your own interest to write legibly and to present the work neatly.
9. **An information sheet with formulae is included at the end of the question paper.**

QUESTION 1

1.1 Solve for x:

1.1.1 $3(2x^2 - 5) = x$ (4)

1.1.2 $3x^2 + x - 5 = 0$ (round off your answer to TWO decimal places) (5)

1.1.3 $\sqrt{x} - 1 = 5$ (2)

1.2 Solve for x and y if they satisfy the following equations simultaneously:

$$x - 2y = 1$$
$$x^2 - 2xy + y^2 - 9 = 0$$ (8)

[19]

QUESTION 2

2.1 The roots of a quadratic equation are $x = 5 \pm \sqrt{2p - 1}$.

2.1.1 Calculate the value of p for which the roots are real. (2)

2.1.2 Give one value of p for which the roots are rational. (2)

2.2 Show that there is no real number x that satisfies the equation $x^2 + x = -1$. (5)

[9]

QUESTION 3

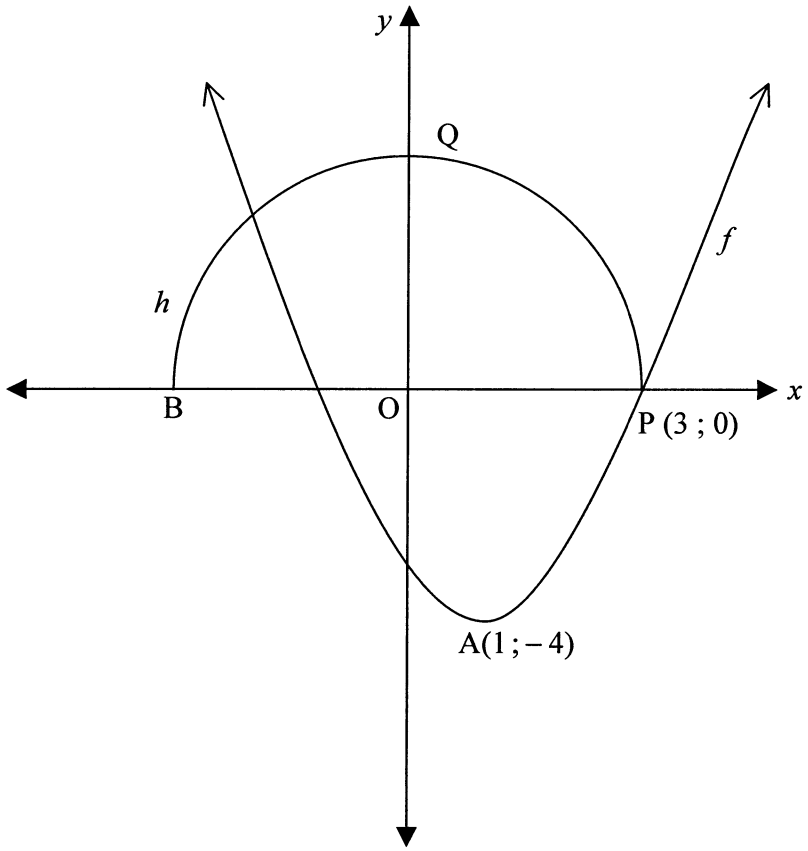
3.1 The remainder is -2 when $f(x) = 2x^2 - mx - 4x + 10$ is divided by $(x - 3)$. Determine the value of m. (4)

3.2 $(x - 2)$ is a common factor of the polynomials $f(x) = 2x^3 - ax + b$ and $g(x) = x^3 - ax^2 - bx - 8$. Determine the values of a and b. (6)

[10]

QUESTION 4

4.1 In the figure below, sketch graphs of f defined by $f(x) = a(x-p)^2 + q$ and the semi-circle h are drawn. A$(1;-4)$ is the turning point of f. P$(3;0)$ and B are points of intersection with the x-axis.



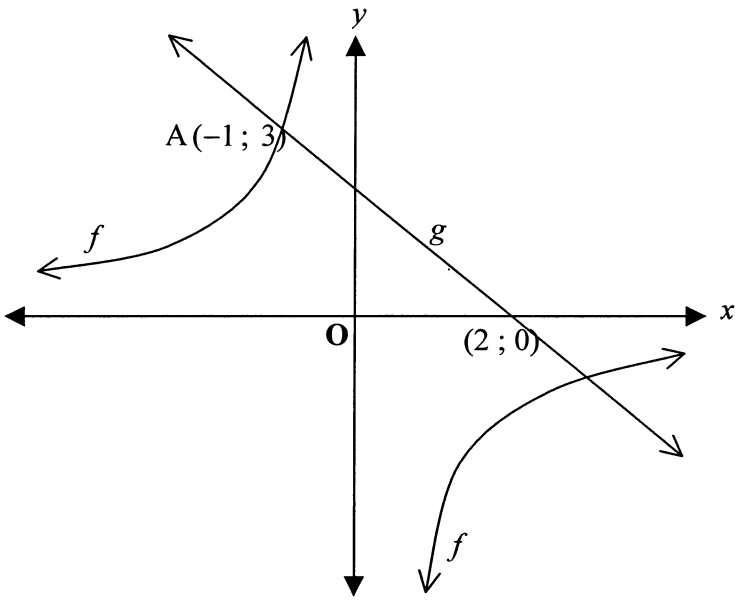
4.1.1 Determine the equation of h. (2)

4.1.2 Write down the values of p and q. (2)

4.1.3 Determine the value of a. (4)

4.1.4 Write down the range of f. (2)

4.2 In the figure below the hyperbola f and the straight line g are represented. The graphs of f and g intersect at the point $A(-1\,;\,3)$.



Determine:

4.2.1 The equation of f (3)

4.2.2 The equation of the straight line g passing through the points A and $(2\,;\,0)$ (5)

4.2.3 The values of x for which $f(x) > g(x)$ and $x < 0$ (2)

[20]

QUESTION 5

5.1 Simplify **fully without using a calculator** :

5.1.1 $$\frac{3^{2-x}-4\left(3^{-x}\right)}{3^{-x+2}}$$ (4)

5.1.2 $$\frac{\log 9}{\log\left(\frac{1}{3}\right)}$$ (4)

5.2 Solve for x, **without using a calculator**:

5.2.1 $$\left(\frac{1}{2}\right)^{x-9}=4^{x+3}$$ (5)

5.2.2 $$\log_4(x-1)+\log_4(x+2)=1$$ (6)

5.3 Given: $12^{x+1}=36(6^x)$

5.3.1 Show that the equation can be written in the form:

$$2^x=3$$ (3)

5.3.2 Solve for x, **correct to two decimal places.** (3)

[25]

QUESTION 6

6.1 Given the arithmetic series: $5 + 9 + 13 + ... + 401$.

Calculate:

6.1.1 The number of terms in the series (4)

6.1.2 The sum of the terms in the series (3)

6.2 Given the sequence: $2 ; x ; 18 ; \ldots.$

Calculate x if this sequence is:

6.2.1 An arithmetic sequence (3)

6.2.2 A geometric sequence (4)

6.3 Given : $\sum_{k=1}^{10} 3(2)^{1+k}$

6.3.1 Write down the first three terms of the series. (3)

6.3.2 Determine the sum of the series. (5)

6.4

In order to encourage South Africans to know their country better and to boost the tourism industry, a travel agent advertises a *"travel now and pay later"* promotion. The travel package costs R6 530 but can be paid back over a period of 18 months at an interest rate of 15% per annum, compounded monthly. What will the travel package eventually cost? (6)

[28]

QUESTION 7

7.1 Calculate the derivative of $f(x) = 4x^2$ from **first principles**. (6)

7.2 Find $\frac{dy}{dx}$ in each of the following :

7.2.1 $y = x^3 - \frac{3}{x}$ (3)

7.2.2 $y = (x+1)(x-3)$ (3)

7.3 The x- coordinate of a point P on the graph of $y = x^2 - 1$ is 2.

7.3.1 What is the y-coordinate? (1)

7.3.2 A tangent to the graph is drawn at P. What is the gradient of this line? (3)

[16]

QUESTION 8

8.1 Given: $f(x) = -x^3 - 3x^2$.

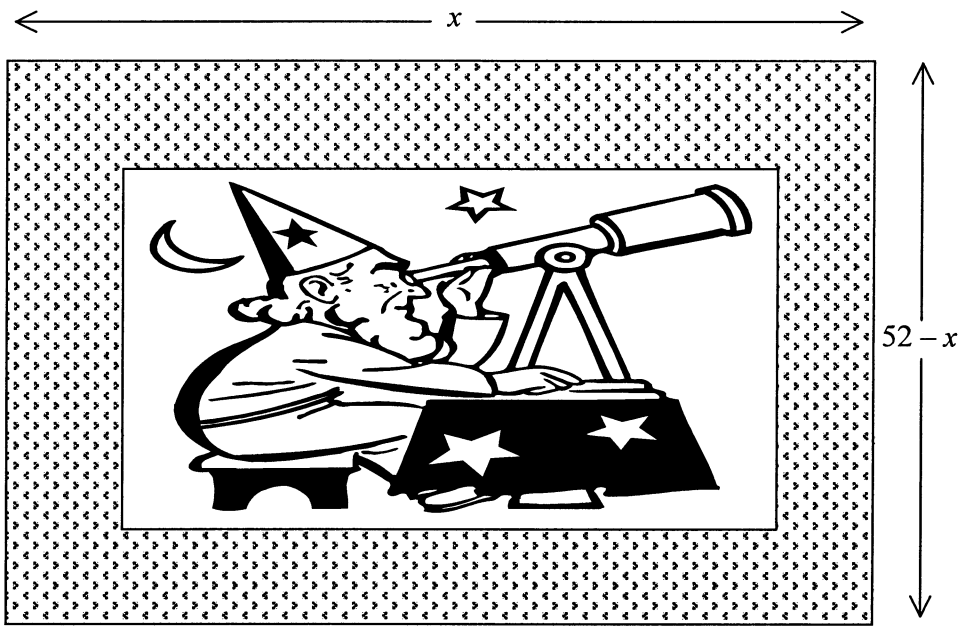
8.1.1 Determine the x- and y-intercepts of the graph of f. (3)

8.1.2 Determine the co-ordinates of the turning points of the graph of f. (6)

8.1.3 Sketch the graph of f. Clearly show all the intercepts with the axes and the turning points on the graph. (4)

8.1.4 For which values of x is f increasing? (2)

8.2 The diagram below represents a rectangular picture surrounded by a frame 1 cm in width. The sides of the frame measured along the **outer edge** are x cm and $(52-x)$ cm as indicated.



8.2.1 Write the measurements of the sides of the picture in terms of x. (2)

8.2.2 Show that the area, A, of the picture is given by

$$A = -x^2 + 52x - 100.$$

(2)

8.2.3 Hence, calculate the maximum area of the picture. (4)

[23]

TOTAL: 150

SENIOR CERTIFICATE EXAMINATION - MARCH 2006

Mathematics Formula Sheet (HG and SG)
Wiskunde Formuleblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}(a + T_n) \qquad \text{or / of} \quad S_n = \frac{n}{2}(a + l)$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(1 - r^n)}{1 - r} \quad (r \neq 1) \qquad S_n = \frac{a(r^n - 1)}{r - 1} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1 - r} \quad (|r| < 1)$$

$$A = P\left(1 + \frac{r}{100}\right)^n \qquad \text{or / of} \qquad A = P\left(1 - \frac{r}{100}\right)^n$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$(x_3; y_3) = \left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$x^2 + y^2 = r^2$$

$$(x - p)^2 + (y - q)^2 = r^2$$

In ΔABC: $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta ABC = \frac{1}{2}ab.\sin C$$