education

Department:
Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATION - 2006

MATHEMATICS PAPER 1
ALGEBRA

STANDARD GRADE

OCTOBER/NOVEMBER 2006

301-2/1E

MATHEMATICS SG: Paper 1

301 2 1E SG

MARKS: 150

TIME: 3 hours

This question paper consists of 7 pages and 1 formula sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions:

1. This question paper consists of 7 questions. Answer ALL the questions.
2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining the answers.
3. An approved calculator (non-programmable and non-graphical) may be used unless stated otherwise.
4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
5. Graph paper is NOT required in this question paper.
6. Number the answers EXACTLY as the questions are numbered.
7. Diagrams are not necessarily drawn to scale.
8. It is in your own interest to write legibly and to present the work neatly.
9. A formula sheet is included at the end of the question paper.

QUESTION 1

1.1 Solve for x:

1.1.1 $2x^2 = 3x + 5$ (3)

1.1.2 $x^2 - 4x + 2 = 0$ (Round off the answer to TWO decimal places.) (4)

1.1.3 $2x - 1 = \sqrt{1 - x}$ (5)

1.2 Solve for x and y if they satisfy the following equations simultaneously:

$$x - y = 1$$
$$x^2 + xy - 5x + 5y - y^2 = 0$$

(8)
[20]

QUESTION 2

2.1 Given that -2 is one root of the quadratic equation $kx^2 + 3x - k = 0$.

Determine:

2.1.1 The value of k (3)

2.1.2 The product of the two roots of the equation (4)

2.2 Given: $f(x) = 4x^2 - 6x + p$

2.2.1 Determine the values of p if $f(x) = 0$ has non-real roots. (6)

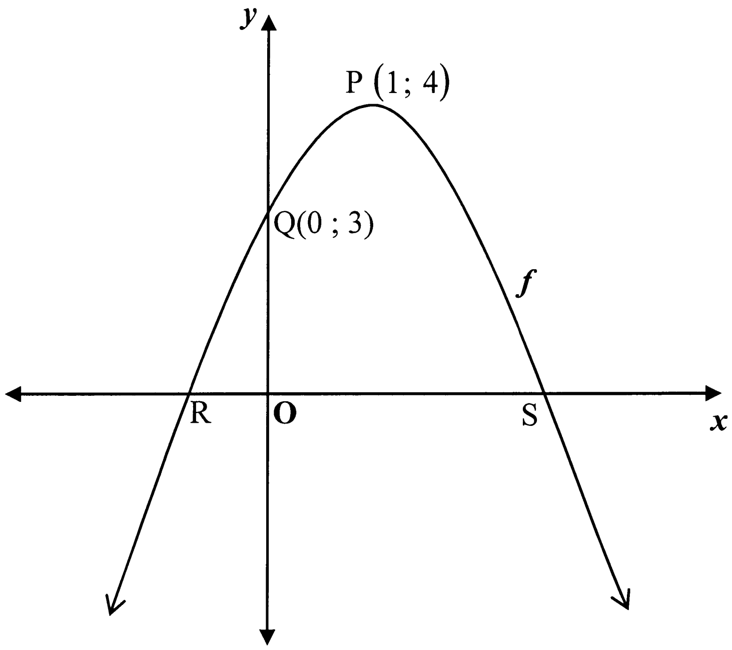
2.2.2 If p is an integer, determine the smallest value of p for which $f(x) = 0$ will have non-real roots. (1)

2.3 Given: $g(x) = ax^3 + bx^2 - 4x + 8$

If $g(x)$ is exactly divisible by $(x - 2)$ and leaves a remainder of 1 when divided by $(x - 1)$, calculate the values of a and b. (8)
[22]

QUESTION 3

3.1 The sketch, not drawn to scale, shows the graph of a parabola f.
The graph of f has a turning point $P(1\,;\,4)$ and intersects the y-axis at $Q(0\,;\,3)$.
f cuts the x-axis at R and S.



Determine:

3.1.1 The values of a, b and c if $f(x) = ax^2 + bx + c$ (6)

3.1.2 The co-ordinates of R and S (Give both co-ordinates in each case.) (5)

3.1.3 The range of f (1)

3.1.4 The equation of the semi-circle above the x-axis with origin as centre and passing through Q in the form $y = \ldots$ (3)

3.2 On the same system of axes draw sketch graphs of:

$f(x) = -\frac{2}{x}$, and $g(x) = -\frac{x}{2}$

Determine the co-ordinates of the points of intersection of the graphs.
Show ALL calculations. (8)

[23]

QUESTION 4

4.1 Simplify completely (without the use of a calculator):

4.1.1 $8^{-\frac{2}{3}}$ (2)

4.1.2 $\left(\sqrt[3]{a}\,.\sqrt{b}\right)^6$ (3)

4.1.3 $\dfrac{2\times 7^{2a-1}+7^{2a+1}}{49^a}$ (4)

4.2 If $\log 2 = a$ and $\log 3 = b$, determine the following in terms of a and b:

4.2.1 $\log(2\times 3)$ (2)

4.2.2 $\log(2+3)$ (4)

4.3 Solve for x, **without using a calculator:**

4.3.1 $12^x \times 4 = 36\times 4^x$ (3)

4.3.2 $\log_7(5x+2)-2\log_7 x = \log_7 3$ (5)

4.4 Solve for x:

$8^x = 160$ (Round off the answer to TWO decimal places.) (4)

[27]

QUESTION 5

5.1 The first term of an arithmetic sequence is -1 and the seventh term is 35.

Determine:

5.1.1 The common difference of the sequence (3)

5.1.2 The number of terms if the last term of the sequence is 473 (3)

5.2 Determine: $\sum_{r=1}^{100}(3r-1)$ (5)

5.3 The first three terms of a geometric sequence are: $\frac{1}{27};\frac{1}{9}$ and $\frac{1}{3}$.

Determine the sum of the first 10 terms of the sequence. (4)

5.4 Mobile D, a cellphone company, advertised their new package where the more you buy the less you pay per 200 air time minutes as shown in the following table:

Air time minutes	200	400	600	...
Amount that you pay in rands	250	350	490	...

5.4.1 Show that the amount that you pay in the first three months forms a geometric sequence. (2)

5.4.2 Use a formula to calculate how much you would pay for 1 000 minutes of air time. (3)

5.5 The value of a certain vehicle depreciates at 9% per annum. Determine the present book value (to the nearest rand) of the vehicle that was bought for R70 000 seven years ago. $\left[\text{Use } A = P\left(1 \pm \frac{r}{100}\right)^n\right]$ (5)

[25]

QUESTION 6

6.1 Given: $f(x) = -x^2$
Use first principles to prove that $f'(x) = -2x$. (5)

6.2 Determine $\frac{dy}{dx}$ if:

6.2.1 $y = 3x^{\frac{4}{3}} - 2x$ (2)

6.2.2 $y = \frac{9x^4 - 6}{3x}$ (4)

6.3 Given: $f(x) = x^3 - 5x^2 + 7x - 3$

6.3.1 It is further given that $(x-1)$ is a factor of $f(x)$. Calculate the co-ordinates of the intercepts with the axes of the graph of *f*. (5)

6.3.2 Prove that $\left(\frac{7}{3}\,;\,-1\frac{5}{27}\right)$ is one of the turning points of *f*. (5)

6.3.3 Draw the graph of *f*. (4)
[25]

QUESTION 7

Just after birth the mass of a baby drops for a few days and then starts to increase again. The average mass of a baby in its first 30 days of life can be approximated by the following equation:

$m(t) = 0{,}02t^3 - 0{,}2t^2 + 3200;\ \ 0 \le t \le 30$

where t is the time in days and $m(t)$ is the mass in grams.

7.1 What is the mass of the baby at birth? (1)

7.2 Calculate on what day the mass reaches a minimum. (5)

7.3 Determine the maximum mass of the baby in the 30-day period. (2)
[8]

TOTAL: 150

Mathematics Formula Sheet (HG and SG)
Wiskunde Formuleblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$T_n = a + (n - 1)d$ $\qquad S_n = \frac{n}{2}(a + T_n)$ or/of $S_n = \frac{n}{2}(a + \ell)$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$T_n = ar^{n-1}$ $\qquad S_n = \frac{a(1 - r^n)}{1 - r}$ $(r \neq 1)$ $\qquad S_n = \frac{a(r^n - 1)}{r - 1}$ $(r \neq 1)$

$$S_\infty = \frac{a}{1 - r} \quad (|r| < 1)$$

$A = P\left(1 + \frac{r}{100}\right)^n$ or/of $A = P\left(1 - \frac{r}{100}\right)^n$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$(x_3; y_3) = \left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$x^2 + y^2 = r^2$$

$$(x - p)^2 + (y - q)^2 = r^2$$

In ΔABC: $\qquad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta ABC = \frac{1}{2}ab.\sin C$$