education

Department:
Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATION - 2006

MATHEMATICS P2 : GEOMETRY

HIGHER GRADE

FEBRUARY/MARCH 2006

301-1/2 E

Marks: 200

3 Hours

This question paper consists of 11 pages 1 formula sheet and 5 diagram sheets.

MATHEMATICS HG: Paper 2

301 1 2E HG

X05

INSTRUCTIONS

1. This question paper consists of **10** questions, a formula sheet and diagram sheets.
2. Use the formula sheet to answer this question paper.
3. Detach the diagram sheets from the question paper and place them inside your **ANSWER BOOK.**
4. The diagrams are not drawn to scale.
5. Answer **ALL** the questions.
6. Number **ALL** the answers correctly and clearly.
7. **ALL** the necessary calculations must be shown.
8. Non-programmable calculators may be used, unless otherwise stated.
9. The number of decimal digits to which answers must be rounded off will be stated in the question where necessary.

ANALYTICAL GEOMETRY

NOTE: - USE ANALYTICAL METHODS IN THIS SECTION.
- CONSTRUCTION AND MEASUREMENT METHODS MAY NOT BY USED.

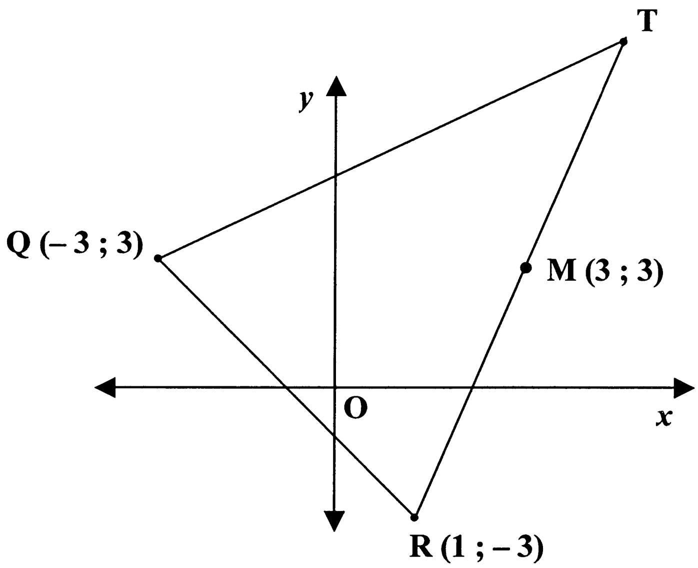
QUESTION 1

In the diagram alongside,

R (1 ; – 3) , Q (– 3 ; 3) and T

are the vertices of ΔTRQ.

M (3 ; 3) is the midpoint of TR.



1.1 Determine:

1.1.1 The length of TR (leave the answer in surd form) (4)

1.1.2 The size of $\hat{R}$, rounded off to ONE decimal digit (6)

1.2 1.2.1 Determine the equation of the median from T to RQ. (9)

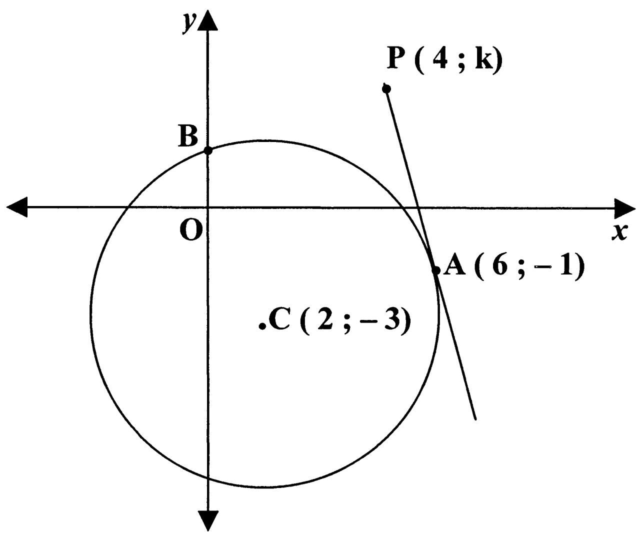
1.2.2 Hence, or otherwise, determine the coordinates of the point of intersection of the medians of Δ TRQ. (4)

[23]

QUESTION 2

2.1 The circle with centre C (2 ; – 3) passes through point A (6 ; – 1) and through point B, which lies on the y-axis.

P (4 ; k) is a point such that PA is a tangent to the circle.



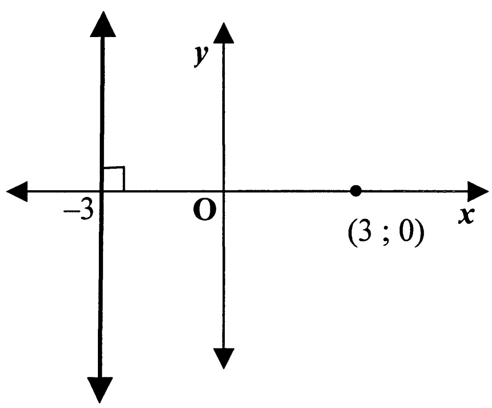
2.1.1 Determine the equation of the circle. (4)

2.1.2 Determine the equation of tangent PA. (4)

2.1.3 Determine the value of k. (2)

2.1.4 Hence, prove analytically that PB is a tangent to the circle. (7)

2.2 In the diagram alongside, a circle with centre P (x ; y) passes through point (3 ; 0) and touches the straight line $x = -3$



2.2.1 Determine the equation of the locus of P. (7)

2.2.2 Hence, name the shape of the locus of P. (1)

[25]

TRIGONOMETRY

QUESTION 3

Answer this question without the use of a calculator.

3.1 Simplify the following to a single trigonometric ratio of θ:

$$\frac{\cos(\theta - 90°)}{\text{cosec}(\theta - 180°)} + \cos(360° + \theta).\ \text{cosec}(90° - \theta)$$ (7)

3.2 If $\cos 61° = p$, express the following in terms of p:

3.2.1 $\sin 209°$ (3)

3.2.2 $\text{cosec}(-421°)$ (3)

3.2.3 $\cos 1°$ (6)

[19]

QUESTION 4

Given: $f(x) = 2\sin x$ and $g(x) = \cos(x + 30°)$

4.1 Show that the equation $2\sin x = \cos(x + 30°)$ can also be expressed as

$$\tan x = \frac{\sqrt{3}}{5}$$ (6)

4.2 Hence, determine the value(s) of $x \in [-90° ; 270°]$, rounded off to ONE decimal digit, where $f(x) = g(x)$ (3)

4.3 Use the system of axes given on the diagram sheet to draw sketch graphs of the curves of f and g for $x \in [-90° ; 270°]$
Clearly show all the coordinates of turning points and intercepts with the axes. (9)

4.4 Use the solution(s) obtained in QUESTION 4.2 and the graphs drawn in QUESTION 4.3 to determine for which value(s) of $x \in [0° ; 270°]$ is:

4.4.1 $f(x) > g(x)$ (2)

4.4.2 $f(x) . g(x) < 0$ (3)

[23]

QUESTION 5

5.1 5.1.1 Write down an expression for $\sin(x + y)$ in terms of the sines and the cosines of x and y. (1)

5.1.2 Hence, using QUESTION 5.1.1, show how to derive an expression for $\cos(x + y)$ in terms of the sines and the cosines of x and y. (3)

5.2 5.2.1 Prove that $\cos(x - y) - \cos(x + y) = 2\sin x \cdot \sin y$ (3)

5.2.2 Hence or otherwise, calculate the numerical value of

$$2\sin 195° \cdot \sin 45°,$$

without the use of a calculator. (6)

5.3 5.3.1 Prove the following identity :

$$\frac{\cos 2\theta + 1}{\sin 2\theta \cdot \tan\theta} = \cot^2\theta$$

(4)

5.3.2 Determine the values of θ for which the identity in QUESTION 5.3.1 is undefined . Give the answer as a general solution. (4)

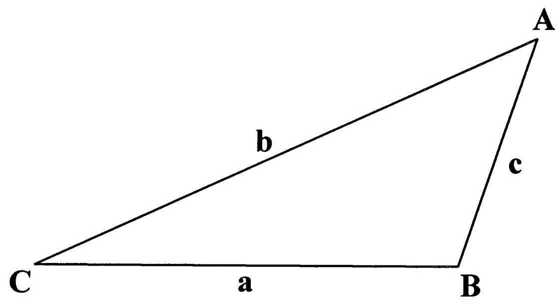
[21]

QUESTION 6

6.1 In the diagram alongside ΔABC is obtuse angled.

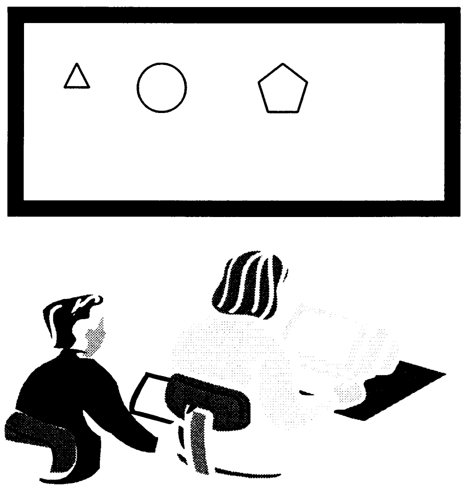
Use the diagram on the diagram sheet, or redraw the diagram in your answer book to prove that:

$b^2 = a^2 + c^2 - 2(a)(c)\cos B$

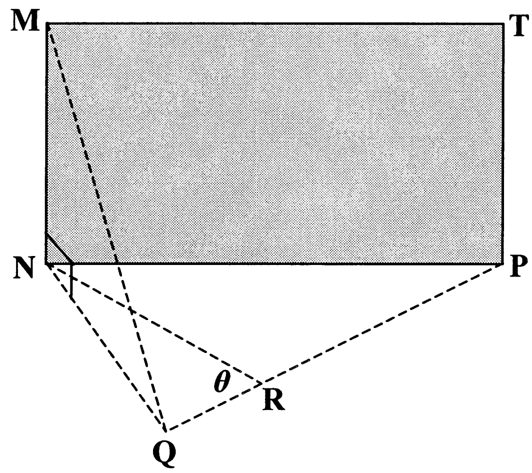


(6)

6.2



The diagram alongside is a representation of the picture above.
MNPT represents the rectangular writing board mounted on a vertical wall in a classroom.
Q and R represent the eyes of two learners sitting at desks facing the writing board.
Points N, Q, R and P lie on the same horizontal plane .



NR = RP = 2RQ = x

$N\hat{R}Q = \theta$ and

NP = y

6.2.1 Prove that $\cos\theta = \dfrac{y^2}{2x^2} - 1$ (5)

6.2.2 If y = 2,3 metres, x = 1,5 metres and $N\hat{Q}M = 38°$ calculate, rounded off to ONE decimal digit:

(a) The value of θ (2)

(b) The length of NQ (5)

(c) The size of $N\hat{Q}R$ (4)

(d) The width MN of the writing board (3)

[25]

EUCLIDEAN GEOMETRY

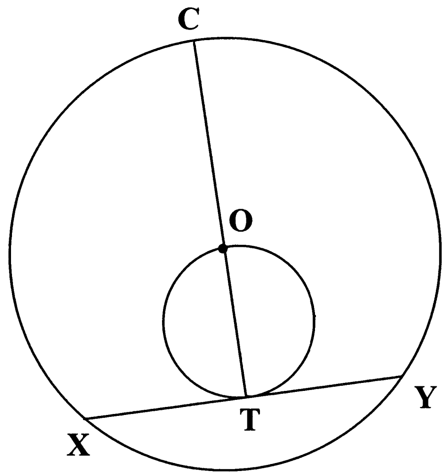
NOTE:
- **DIAGRAMS FOR PROVING THEORY MAY BE USED ON THE DIAGRAM SHEETS OR REDRAWN IN YOUR ANSWER BOOK.**
- **DETACH THE DIAGRAM SHEETS FROM THE QUESTION PAPER AND PLACE THEM IN YOUR ANSWER BOOK.**
- **GIVE A REASON FOR EACH STATEMENT, UNLESS OTHERWISE STATED.**

QUESTION 7

In the diagram alongside, O is the centre of the larger circle and OT the diameter of the smaller circle.

Chord XY of the larger circle is a tangent to the smaller circle at T.

COT is a straight line.



If $OC = r$ and $XY = \frac{3r}{2}$, show stating reasons that:

$$CT = \frac{(4+\sqrt{7})r}{4}$$

[7]

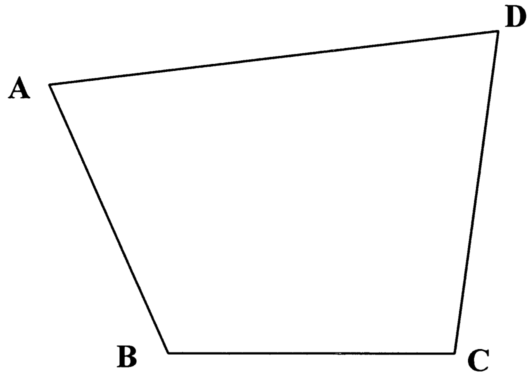
QUESTION 8

8.1 In the diagram alongside, ABCD is a quadrilateral.

Use the diagram on the diagram sheet or redraw the diagram in your answer book to prove the theorem which states that :

If $\hat{B} + \hat{D} = 180°$, then

ABCD is a cyclic quadrilateral. (6)



8.2 Write down the statement of the converse of the following theorem:
'The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment.' (2)

8.3 In the diagram below, two circles PTRQ and PQB intersect at P and Q.

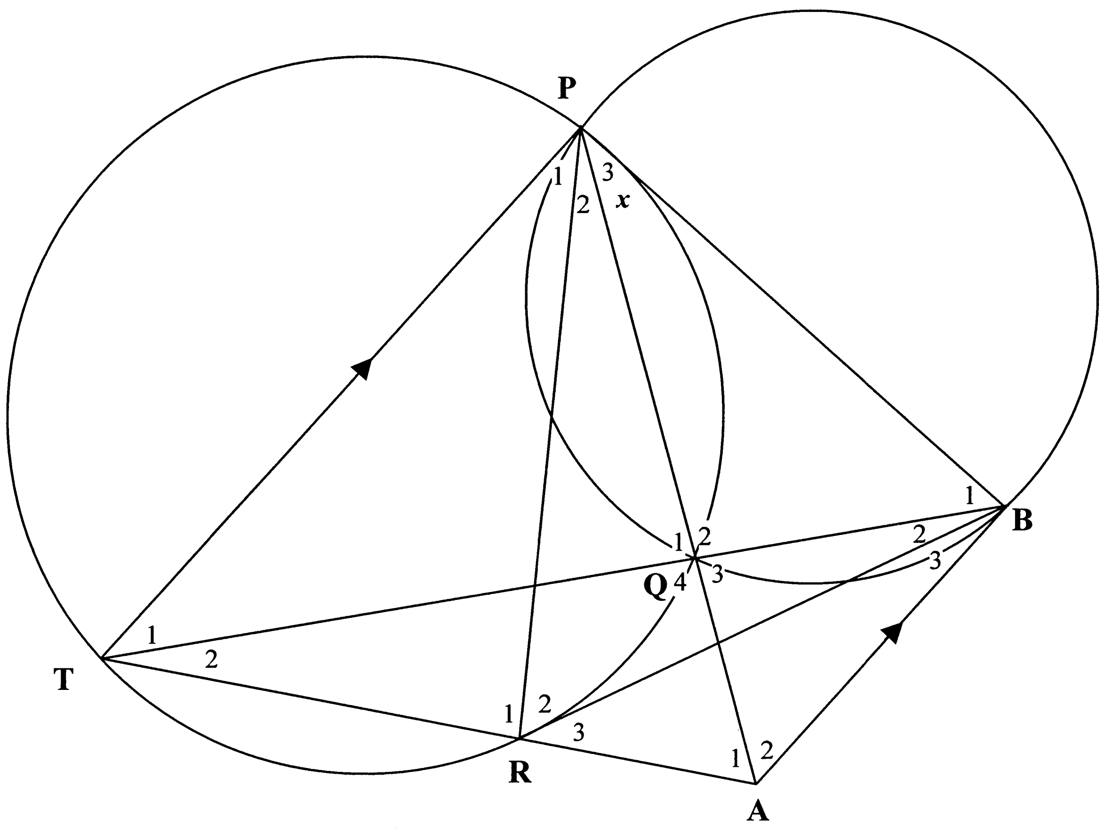
AB is a tangent to the smaller circle, with PQA a straight line.

BQ produced meets the larger circle at T such that PT || BA.

TA intersects the larger circle at R.

PR, PB and RB are drawn.

Let $\hat{P}_3 = x$



8.3.1 Name, stating reasons, TWO other angles each equal to x. (3)

8.3.2 Prove that:

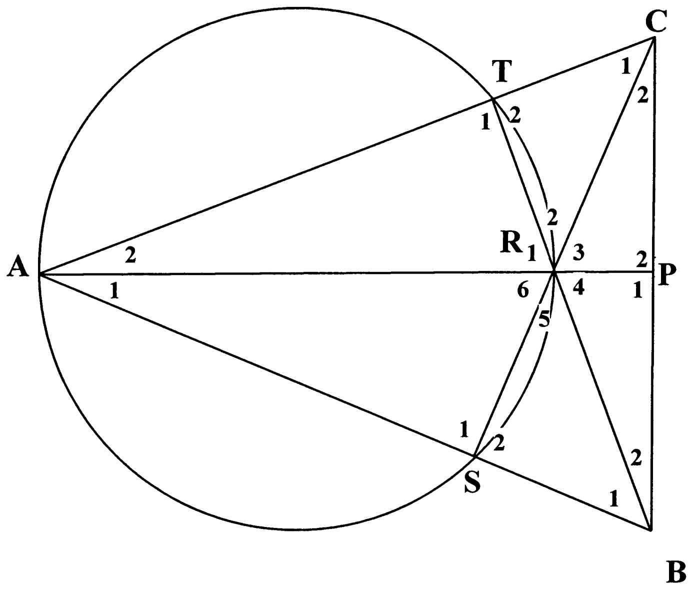
(a) PRAB is a cyclic quadrilateral (5)

(b) AB is a tangent to circle TRB (5)

[21]

QUESTION 9

In the diagram alongside, AR is a diameter of circle ASRT. AS, AR and AT are produced to B, P and C respectively so that BPC is a straight line. SC and TB intersect at R.



9.1 Prove that AP is an altitude of Δ ACB. (4)

9.2 If it is further given that AP is the bisector of $B\hat{A}C$, then prove that TS || CB. (8)

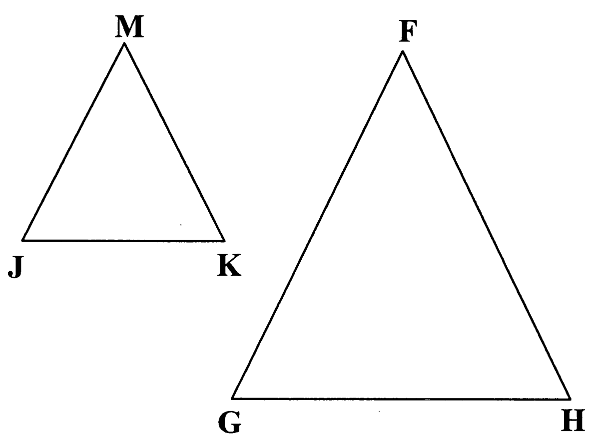
[12]

QUESTION 10

10.1 In the diagram alongside, Δ MJK and Δ FGH are given. Use the diagram on the diagram sheet, or redraw the diagram in your answer book to prove the theorem which states that:

If $\hat{M} = \hat{F}$, $\hat{J} = \hat{G}$ and $\hat{K} = \hat{H}$,

then $\frac{GH}{JK} = \frac{FH}{MK}$



(7)

10.2 In the diagram alongside AB is the diameter of the circle with centre O.

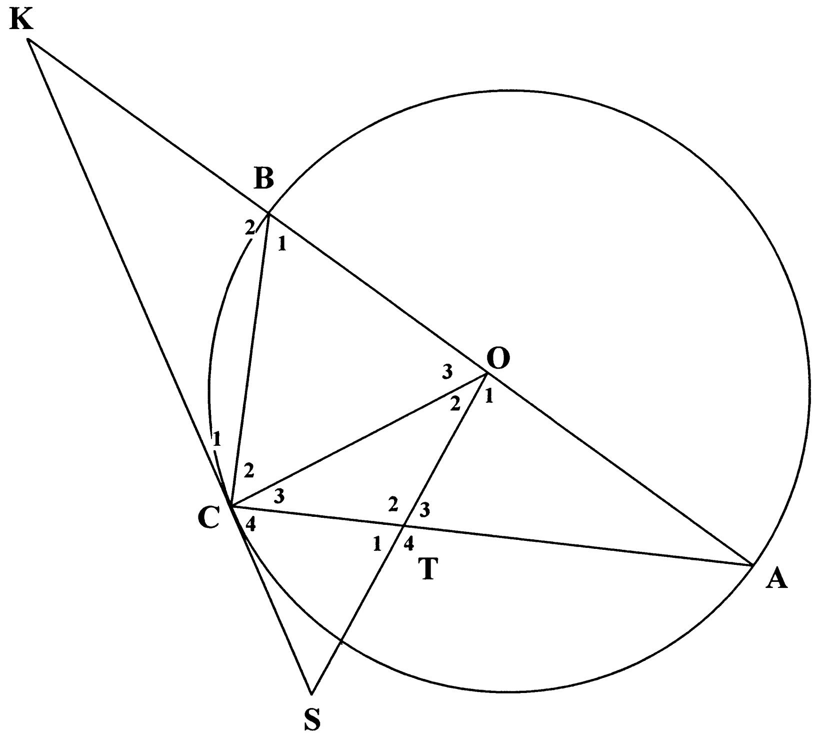
SK is a tangent to the circle at C.

SO ⊥ AB

CA and SO intersect at T.

KBOA is a straight line.

Let $\hat{A} = x$



Prove that :

10.2.1 $K\hat{C}T = \hat{T}_2$ (6)

10.2.2 Δ CKB ||| Δ AKC ||| Δ COT (6)

10.2.3 $BK \cdot AK = \dfrac{OT^2 \cdot CA^2}{CT^2}$ (5)

[24]

TOTAL: 200

Mathematics Formula Sheet (HG and SG)
Wiskunde Formuleblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}(a + T_n)$ $\quad S_n = \frac{n}{2}(a + \ell)$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = a.r^{n-1}$ $\quad S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$ $\quad S_n = \frac{a(r^n - 1)}{r-1} \quad (r \neq 1)$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

$A = P\left(1 + \frac{r}{100}\right)^n$ OR / OF $A = P\left(1 - \frac{r}{100}\right)^n$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$(x_3 \; ; \; y_3) = \left(\frac{x_1 + x_2}{2} \; ; \; \frac{y_1 + y_2}{2}\right)$$

$$x^2 + y^2 = r^2$$

$$(x-p)^2 + (y-q)^2 = r^2$$

In Δ ABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta ABC = \tfrac{1}{2} ab.\sin C$$

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SENIOR CERTIFICATE EXAMINATION/SENIORSERTIFIKAAT-EKSAMEN
MATHEMATICS HG/WISKUNDE HG
PAPER II/VRAESTEL II
FEBRUARY/MARCH 2006

DIAGRAM SHEET/DIAGRAMVEL

INSTRUCTION

This diagram sheet must be handed in with your answer book. Please ensure that your details are complete.

INSTRUKSIE

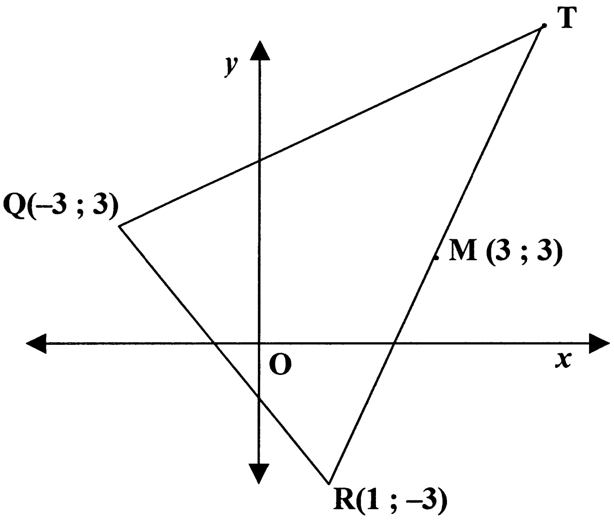
Hierdie diagramvel moet saam met jou antwoordeboek ingelewer word. Maak asseblief seker dat jou besonderhede volledig ingevul is.

EXAMINATION NUMBER **EKSAMENNOMMER**														

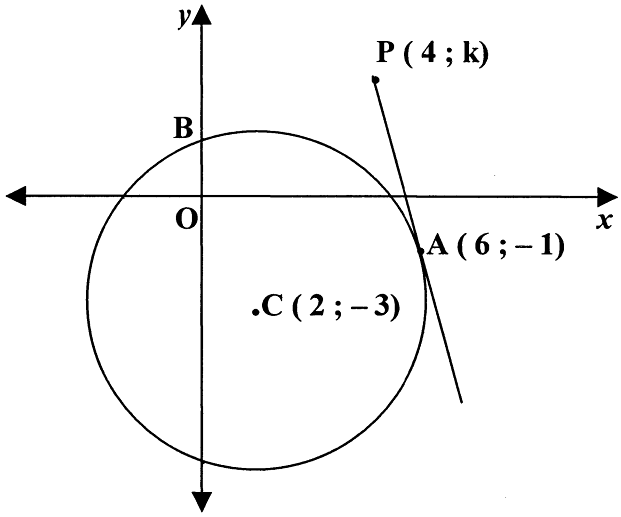
CENTRE NUMBER **SENTRUMNOMMER**								

EXAMINATION NUMBER
EKSAMENNOMMER

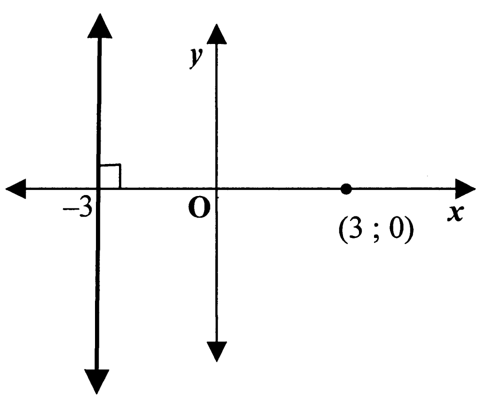
QUESTION 1 / VRAAG 1



QUESTION 2.1 / VRAAG 2.1



QUESTION 2.2 / VRAAG 2.2



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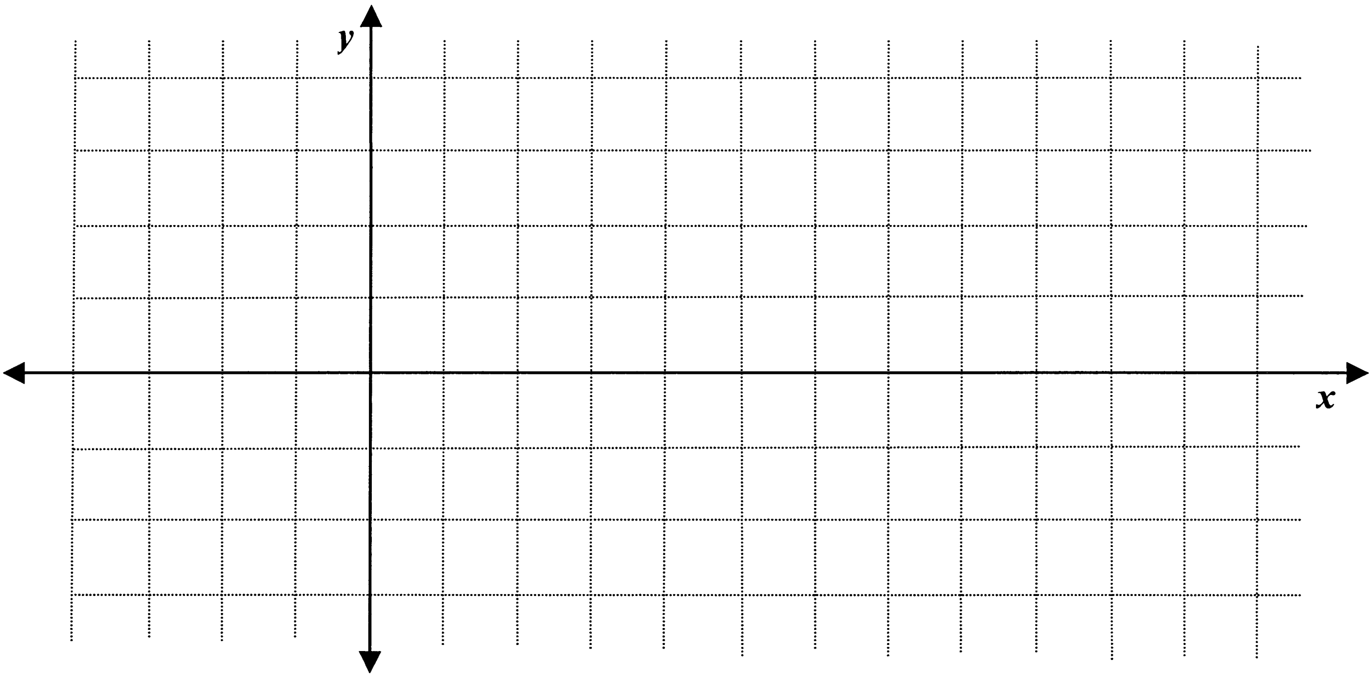
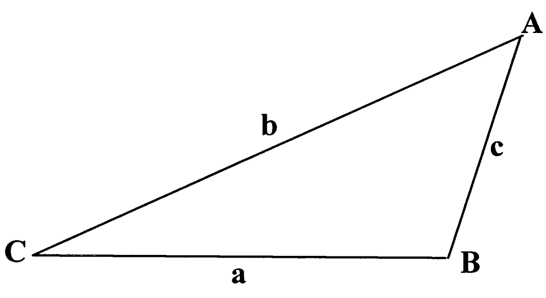
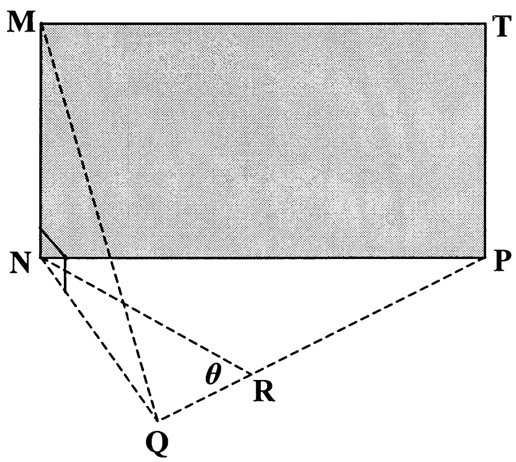
QUESTION 4.3 / VRAAG 4.3



QUESTION 6.1 / VRAAG 6.1



QUESTION 6.2 / VRAAG 6.2



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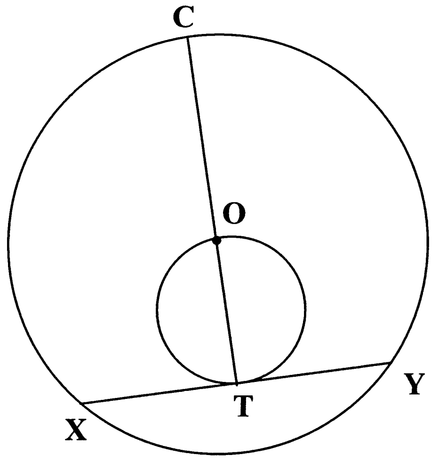
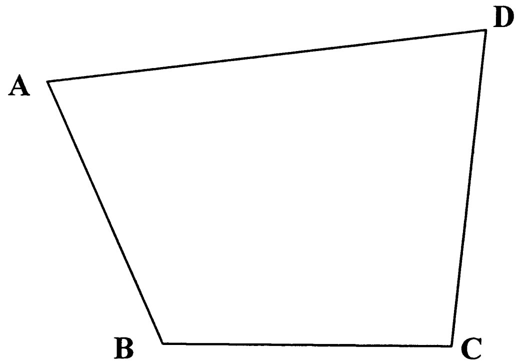
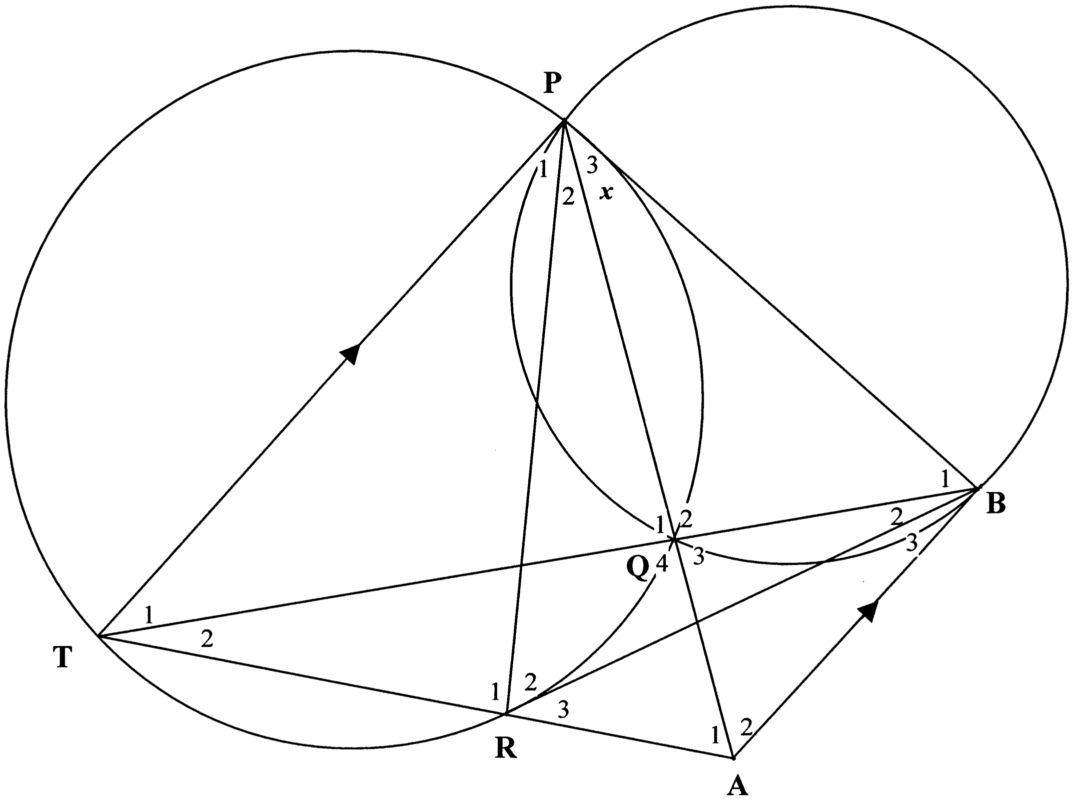
QUESTION 7 / VRAAG 7



QUESTION 8.1 / VRAAG 8.1



QUESTION 8.3 / VRAAG 8.3



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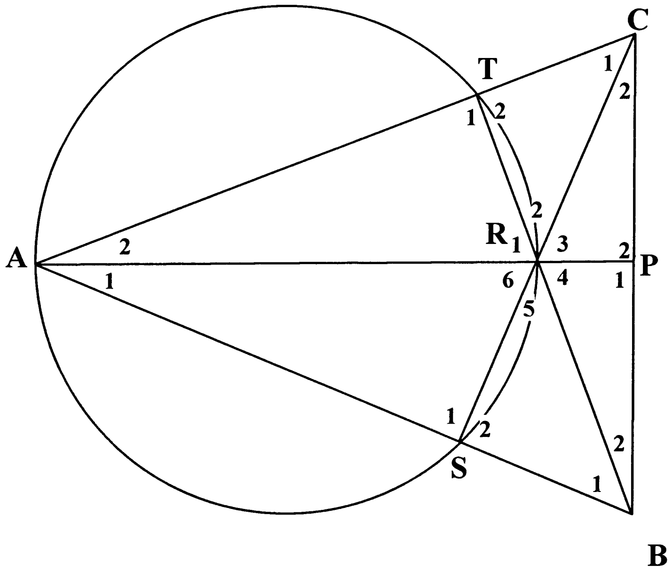
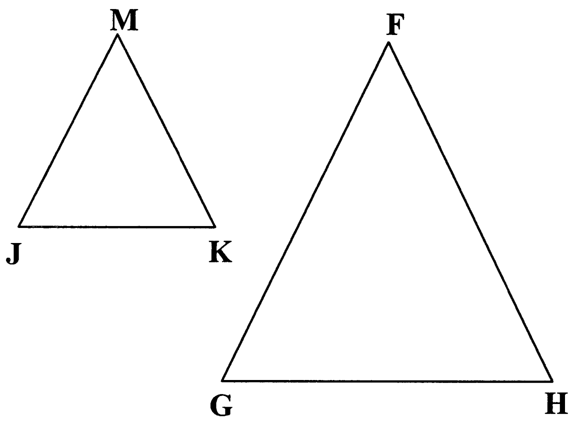
QUESTION 9 / VRAAG 9



QUESTION 10.1 / VRAAG 10.1



QUESTION 10.2 / VRAAG 10.2