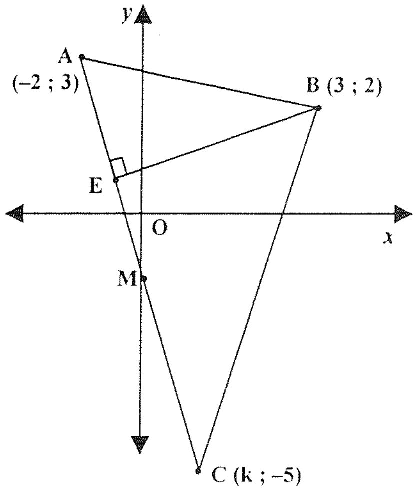
MATHEMATICS HIGHER GRADE PAPER TWO NOVEMBER 2006

QUESTION 1 [24]

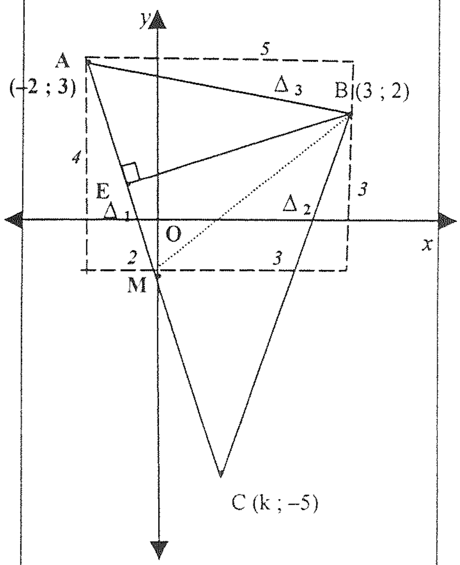
INCORRECT FORMULA NO MARKS
NO MARKS GIVEN FOR FORMULA ONLY

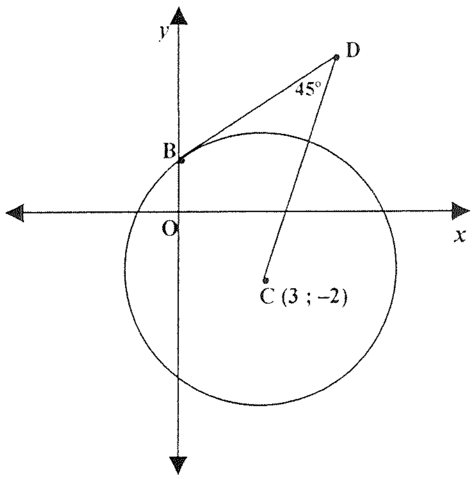


1.1	$k = 2$ ✓A (1)	1A correct value
1.2	$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{2-3}{3+2}$ ✓M $= -\frac{1}{5}$ ✓A $m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-5-3}{2+2}$ $= -2$ ✓CA $\tan\alpha = m$ ✓M **OR** $\tan A = \frac{m_{AB} - m_{AC}}{1 + m_{AB}.m_{AC}}$ ✓M inclination AC $= 116{,}6^\circ$ ✓CA $\qquad = \frac{\left(-\frac{1}{5}\right)-(-2)}{1+\left(-\frac{1}{5}\right)(-2)}$ ✓CA inclination AB $= 168{,}7^\circ$ ✓CA $\qquad = \frac{1\frac{4}{5}}{1+\frac{2}{5}}$ ✓CA $\hat{A} = 168{,}7^\circ - 116{,}6^\circ \qquad = \frac{9}{7}$ $= 52{,}1^\circ$ ✓CA $\qquad \hat{A} = 52{,}1^\circ$ ✓CA **OR**	**First rounding off Penalty 1** 1M use of gradient formula 1A m_{AB} 1CA on 1.1 1M use of inclination/tan formula 1CA inclination AC/substitution 1CA inclination AB/simplification 1CA correct size

	OR	
	$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ ✓M	1M use of distance formula
	$= \sqrt{(-2-3)^2 + (3-2)^2}$	
	$= \sqrt{26}$ ✓A	1A correct value
	$BC = \sqrt{(3-2)^2 + (2-(-5))^2}$	
	$= \sqrt{50}$ ✓CA	1CA correct value
	$AC = \sqrt{(-2-2)^2 + (3-(-5))^2}$	
	$= \sqrt{80}$ ✓CA	1CA correct value
	$\cos A = \dfrac{26+80-50}{2\sqrt{26}.\sqrt{80}}$ ✓M ✓CA	1M , 1CA subst. in any cos formula
	$\hat{A} = 52,1°$ ✓CA (7)	1CA correct size
1.3	$m_{alt.} = \dfrac{1}{2}$ ✓M	1M gradient of altitude
	$y = mx + c$ **or** $y - y_1 = m(x - x_1)$	
	$2 = \dfrac{1}{2}(3) + c$ $y - 2 = \dfrac{1}{2}(x - 3)$ ✓M ✓A	1 M equation of str. line 1A substitution
	$c = \dfrac{1}{2}$ $2y - 4 = x - 3$	
	$y = \dfrac{1}{2}x + \dfrac{1}{2}$ **or** $2y = x + 1$ ✓CA (4)	1 CA equation
1.4	For E: Eq BE $y = \dfrac{1}{2}x + \dfrac{1}{2}$ ....(1) **or** $2y = x + 1$	
	✓M ✓CA ✓A	1 M equation of straight line 1 CA gradient 1 A y–intercept
	Eq. AEC: $y = -2x - 1$ ....(2) **or** $2y = -4x - 2$	
	Subst. (2) into (1)	
	$\dfrac{1}{2}x + \dfrac{1}{2} = -2x - 1$ ✓M **or** $x + 1 = -4x - 2$	1M equating
	$\dfrac{5}{2}x = -\dfrac{3}{2}$ **or** $5x = -3$	
	$x_E = -\dfrac{3}{5}$ ✓CA	1CA x–value
	Sub. in (1) $y_E = -2(-\dfrac{3}{5}) - 1$	
	$= \dfrac{1}{5}$ ✓CA	1CA y– value
	$E\left(-\dfrac{3}{5} ; \dfrac{1}{5}\right)$ (6)	

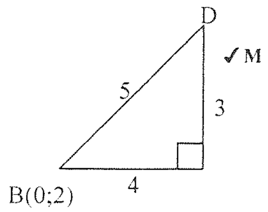
1.5	$BE = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(3 + \frac{3}{5})^2 + (2 - \frac{1}{5})^2}$ ✓M $= \sqrt{\frac{405}{25}}$ **or** $\sqrt{\frac{81}{5}}$ **or** $\frac{9}{\sqrt{5}}$ **or** $\frac{9\sqrt{5}}{5}$ **or** 4,025... ✓CA $AM = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **or** $AC = \sqrt{80}$ $= \sqrt{(0 + 2)^2 + (-1 - 3)^2}$ ✓M $= \sqrt{20}$ **or** $2\sqrt{5}$ **or** 4,47. ✓CA Area Δ ABM $= \frac{1}{2}$ BE x AM or $\left(\frac{1}{2}BE.\frac{1}{2}AC\right)$ $= \frac{1}{2}(\frac{9}{\sqrt{5}})(2\sqrt{5})$ ✓M $= \frac{1}{4}\cdot\frac{\sqrt{405}}{5}.\sqrt{80}$ = 9 units^2 ✓CA **OR** 8,99... units^2 **OR** $AB = \sqrt{(-2 - 3)^2 + (3 - 2)^2}$ $= \sqrt{26}$ ✓A Area Δ ABM $= \frac{1}{2}$ Δ ACB $= \frac{1}{2}\left[\frac{1}{2}(AB)(AC)\sin A\right]$ ✓M ✓CA ✓A ✓CA $= \frac{1}{2}\left[\frac{1}{2}(\sqrt{26})(2\sqrt{20})\sin 52{,}1°\right]$ = 8.99... units^2 ✓CA **OR** $AM = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ ✓M $= \sqrt{(0 + 2)^2 + (-1 - 3)^2} = \sqrt{20}$ ✓CA $AB = \sqrt{(-2 - 3)^2 + (3 - 2)^2}$ $= \sqrt{26}$ ✓A Area Δ ABM $= \frac{1}{2}$ AM. AB sin A ✓M $= \frac{1}{2}\sqrt{20}\ \sqrt{26}\ \sin 52{,}1°$ ✓CA = 8,99... units^2 ✓CA	**No penalty for rounding off** 1 M substitution in dist. form. 1 CA simplification 1 M value 1 CA value 1 M substitution in area form 1 CA value 1A length of AB 1M use of area formula as in diagram 3 CA substitution 1CA value 1 M substitution in dist. form. 1CA length 1A length 1M use of area formula 1CA substitution 1CA value

	OR Area Δ ABM = Rectangle – Δ_1 – Δ_2 – Δ_3 ✓M $= (5)(4) - \frac{1}{2}(4)(2) - \frac{1}{2}(3)(3) - \frac{1}{2}(1)(5)$ ✓A ✓CA ✓CA ✓A $= 20 - 4 - 4\frac{1}{2} - 2\frac{1}{2}$ = 9 units ✓CA (6)	

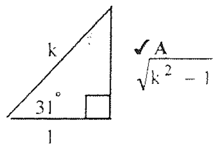
QUESTION 2 [27]		
		
2.1.1	For B, x = 0 $-4y = -8$ ✓A $y = 2$ ✓A B (0 ; 2) (2)	1 A x-value B 1A y-value B
2.1.2	$r^2 = (3-0)^2 + (-2-2)^2$ ✓M $= 25$ ✓CA $(x-3)^2 + (y+2)^2 = 25$ ✓M ✓CA $x^2 - 6x + 9 + y^2 + 4y + 4 - 25 = 0$ $x^2 - 6x + y^2 + 4y - 12 = 0$ **OR**	1M subst. 1CA value of r^2 1M subst. in circ. 1 CA expansion

	OR $x^2 - 6x + 9 + y^2 + 4y + 4 = 12 + 9 + 4$ $(x-3)^2 + (y+2)^2 = 25$ ✓M C (3 ; – 2) is the centre ✓CA Sub. B (0 ; 2) ✓M LHS: $(0)^2 - 6(0) + (2)^2 + 4(2) - 12 = 0$ = RHS B satisfies the given equation ✓CA (4)	1M completing the squares 1CA centre = C 1M substitution 1CA conclusion
2.1.3	For tangents: $x = 3 - 5$ ✓M ✓A or $x = 3 + 5$ $= -2$ $= 8$ ✓CA $\therefore$ q = 2 or q = – 8 ✓CA **OR** If (x + q) = 0 is a tangent, x = – q $\therefore \Delta = 0$ ✓M For $q^2 + 6q + y^2 + 4y - 12 = 0$ $4^2 - 4(1)(q^2 + 6q - 12) = 0$ ✓CA $16 - 4q^2 - 24q - 48 = 0$ $q^2 + 6q - 16 = 0$ ✓CA $(q-2)(q+8) = 0$ $\therefore$ q = 2 or q = – 8 ✓CA **OR** y-coordinate of each tangent point is y = –2 ✓M Subst. y = -2 in equation of circle : $x^2 - 6x + 4 - 8 - 12 = 0$ ✓CA $x^2 - 6x - 16 = 0$ $(x-8)(x+2) = 0$ ✓CA x = 8 or x = –2 $\therefore$ q = 2 or q = – 8 ✓CA (4)	**Answer only full marks** 1 M equation of tangent 1A use of r 1CA second tangent 1 CA values of q **Negatives of q $\frac{2}{4}$** 1M for equal roots, $\Delta = 0$ 1CA substitution into $\Delta = 0$ a = 1, b = 4 and c = $q^2 + 6q - 12$ 1CA factorizing 1CA values of q 1M 1CA substitution 1CA factorizing 1CA values of q
2.1.4 (a)	BD = BC = 5 ✓CA (1)	1 CA for length

2.1.4 (b)	Let A be a point on the y–axis with AD // x–axis. For $\tan B\hat{D}A = \frac{3}{4}$ ✓A $BD = 5$ $y_A = 2 + 3 = 5$ ✓M ✓CA A (0 ; 5) ✓CA For D ; $y = 5$ ✓A $3x = 4(5) - 8$ $x = 4$ ✓CA D(4 ; 5) **OR** $BD^2 = 25$ Let D be (x ; y) $(x-0)^2 + (y-2)^2 = 25$ ✓M $x^2 + y^2 - 4y + 4 = 25$ ✓CA $x^2 = -y^2 + 4y + 21$ … (1) $4y = 3x + 8$ ….. (2) Sub. (2) into (1) $x^2 + [\frac{1}{4}(3x+8)]^2 = 3x + 8 + 21$ ✓M $16x^2 + 9x^2 + 48x + 64 - 48x - 29(16) = 0$ $25x^2 - 400 = 0$ $x^2 - 16 = 0$ $(x-4)(x+4) = 0$ ✓CA $x = 4$ or $x = -4$ For D: $x = 4$ ✓CA $y = 7(4) - 23$ $= 5$ ✓CA D (4 ; 5) **OR** D ($r\cos\theta$; $2 + r\sin\theta$) ✓M ✓CA Where $r = 5$; $\cos\theta = \frac{4}{5}$; $\sin\theta = \frac{3}{5}$ ✓CA $D\left(5(\frac{4}{5})\ ;\ 2+5(\frac{3}{5})\right)$ ✓M D (4 ; 5) ✓CA ✓CA **OR**	1A 1M finding y_A 1CA substitution 1CA coordinates of A 1 A $y_D = y_A$ 1CA x_D **Correct Answer Full marks** **x OR y correct $\frac{3}{6}$ (coordinates reversed 0)** 1M subst. into distance formula 1CA calculating length of BD 1M sub. (1) into (2) 1 CA factorising 1 CA x–value 1CA y – value 1 M 1A 1 CA 1M substitution 1 CA x–value 1 CA x–value

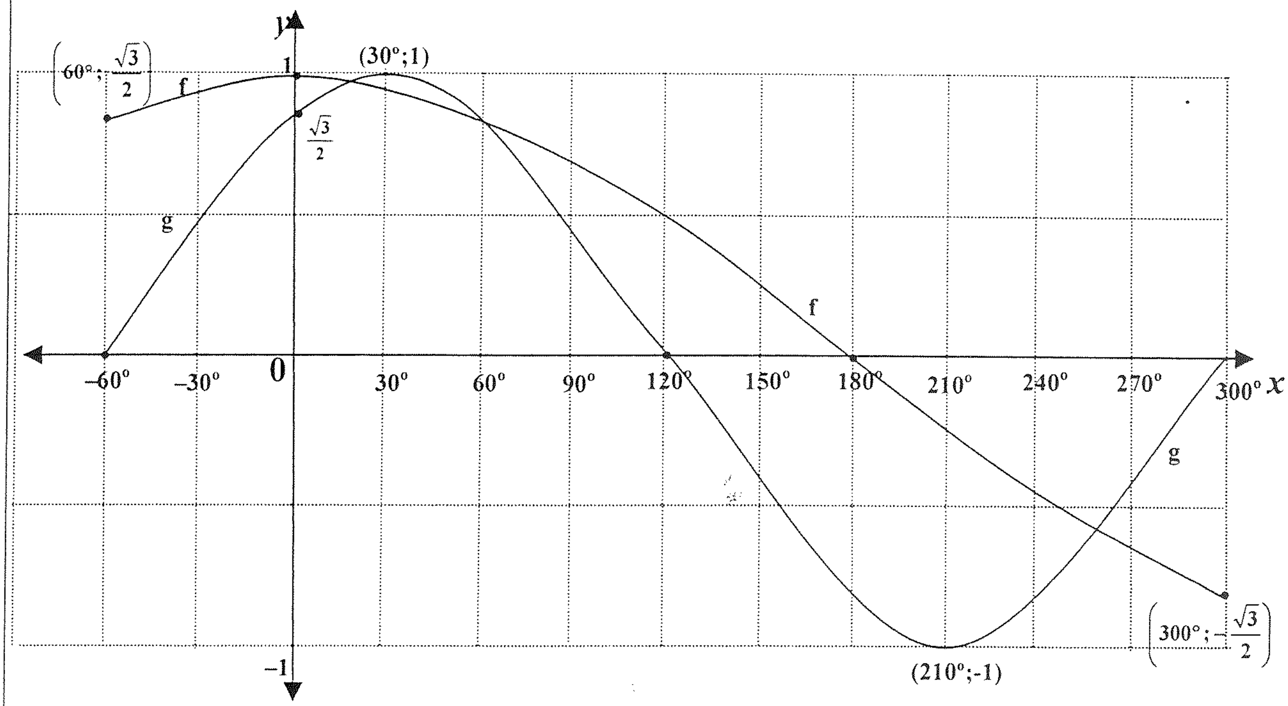
2.1.4 (b)	**OR**	
	$BD = 5$ $(x-0)^2 + (y-2)^2 = 25$ ✓M	1M subst. into distance formula
	$x^2 + y^2 - 4y + 4 = 25$ ✓CA $x^2 = -y^2 + 4y + 21$ … (1) $3x = 4y + 8$ ….. (2) $\therefore \quad x = \left(\frac{4y-8}{3}\right)$ …… (3)	1CA calculating length of BD
	Sub. (3) into (1) ✓M $\left(\frac{4y-8}{3}\right)^2 = -y^2 + 4y + 21$ $\frac{16y^2 - 64y + 64}{9} = -y^2 + 4y + 21$	1M sub. (3) into (2)
	$16y^2 - 64y + 64 + 9y^2 - 36y + 36 = 225$ $25y^2 - 100y - 125 = 0$ $y^2 - 4y - 5 = 0$ $(y-5)(y+1) = 0$ ✓CA $y = 5$ or -1 ✓CA	1 CA factorising 1 CA y–value
	For D: $y = 5, \quad x = \frac{4(5)-8}{3} = 4$ ✓CA D (4 ; 5)	1CA x – value
	OR	
	Equation of BD is $3x - 4y + 8 = 0$ $\therefore \quad y = \frac{3}{4}x + 2$ $\therefore \quad m_{BD} = \frac{3}{4}$ ✓A and $BD = 5$ $\therefore x_D = 0 + 4 = 4$ ✓M ✓CA and $y_D = 2 + 3 = 5$ ✓M ✓CA $\therefore$ D (4 ; 5)	
	OR	
	Equation of BD : $y = \frac{3}{4}x + 2$ $m_{BD} = \tan\theta = \frac{3}{4}$ ✓A $m_{CD} = \tan\alpha = \tan(45° + \theta)$ ✓M $= \frac{\tan 45° + \tan\theta}{1 - \tan 45° \tan\theta}$ $= \frac{1 + \frac{3}{4}}{1 - \frac{3}{4}} = 7$ ✓A ✓M Equation of CD : $y + 2 = 7(x - 3)$ $y = 7x - 23$ ……………(i) Equation of BD : $4y = 3x + 8$ …………..(ii) Finding point of intersection (ii) – 4(i) $25x - 100 = 0$ $x = 4$ ✓CA and $y = 5$ ✓CA $\therefore$ D (4 ; 5)	
	(6)	

2.1.5	$m_{CD} = 7$ $\quad\quad$ $m_{CD} = 7$ $m_{CE} = \frac{y_2 - y_1}{x_2 - x_1}$ **OR** $m_{DE} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-2+9}{3-2}$ ✓M $\quad\quad$ $= \frac{-2-5}{3-4}$ $= 7 = m_{CD}$ ✓CA $\quad\quad$ $= 7$ D, C and E are collinear ✓CA **OR** $EC = \sqrt{(2-3)^2 + (-9+2)^2} = \sqrt{50}$ ✓M $CD = \sqrt{50}$ $ED = \sqrt{(2-4)^2 + (-9-5)^2} = \sqrt{200}$ $ED = EC + CD$ ✓CA E, C and D are collinear ✓CA **OR** $m_{CD} = \frac{5+2}{4-3} = 7$ $y + 2 = 7(x-3)$ ✓M $y = 7x - 23$ Subst $x = 2$ $y = 7(2) - 23 = -9 =$ LHS ✓CA E, C and D are collinear ✓CA (3)	1 M substitution 1 CA 1 CA conclusion 1M finding lengths 1CA equating lengths 1 CA conclusion 1 M equation of CD 1CA substitution 1 CA conclusion
2.2	$PR = 2PT$ ✓M $PR^2 = 4PT^2$ $(x-1)^2 + (y+4)^2 = 4[(x+2)^2 + (y+1)^2]$ ✓A ✓CA $x^2 - 2x + 1 + y^2 + 8y + 16 = 4[x^2 + 4x + 4 + y^2 + 2y + 1]$ ✓CA ✓CA $x^2 - 2x + y^2 + 8y + 17 = 4x^2 + 16x + 4y^2 + 8y + 20$ ✓CA $3x^2 + 18x + 3y^2 = -3$ ✓CA **OR** $x^2 + 6x + y^2 = -1$ (7) **Max $\frac{4}{7}$ if $2PR^2 = PT^2$** **Max $\frac{6}{7}$ if $PR^2 = 2PT^2$**	1 M 1A 1 CA Sub. 2 CA manipulation of squares 1CA simplification 1 CA equation **Max $\frac{5}{7}$ if $PT = 2PR$** **Max $\frac{3}{7}$ if $PR = PT$**

QUESTION 3 [18]		
3.1.1	$\sec 751^\circ = \sec 31^\circ = k$ ✓A $\cos 31^\circ = \frac{1}{k}$ ✓CA (2)	1 A $\cos 31^\circ$ 1 CA substitution **answer only full marks**
3.1.2	$2\ \text{cosec}(-121^\circ) = -2\ \text{cosec}\ 59^\circ$ ✓M $= -2 \sec 31^\circ$ ✓A $= -2k$ ✓CA (3) **OR** $2\ \text{cosec}(-121^\circ) = -2\ \text{cosec}\ 59^\circ$ ✓M $= -\frac{2}{\sin 59^\circ}$ $= -\frac{2}{\cos 31^\circ}$ ✓A $= -\frac{2}{\frac{1}{k}}$ $= -2k$ ✓CA	1 M reduction 1A identity 1 CA for substitution **answer only full marks** 1 M reduction 1A identity 1 CA for substitution **answer only full marks**
3.1.3	$\tan 329^\circ = -\tan 31^\circ$ ✓M $= -\sqrt{k^2 - 1}$ ✓CA  **OR** $\tan 329^\circ = -\tan 31^\circ$ ✓M $= -\sqrt{\sec^2 31^\circ - 1}$ ✓A $= -\sqrt{k^2 - 1}$ ✓CA (3)	1 M reduction 1 A for $\sqrt{k^2 - 1}$ 1 CA for substitution 1 M reduction 1A identity 1 CA for substitution **answer only full marks**

3.2	$\sqrt{\tan(-207°)\,.\,\cot 333° - \frac{\sin^2(x-360°)\,.\,\text{cosec}(x-90°)}{\cos x}}$	
	$= \sqrt{-\tan 27° \,.\, -\cot 27° - \frac{\sin^2 x\,.\,(-\sec x)}{\cos x}}$ ✓A ✓A ✓A ✓A	4 A reductions
	$= \sqrt{\tan 27°\,.\,\frac{1}{\tan 27°} + \frac{\sin^2 x\,.\,\frac{1}{\cos x}}{\cos x}}$ ✓CA ✓CA	2CA identities
	$= \sqrt{1 + \frac{\sin^2 x}{\cos^2 x}}$ ✓CA	1 CA simplification
	$= \sqrt{1+\tan^2 x}$ ✓CA **or** $= \sqrt{\frac{\cos^2 x + \sin^2 x}{\cos^2 x}}$ ✓CA	1 CA simplification
	$= \sqrt{\sec^2 x}$ ✓CA $= \sqrt{\frac{1}{\cos^2 x}}$ ✓CA	1 CA identity
	$= \sec x$ ✓CA $= \frac{1}{\cos x}$ ✓CA	1 CA simplification
	(10)	**Calculator used to simplify tan (-207°). cot 333°** $\boxed{\frac{1}{3}}$
QUESTION 4	**[23]**	
4.1	$\sin(x+60°) = \cos\frac{1}{2}x$	**Correct reading from graph full marks**
		Answer only full marks: x = 20° (2) x = 260° (3) x = 60° (2)
	$\sin(x+60°) = \sin\left(90° - \frac{1}{2}x\right)$ ✓A **OR** $\sin(x+60°) = \sin\left(180° - 90° + \frac{1}{2}x\right)$ ✓M	1A co–function 1 M
	$x + 60° = 90° - \frac{1}{2}x + k.360°$ ✓CA $x + 60° = (90° + \frac{1}{2}x) + k.360°$	1CA
	$\frac{3}{2}x = 30° + k.360°$ $\frac{1}{2}x = 30° + k.360°$ ✓CA	1CA
	$x = 20° + k.240°$ $x = 60° + k.720°$	
	$x = 20°$ ✓CA **OR** $x = 260°$ ✓CA **OR** $x = 60°$ ✓CA	1CA 1 CA 1CA
	OR	**NOTE: General soln. not necessary**
	$\sin(x+60°) = \cos\frac{1}{2}x$	**Any extra feasible values Penalty 1**
	$\cos(30° - x) = \cos\frac{1}{2}x$ ✓A	1A co–function
	$30° - x = \frac{1}{2}x + k.360°$ ✓CA or $30° - x = -\frac{1}{2}x) + k.360°$ ✓M	1CA 1 M
	$-\frac{3}{2}x = -30° + k.360°$ $-\frac{1}{2}x = -30° + k.360°$ ✓CA	1CA
	$x = 20° + k.240°$ $x = 60° + k.720°$	
	$x = 20°$ ✓CA **or** $x = 260°$ ✓CA **or** $x = 60°$ ✓CA (7)	1CA 1 CA 1CA

4.2



For graph	f: $y = \cos\frac{1}{2}x$	g: $y = \sin(x + 60°)$
x–intercepts	✓A	✓A
y intercept	✓A	✓A
end points	✓A ✓A	
curvature	✓A	✓A
turning points	✓A	✓A

Graphs drawn outside given domain Penalty 1

If reflected in x–axis f : max $(\frac{3}{6})$; g: max $(\frac{2}{4})$

If g is shifted in wrong direction max $\frac{2}{4}$

(10)

4.3.1	$x \in (20°; 60°)$ ✓CA or $x \in (260°; 300°]$ ✓CA ✓A **OR** $20° < x < 60°$ ✓CA ✓CA or $260° < x \leq 300°$ ✓A $(x > 260°)$ (3)	2 X 1CA each interval 1A notation
4.3.2	$x \in [120°; 180°]$ ✓CA ✓A or $x = -60°$ or $x = 300°$ ✓CA **OR** $120° \leq x \leq 180°$ ✓A ✓A or $x = -60°$ or $x = 300°$ ✓A (3)	1CA interval 1 A notation 1CA x–intercepts of graphs

QUESTION 5 [22]		
5.1	$2\sin x + \text{cosec}\, x - 3 = 0$ $2\sin x + \frac{1}{\sin x} - 3 = 0$ ✓A $2\sin^2 x - 3\sin x + 1 = 0$ ✓A $(2\sin x - 1)(\sin x - 1) = 0$ ✓M $\sin x = \frac{1}{2}$ ✓CA or $\sin x = 1$ ✓CA $x = 30^\circ + k.360^\circ$ ✓CA or $150^\circ + k.360^\circ$ ✓CA or $90^\circ + k.360^\circ$ ✓CA $k \in Z$ ✓A (9)	1A identity 1A quadratic equation 1 M factorisation 2 CA sin x 3 CA values of x 1A general soln. notation.
	OR	**Penalty of 1 no general solution**
	$\frac{2}{\text{cosec}\, x} + \text{cosec}\, x - 3 = 0$ ✓A $\text{cosec}^2 x - 3\text{cosec}\, x + 2 = 0$ ✓A $(\text{cosec}\, x - 2)(\text{cosec}\, x - 1) = 0$ ✓M $\text{cosec}\, x = 2$ or $\text{cosec}\, x = 1$ $\sin x = \frac{1}{2}$ ✓CA or $\sin x = 1$ ✓CA $x = 30^\circ + k.360^\circ$ ✓CA or $150^\circ + k.360^\circ$ ✓CA or $90^\circ + k.360^\circ$ ✓CA $k \in Z$ ✓A (9)	1A identity 1A quadratic equation 1 M factorisation 2 CA sin x 3 CA values of x 1A general soln. notation
5.2.1	$\cos 2\theta = 2\cos^2\theta - 1$ ✓A (1)	1A
5.2.2	LHS: $2\cos\theta\cos 2\theta + \sec\theta(\sin 2\theta)^2$ $= 2\cos\theta(2\cos^2\theta - 1)$ ✓A $+ \frac{1}{\cos\theta}$ ✓A $(2\sin\theta.\cos\theta)^2$ ✓A $= 4\cos^3\theta - 2\cos\theta$ ✓CA $+ \frac{1}{\cos\theta} 4\sin^2\theta\cos^2\theta$ ✓CA $= 4\cos^3\theta - 2\cos\theta + 4(1 - \cos^2\theta)\cos\theta$ ✓CA $= 4\cos^3\theta - 2\cos\theta + 4\cos\theta - 4\cos^3\theta$ ✓CA $= 2\cos\theta =$ **RHS**	3 A (identities $\cos 2\theta$, $\sin 2\theta$, $\sec\theta$) 2 CA calculation 1 CA (identity) 1 CA simplification
	OR	
	LHS: $2\cos\theta\cos 2\theta + \sec\theta(\sin 2\theta)^2$ ✓A $= 2\cos\theta(2\cos^2\theta - 1) + \frac{1}{\cos\theta}$ ✓A $(2\sin\theta\cos\theta)^2$ ✓A $= 4\cos^3\theta - 2\cos\theta$ ✓CA $+ \frac{1}{\cos\theta} 4\sin^2\theta\cos^2\theta$ ✓CA $= 2\cos\theta(2\cos^2\theta - 1 + 2\sin^2\theta)$ ✓CA $= 2\cos\theta(2 - 1)$ ✓CA $= 2\cos\theta =$ **RHS**	3 A (identities $\cos 2\theta$, $\sin 2\theta$, $\sec\theta$) 2 CA calculation 1 CA (identity) 1 CA simplification
	OR	

	LHS: $2.\cos\theta\ \cos 2\theta + \sec\theta\ (\sin 2\theta)^2$ $= 2.\cos\theta.(\cos^2\theta - \sin^2\theta) + \frac{1}{\cos\theta}.(2\sin\theta.\cos\theta)^2$ ✓A ✓A ✓A $= 2.\cos^3\theta - 2\cos\theta\ \sin^2\theta + 4\sin^2\theta.\cos\theta$ ✓CA ✓CA $= 2.\cos^3\theta + 2\sin^2\theta.\cos\theta$ $= 2\cos\theta\ (\cos^2\theta + \sin^2\theta)$ ✓CA $= 2\cos\theta\ (1)$ ✓CA $= 2\cos\theta$ = **RHS** **OR** $2\cos\theta\ (1 - 2\sin^2\theta) + \frac{1}{\cos\theta}(2\sin\theta.\cos\theta)^2$ ✓A ✓A ✓A $= 2\cos\theta\ (1 - 2\sin^2\theta) + \frac{1}{\cos\theta}4\sin^2\theta.\cos^2\theta$ ✓CA $= 2\cos\theta\ (1 - 2\sin^2\theta) + 4\sin^2\theta.\cos\theta$ $= 2\cos\theta\ (1 - 2\sin^2\theta + 2\sin^2\theta)$ ✓CA ✓CA $= 2\cos\theta\ (1)$ ✓CA $= 2\cos\theta$ = RHS (7)	3 A (identities $\cos 2\theta$, $\sin 2\theta$, $\sec\theta$) 2 CA calculation 1 CA (identity) 1 CA simplification **Penalty of 1 if using LHS = RHS** **B/D if an incorrect identity is used Max $\frac{3}{7}$** **Penalty of 1 if θ left out**
5.3.1	$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A.\tan B}$ ✓A (1)	
5.3.2	$\tan(A + B) = \theta$ $4 = \frac{1 + \tan B}{1 - 1.\tan B}$ ✓M ✓A $1 + \tan B = 4 - 4\tan B$ ✓CA $5\tan B = 3$ $\tan B = \frac{3}{5}$ ✓CA (4)	1 M use of correct tan identity 1A correct substitution 1 CA simplification 1 CA simplification **Answer only Max $\frac{1}{4}$** **B/D Max $\frac{2}{4}$** **If $\frac{\tan A - \tan B}{1 + \tan A.\tan B}$ used max. $\frac{3}{4}$**

QUESTION 6 [22]

6.1

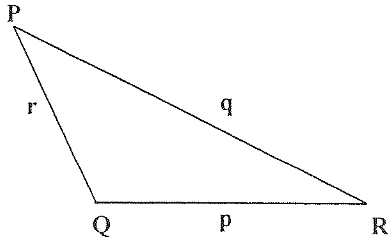
Area Δ PQR $= \frac{1}{2}$ p.r. sin Q ✓A

Area Δ PQR $= \frac{1}{2}$ q.r sin P ✓A

$\frac{1}{2}$ p.r. sin Q $= \frac{1}{2}$ q.r.sin P

$$\frac{\frac{1}{2}\text{p.r} \sin Q}{\frac{1}{2}.\text{p.r.q}} = \frac{\frac{1}{2}\text{q.r} \sin P}{\frac{1}{2}.\text{p.r.q}}$$ ✓M

$$\frac{\sin Q}{q} = \frac{\sin P}{p}$$



OR

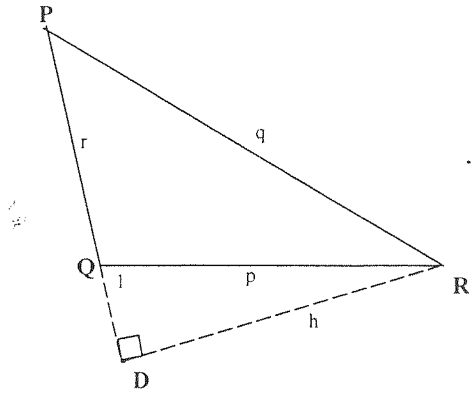
Constr. Draw RD, the height (h) of ΔPQR.
Proof:

In ΔPQD: $\frac{h}{p} = \sin Q_1 = \sin Q$ ✓M

$\therefore h = p \sin Q$ ✓A

similarly $h = q \sin P$ ✓A

$$\therefore \frac{\sin Q}{q} = \frac{\sin P}{p}$$ (3)



6.2

$$\frac{k}{\sin K} = \frac{n}{\sin N}$$ ✓M

$$\frac{k}{\sin 2\theta} = \frac{n}{\sin(90° - \theta)}$$ ✓A ✓A

$$k = \frac{n.2\sin\theta\cos\theta}{\cos\theta}$$ ✓A ✓A

$k = 2n . \sin\theta$

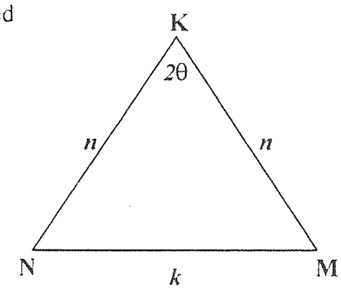
1M sine rule **OR** implied

1A sub. in sine rule

1A $\hat{N}$

1A expansion sin 2A

1A co–function ratio



OR

$k^2 = n^2 + m^2 - 2\,n.\,m.\cos K$ ✓M
$= 2n^2 - 2n^2.\cos 2\theta$ ✓A
$= 2n^2 [\,1 - (\,1 - 2\sin^2\theta\,)]$ ✓A ✓A
$= 2n^2\,(\,2\sin^2\theta\,)$ ✓A
$= 4n^2 . \sin^2\theta$
$k = 2n . \sin\theta$

1M cos rule **OR** implied
1A substitution
1A factorizing
1A expansion sin 2θ

1A square root

OR

Area Δ KMN $= \frac{1}{2}k.n.\sin(90° - \theta)$ ✓M
$= \frac{1}{2}k.n.\cos\theta$ ✓A

Area Δ KMN $= \frac{1}{2}n.n.\sin 2\theta$ ✓A
$= \frac{1}{2}n.^2\;2\sin\theta.\cos\theta$ ✓A

$\frac{1}{2}k.n.\cos\theta = n.^2\sin\theta.\cos\theta$ ✓A

$k = 2n . \sin\theta$

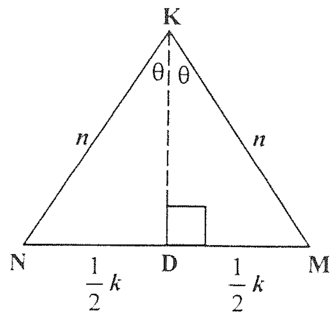
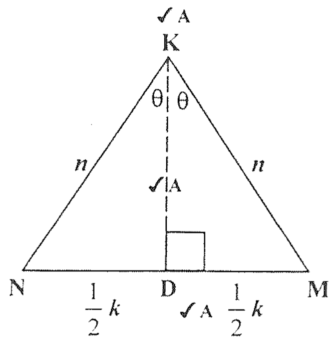
1M area rule **OR** implied

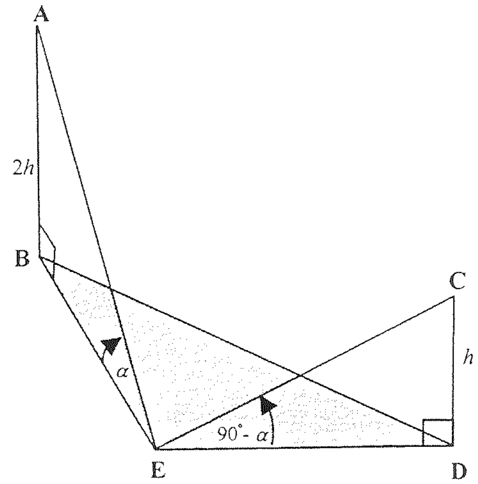
1 A substitution

1 A substitution
1A expansion of sin 2θ

1A equating

OR

	OR Draw perpendicular KD ✓M $ND = DM = \frac{1}{2}k$ ✓A $\cos N = \frac{\frac{1}{2}k}{n}$ ✓A $n\cos(90° - \theta) = \frac{k}{2}$ ✓A $2n.\sin\theta = k$ ✓A	
	OR RHS: $2n.\sin\theta = 2n\left(\frac{\frac{1}{2}k}{n}\right)$ ✓M ✓A $= k =$ LHS	
	OR $\hat{M} = \hat{N} = 90° - \theta$ ✓M $n^2 = k^2 + n^2 - 2kn.\cos(90° - \theta)$ ✓M ✓A $n^2 = k^2 + n^2 - 2kn.\sin\theta$ ✓A $k^2 = 2k.n\sin\theta$ ✓A $k = 2n.\sin\theta$ (5)	

6.3 6.3.1	In Δ ABE: $\tan\alpha = \frac{2h}{BE}$ ✓M $BE = 2h.\cot\alpha$ ✓A **OR** $BE = \frac{2h}{\tan\alpha}$ (2)	1M 1A 
6.3.2	In Δ CED: $\tan(90° - \alpha) = \frac{h}{DE}$ ✓M $ED = h \tan\alpha$ ✓A In Δ BDE : $BD^2 = BE^2 + ED^2 - 2(BE)(ED).\cos E$ ✓M $= (2h.\cot\alpha)^2 + (h \tan\alpha)^2 - 2(2h.\cot\alpha)(h \tan\alpha)\cos 120°$ ✓A $= 4h^2.\cot^2\alpha + h^2.\tan^2\alpha - 4h^2(\cot\alpha.\tan\alpha)(-\frac{1}{2})$ ✓A $= h^2(4\cot^2\alpha + \tan^2\alpha + 2)$ ✓CA $= h^2\left(\frac{4}{\tan^2\alpha} + \tan^2\alpha + 2\right)$ ✓CA ✓A $= \frac{h^2(\tan^4\alpha + 2\tan^2\alpha + 4)}{\tan^2\alpha}$ $BD = \frac{h\sqrt{\tan^4\alpha + 2\tan^2\alpha + 4}}{\tan\alpha}$ (8)	1M 1A (value of ED) 1M (application of cos rule) can be implied 1CA (substitution) 1A (cos 120°) 1CA (simplificaion) 1CA grouping 1A (identity) **NB: No conclusion Penalty 1**
6.3.3	$h = \frac{BD\tan\alpha}{\sqrt{\tan^4\alpha + 2\tan^2\alpha + 4}}$ ✓M $= \frac{509.\tan 48°}{\sqrt{\tan^4 48° + 2\tan^2 48° + 4}}$ ✓A CD = 200 m ✓CA ✓A (4)	1M making h subject 1A substitution (can be done first) 1A value of CD **1A rounding off** **answer only full marks**

QUESTION 7 [22]

7.1

Proof: Join MO and KO ✓M

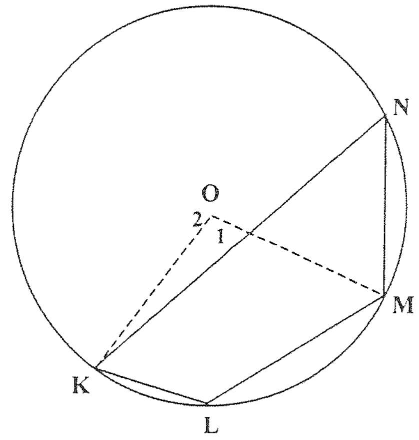
$\hat{O}_1 = 2\hat{N}$ ✓S (∠ at centre = 2∠ at circ.) ✓R

$\hat{O}_2 = 2\hat{L}$ ✓S (∠ at centre = 2∠ at circ.)

$\hat{O}_1 + \hat{O}_2 = 360^\circ$ (∠'s around a point) ✓S/R

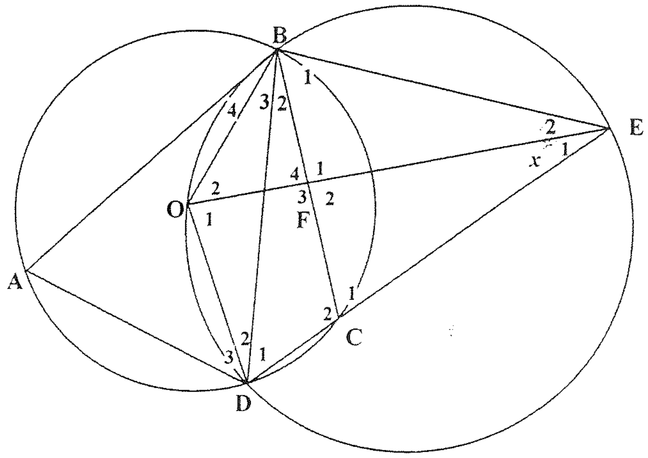
$2\hat{N} + 2\hat{L} = 360^\circ$ ✓S

$\therefore \hat{N} + \hat{L} = 180^\circ$ (6)



Penalty of 1 if diagram simplified

7.2



7.2.1

$\hat{B}_3 = \hat{E}_1 = x$ ✓S (∠'s in same segm) ✓R

$\hat{B}_3 = \hat{D}_2 = x$ (∠'s opp. = s's) ✓S/R

$B\hat{O}D == 180^\circ - 2x$ ✓S (sum ∠'s Δ)

$\hat{A} = 90^\circ - x$ ✓S (∠ cen. = 2 ∠ at circ.) ✓R

OR

OB = OD ✓S (radii)

Let $\hat{E}_1 = x$

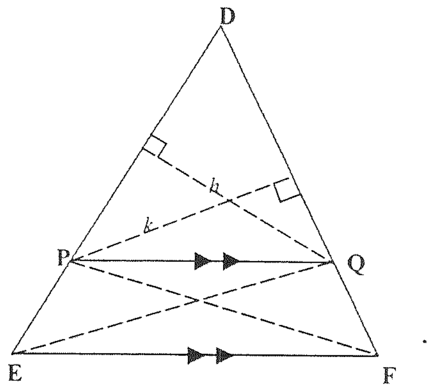
$\therefore \hat{E}_2 = x$ ✓S (equal chords subtend equal angles) ✓R

$\therefore B\hat{O}D == 180^\circ - 2x$ ✓S (sum ∠'s Δ)

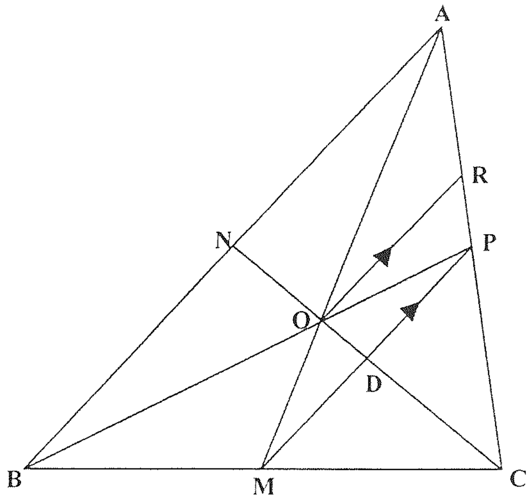
$\therefore \hat{A} = 90^\circ - x$ ✓S (∠ cen. = 2 ∠ at circ.) ✓R

(6)

7.2.2 (a)	$\hat{C}_1 = \hat{A} = 90^\circ - x$ ✓S (ext. ∠ of cycl. quad) ✓R $\hat{F}_2 = 180^\circ - (x + 90^\circ - x)$ ✓S (sum ∠'s Δ) $= 90^\circ$ In ΔBEF and Δ CEF $\hat{F}_1 = \hat{F}_2 = 90^\circ$ ✓S (Adj. ∠ s str line) BF = FC ✓S (line from cent. ⊥bisects chord) ✓R FE is common ΔBEF ≡ Δ CEF (S, ∠, S) ✓S/R BE = EC (≡) **OR** $\hat{C}_1 = 90^\circ - x$ ✓S (ext. ∠ of cycl. quad) ✓R $\hat{E}_2 = \hat{E}_1$ ✓S (= chords subt. = ∠'s) ✓R $\hat{B}_1 = 90^\circ - x$ (sum ∠'s Δ) ✓S ∴ $\hat{B}_1 = \hat{C}_1$ ✓S ∴ CE = BE (isosceles Δ) ✓R (7)	**OR** $\hat{C}_1 = \hat{A} = 90^\circ - x$ ✓S (ext. ∠ of cycl. quad) ✓R $\hat{F}_2 = 180^\circ - (x + 90^\circ - x)$ ✓S (sum ∠'s Δ) $= 90^\circ$ $= \hat{F}_1$ (adj. supp. ∠'s) ✓S $\hat{E}_2 = \hat{E}_1$ ✓S (= chords subt. = ∠'s) ✓R $\hat{B}_1 = 90^\circ - x$ (sum ∠'s Δ) ✓S $= \hat{C}_1$ BE = EC (sides opp = ∠'s) If assume EF ⊥ EC, B/D max $\frac{3}{7}$
7.2.2 (b)	BE = EC (proved) ✓S But EC is a secant (not a tangent) ✓S ∴ BE is not a tangent (tang. from common pt =) ✓R **OR** $\hat{B}_1 = 90^\circ - x$ ✓S (sum ∠'s Δ) ∴ $\hat{B}_1 = \hat{A}$ ✓S ∴ BE is not a tangent ($\hat{B}_1 + \hat{B}_2 \neq \hat{A}$) ✓R (3)	**OR** Check working for alternatives e.g. $\hat{B}_1 = 90^\circ - x$ ✓S (sum ∠'s Δ) $\hat{B}_1 + \hat{B}_2 + \hat{B}_3 \neq 90^\circ$ ✓S (line is not ⊥ rad.) ✓R **OR** $\hat{B}_1 = 90^\circ - x$ ✓S (sum ∠'s Δ) $\neq \hat{D}_1$ ✓S ∴ BE is not a tangent (∠ betw. line & chord ≠ ∠ subt. by chord.) ✓R

QUESTION 8 [18]		
8.1	Constr: Join PF and QE ✓S OR on diagram sheet Proof: $\frac{\text{area } \Delta DPQ}{\text{area } \Delta DEQ} = \frac{\frac{1}{2} DP \cdot h}{\frac{1}{2} DE \cdot h}$ ✓S ✓R (or same height) $= \frac{DP}{DE}$ $\frac{\text{area } \Delta DPQ}{\text{area } \Delta DPF} = \frac{\frac{1}{2} DQ \cdot k}{\frac{1}{2} DF \cdot k}$ (or same height) $= \frac{DQ}{DF}$ ✓S but, area Δ PQE = area Δ PQF (same base, same height) ✓S/R ∴ area Δ DEQ = area Δ DPF ✓S (Δ DPF common) $\therefore \frac{\text{area } \Delta DPQ}{\text{area } \Delta DEQ} = \frac{\text{area } \Delta DPQ}{\text{area } \Delta DPF}$ ✓S $\frac{DP}{DE} = \frac{DQ}{DF}$ (7)	 Term area omitted no Penalty Must show this for full marks

8.2



8.2.1

P is midpoint of AC ✓S (medians concur) ✓R

AB // PM (midpt th.) ✓S/R or (line // 1 side Δ)

In Δ BNC :

$$\frac{ND}{NC} = \frac{BM}{BC} = \frac{AP}{AC}$$ ✓S (line // 1 side Δ or midpt th) ✓R

$$= \frac{BM}{2BM} = \frac{1}{2}$$ ✓A

OR

$$\frac{AO}{OM} = \frac{2}{1}$$ ✓A

$$= \frac{AR}{RP}$$ ✓S (OR // MP)

PC = 3RP (median AP = PC) ✓R

$$\frac{RP}{PC} = \frac{1}{3}$$ ✓S

$$= \frac{OD}{DC}$$ (// lines)

NO = 2OP ✓S/R (median)

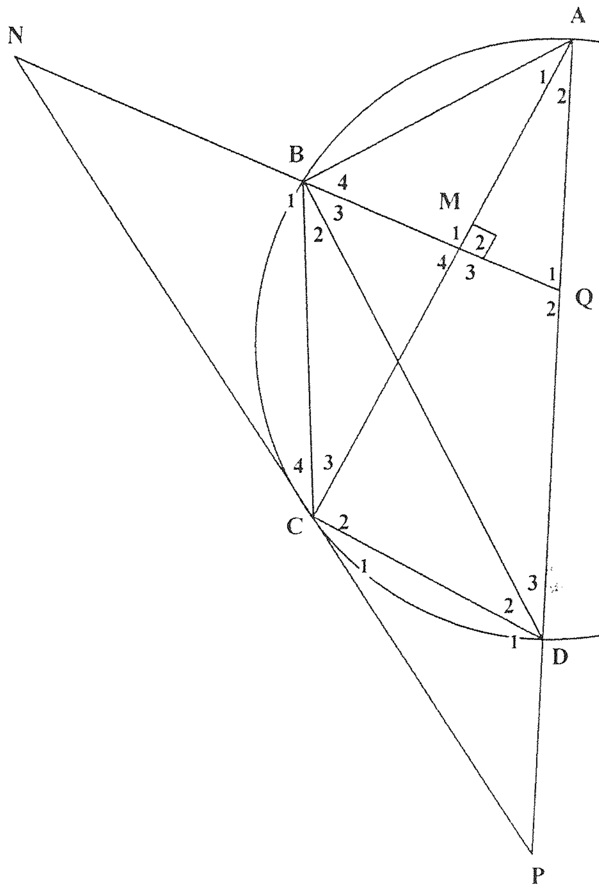
$$\frac{ND}{DC} = \frac{3}{6}$$

$$= \frac{1}{2}$$ ✓S

(6)

Answer only $\frac{3}{6}$

8.2.2	In Δ AMP: ✓S $\frac{AO}{OM} = \frac{2OM}{OM}$ $\frac{RP}{PC} = \frac{RP}{AP}$ ✓S (BP is a median) $= \frac{OM}{AM}$ ✓S (line // 1 side Δ) ✓R $= \frac{OM}{3OM}$ $= \frac{1}{3}$ ✓A **OR** $\frac{AO}{AM}$ ✓S $= \frac{2}{3} = \frac{AR}{AP}$ (line // 1 side Δ) ✓R $\therefore$ AR = 3k $\therefore$ PC = 3k ✓S $\therefore$ RP = k ✓S $\therefore \frac{RP}{PC} = \frac{1}{3}$ ✓A (5)	Answer only $\frac{4}{5}$

QUESTION 9 [24]		
9		
9.1	$\hat{C}_2 = 90°$ ✓S (∠ in semi-circle)✓R $\hat{M}_2 = 90°$ (AM ⊥ NM) ∴ NQ // CD (corresp. ∠'s =) ✓R (3)	(or alt ∠s or coint ∠s)
9.2	$\hat{C}_1 = \hat{N}$ ✓S/R (// lines, corresp. ∠'s) $\hat{A}_2 = \hat{C}_1$ ✓S (tan–chord) ✓R $= \hat{N}$ ∴ ANCQ is a cyclic quad. (∠'s subt. by same line segm) ✓R **OR** $N\hat{C}A = C\hat{D}A$ ✓S (tan – chord)✓R $= \hat{Q}_1$ (// lines, corresp. ∠'s)✓S/R ∴ ANCQ is a cyclic quad. (∠'s subt. by same line segm) ✓R (4)	