education

Department:
Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATION - 2006

MATHEMATICS P1 : ALGEBRA

HIGHER GRADE

FEBRUARY/MARCH 2006

301-1/1 E

Marks: 200

3 Hours

This question paper consists of 9 pages, 1 graph paper and 1 information sheet.

MATHEMATICS HG: Paper 1

301 1 1E HG

X05

INSTRUCTIONS

Read the following instructions carefully before answering the questions:

1. This paper consists of **8** questions. Answer **ALL** the questions.
2. Clearly show **ALL** calculations, diagrams, graphs, et cetera you have used in determining the answers.
3. An approved calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
4. If necessary, answers should be rounded off to **TWO** decimal places, unless stated otherwise.
5. The attached graph paper must be used only for **QUESTION 8**. Detach it from your question paper, fill in your examination number and centre number and insert it in the **FRONT** of the answer book.
6. Number the answers **EXACTLY** as the questions are numbered.
7. Diagrams are not necessarily drawn to scale.
8. It is in your own interest to write legibly and to present the work neatly.
9. **An information sheet with formulae is included at the end of the question paper.**

QUESTION 1

1.1 Solve for x:

1.1.1 $-3x^2+5x+2=0$ (2)

1.1.2 $|x+3|<9$ (3)

1.1.3 $x-7-\sqrt{x-5}=0$ (6)

1.1.4 $4(2^x)+3=\dfrac{1}{2^x}$ (5)

1.2 Given: $(x-2)(x-k)=-4$.

1.2.1 For which values of k will the equation have real roots? (7)

1.2.2 Find a value of k for which the roots are rational and unequal. (3)

1.3 Trevor employs a certain number of workers and pays each worker the same wage. His current daily wage bill is R5 880. A labour dispute has resulted in his workers demanding a wage increase of R10 per day. Trevor claims that he cannot afford this. He claims that only if he retrenches 4 workers will he be able to give them the increase that they demand. His daily wage-bill would then be R5 850.

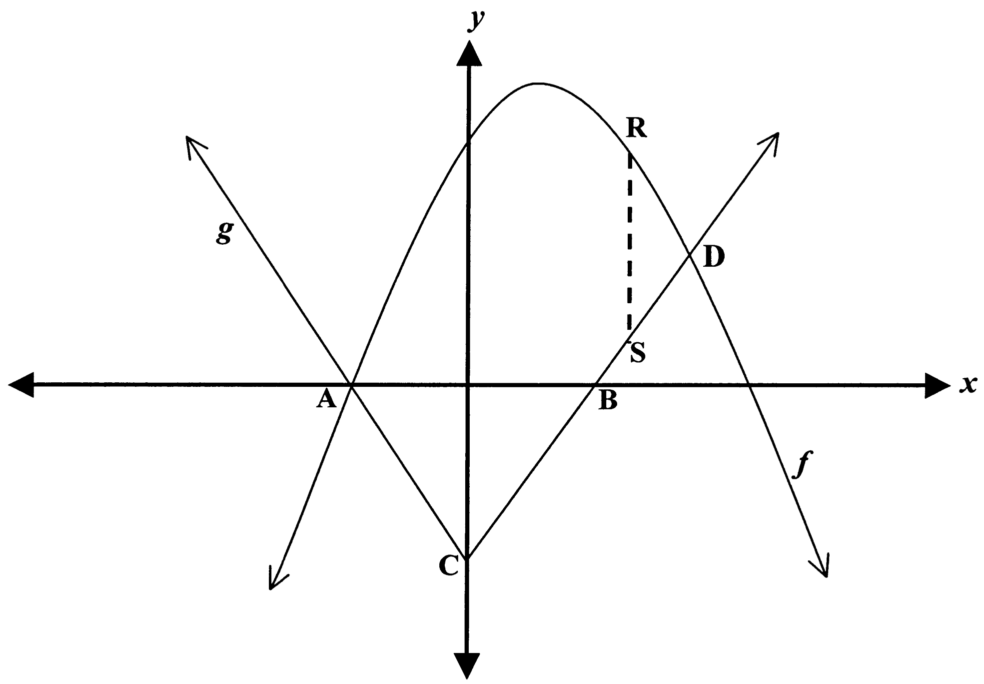
1.3.1 Calculate how many workers Trevor employs. (7)

1.3.2 How much does each worker earn per day, presently? (2)

[35]

QUESTION 2

2.1 The sketch below shows the graphs of the parabola defined by $f(x) = -x^2 + bx + c$ and the absolute value function defined by $g(x) = |x| - 3$.
The points A, B and C are the x- and y-intercepts of the graph of g. A and D are on both graphs.



2.1.1 Write down the co-ordinates of A. (2)

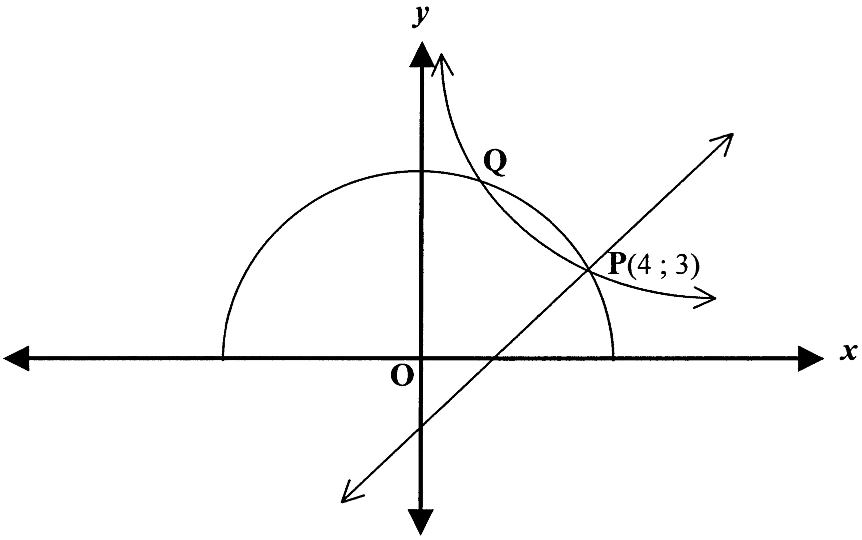
2.1.2 Given that the equation of the axis of symmetry of f is $x = 1$, show that the equation of the parabola is $y = -x^2 + 2x + 15$. (5)

2.1.3 It is further given that R and S are variable points on f and g, and that the straight line RS is parallel to the y-axis.

(a) If S moves between C and D, write down an expression for the length of RS in terms of x. (3)

(b) Determine the coordinates of R if RS is as large as possible. (5)

2.2 The sketch represents graphs of $xy = k\ (x > 0)$, $x^2 + y^2 = r^2\ (y \geqslant 0)$ and $y = mx + c$. All three graphs intersect at P(4 ; 3). The straight line has the same gradient as the axis of symmetry of the hyperbola.



2.2.1 Determine the values of k, r, m and c. (6)

2.2.2 Write down the co-ordinates of Q. (2)

2.3 $S\left(\frac{1}{2};\frac{1}{2}\right)$ is a point on the graph of f defined by $f(x) = a^x\ (a > 0)$.

2.3.1 Prove that $a = \frac{1}{4}$. (2)

2.3.2 Determine f^{-1} in the form $f^{-1}(x) = \ldots$ (2)

2.3.3 Calculate the value of x if $f^{-1}(x) = -1{,}5$. (3)

2.3.4 Sketch the graph of f and clearly indicate the co-ordinates of the intercepts with any of the axes. (2)

[32]

QUESTION 3

3.1 If $(x+2)$ is a common factor of $f(x) = x^3 + ax^2 + 2b$ and $g(x) = x^3 + ax - 4b$, determine the values of a and b. (5)

3.2 A polynomial $f(x)$ can be written in the form $f(x) = (x+k).q(x) - 12$. Calculate the value of k if $(x-3)$ is a factor of $f(x)$ and $q(x)$ leaves a remainder of 3 when divided by $(x-3)$. (4)

[9]

QUESTION 4

4.1 Simplify to a single number **without using a calculator**:

4.1.1 $\sqrt[3]{\left(\sqrt{13}-\sqrt{5}\right)^6} \,.\, \sqrt[3]{\left(\sqrt{13}+\sqrt{5}\right)^6}$ (4)

4.1.2 $3\log \sqrt[3]{40} - 2\log \frac{1}{5}$ (4)

4.2 Solve for x:

4.2.1 $3^{x+1} - 3^{x-1} = 24\sqrt{3}$ (5)

4.2.2 $7^x = 126\left(5^x\right)$ (round off to **two decimal** places) (3)

4.3 4.3.1 Prove that $\log_{\frac{1}{a}} x = -\log_a x$, for any $a > 0$ (3)

4.3.2 Solve for x: $\log_{10}(2x-5) \leq \log_{\frac{1}{10}}(x-3)$ (9)

[28]

QUESTION 5

5.1 The sum of the first 50 terms of an arithmetic series is 1 275. Calculate the sum of the 25^{th} and 26^{th} terms of this series. (6)

5.2 The sum of the first n terms of an arithmetic series is: $S_n = \dfrac{3n^2 - n}{2}$.

5.2.1 Determine S_{10}. (2)

5.2.2 Calculate the value of $\sum_{r=5}^{10} T_r$, where T_r is the r^{th} term of the series. (3)

5.3 The first term of a geometric sequence is 3 and the sum of the first 4 terms is 5 times the sum of the first 2 terms. The common ratio is greater than 1.

Calculate:

5.3.1 The first three terms of the sequence (7)

5.3.2 The value of n for which the sum to n terms will be 765 (4)

5.4 The first two terms of a convergent geometric series are m $(m \neq 0)$ and 6, in that order. The sum of the infinite series is 25. Calculate the values of m. (Check that these values are acceptable.) (7)

[29]

QUESTION 6

6.1 Determine $\lim\limits_{h \to 4} \dfrac{h^2 - h - 12}{16 - h^2}$ (3)

6.2 Given: $f(x) = -\dfrac{x^2}{2} + x$

6.2.1 Determine $f'(x)$, using the **definition of the derivative.** (6)

6.2.2 Use your answer in QUESTION 6.2.1 to determine the value of $\lim\limits_{h \to 0} \dfrac{f(1+h) - f(h)}{h}$ (2)

6.2.3 A tangent to the graph of f has gradient -5, and x-intercept $(a\ ;\ 0)$. Determine a. (6)

6.3 Determine $\dfrac{dy}{dx}$ if $y = \dfrac{5x^5 - 6x^{\frac{3}{2}} + 5}{x}$ (5)

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QUESTION 7

7.1 Given: $f(x) = -x^3 + 3x^2 - 4$

7.1.1 Determine the x- and y-intercepts of the graph of f. (7)

7.1.2 Determine the coordinates of the turning points of f. (5)

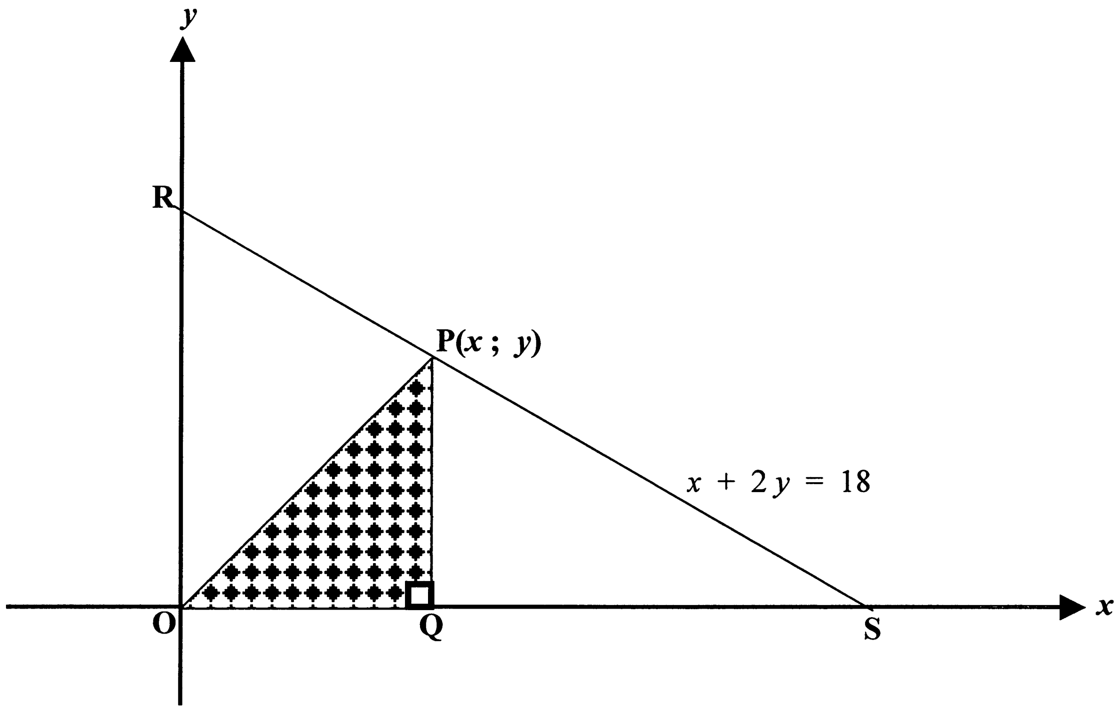
7.1.3 Sketch the graph of f. Show clearly all the turning points as well as the intercepts on the axes. (4)

7.1.4 For which values of x is f increasing? (2)

7.1.5 What is the maximum value of $-x^3 + 3x^2 - 4$ if $0 \leqslant x \leqslant 3$? (1)

7.1.6 How many solutions does the equation $f(x) = -5$ have? (1)

7.2 A point P lies on the line segment as shown.



If the equation of RS is given by $x + 2y = 18$, $0 \leqslant x \leqslant 18$, and A is the area of the right-angled triangle OPQ, determine the coordinates of P so that the area of ΔOPQ is as large as possible. (8)

[28]

QUESTION 8

The owner of a pleasure boat is prepared to take a school group consisting of learners and adults on a cruise, provided that the group consists of not more than 60 people. In addition:

(i) There must be at least 35 people in the group
(ii) There must be at least 6 adults in the group
(iii) There must be not more than 14 adults

Let x be the number of learners, and y the number of adults.

8.1 Give all the constraints in terms of x and y. (5)

8.2 If the group has 25 learners, what is the minimum number of adults that must accompany them? (1)

8.3 Eight adults offer to go on the cruise. What is the maximum number of learners that can be accommodated on the boat? (1)

8.4 If T is the amount in rand paid by the whole group, what is the cost per learner if
$$T = 30x + 50y?$$ (2)

8.5 Now represent the constraints graphically on the graph paper provided and indicate the feasible region clearly. (5)

8.6 What is the composition of the group if the owner's income is as large as possible? (3)

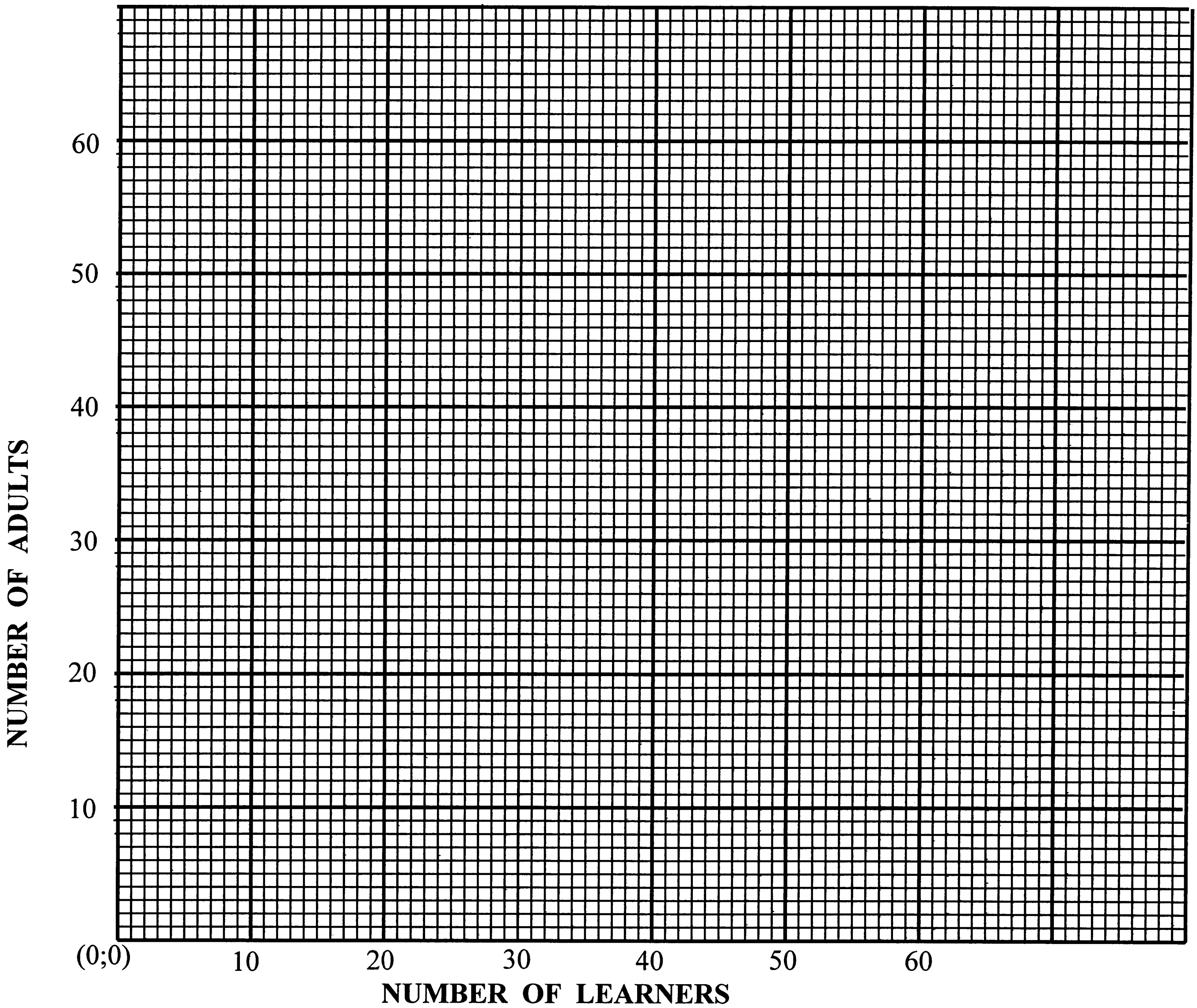
[17]

TOTAL: 200

QUESTION 8

EXAMINATION NUMBER

CENTRE NUMBER



Mathematics Formula Sheet (HG and SG)
Wiskunde Formuleblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$T_n = a + (n - 1)d$ $\quad S_n = \frac{n}{2}(a + T_n)$ or / of $S_n = \frac{n}{2}(a + l)$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(1 - r^n)}{1 - r}$ $(r \neq 1)$ $\quad S_n = \frac{a(r^n - 1)}{r - 1}$ $(r \neq 1)$

$$S_\infty = \frac{a}{1 - r} \quad (|r| < 1)$$

$A = P\left(1 + \frac{r}{100}\right)^n$ or / of $A = P\left(1 - \frac{r}{100}\right)^n$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$(x_3; y_3) = \left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$x^2 + y^2 = r^2$$

$$(x - p)^2 + (y - q)^2 = r^2$$

In $\triangle ABC$: $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \triangle ABC = \frac{1}{2}ab.\sin C$$