

DEPARTMENT OF EDUCATION REPUBLIC OF SOUTH AFRICA

## SENIOR CERTIFICATE EXAMINATION - 2005



Marks: 150
3 Hours
This question paper consists of 8 pages and 1 information sheet.


## INSTRUCTIONS

Read the following instructions carefully before answering the questions.

1. This paper consists of EIGHT questions. Answer ALL the questions.
2. Clearly show ALL calculations, diagrams, graphs, et cetera you have used in determining the answers.
3. An approved calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
5. Graph paper is NOT required in this question paper.
6. Number the answers EXACTLY as the questions are numbered.
7. Diagrams are not necessarily drawn to scale.
8. It is in your own interest to write legibly and to present the work neatly.
9. An information sheet with formulae is included at the end of this question paper.

## QUESTION 1

1.1 Given: $f(x)=x(x+2)-4$

Determine:
1.1.1 $\quad f(-1)$
1.1.2 $x$ if $f(x)=0 \quad$ (Give your answer correct to TWO decimal places.)
1.2 For which values of $p$ will the following equation have non-real roots:

$$
\begin{equation*}
3 x^{2}+2 x+2+p=0 \tag{7}
\end{equation*}
$$

1.3 Senami calculated the discriminant of a quadratic equation and determined the following:

$$
\begin{equation*}
\Delta=(2 k-9)(2 k-1) \tag{3}
\end{equation*}
$$

Describe the nature of the roots of the equation if $k=6$.
1.4 Solve for $x$ and $y$ if they satisfy the following equations simultaneously:

$$
\begin{equation*}
y+7=2 x \tag{8}
\end{equation*}
$$

$x^{2}+x y+y^{2}=21$

## QUESTION 2

2.1 If $f(x)=a x^{3}-5 x^{2}-2 x+5$ is divided by ( $x-2$ ), the remainder is -3 .

Find the value of $a$.
2.2 Solve the following equation:

$$
\begin{equation*}
2 x^{3}-3 x^{2}-5 x+6=0 \tag{6}
\end{equation*}
$$

## QUESTION 3

3.1

Given: $\quad f(x)=-x^{2}+4 x-3$
3.1.1 Calculate the $x$ - and $y$-intercepts of the graph of $f$.
3.1.2 Calculate the co-ordinates of the turning point of $f$.
3.1.3 What is the largest possible value of $-x^{2}+4 x-3$ ?
3.1.4 Make a neat sketch graph of $f$. Indicate the co-ordinates of the intercepts on the axes and of the turning point of the graph.
3.1.5 On the same system of axes, draw a straight line which will help you to solve the equation $-x^{2}+4 x-3=-1$.
3.1.6 Use the graph to determine the values of $x$ for which $-x^{2}+4 x-3 \geq 0$.
3.2 The graphs of a straight line $f$ and the semi-circle $h$ are sketched below. A and $\mathrm{B}(-3 ; 0)$ are intercepts of the graphs on the co-ordinate axes.

3.2.1 Determine the equation of $h$.
3.2.2 Determine the equation of $f$.

## QUESTION 4

4.1 Without using a calculator, calculate the value of each of the following in its simplest form:

$$
\begin{array}{ll}
\text { 4.1.1 } & \left(3^{-1}+2^{-1}\right)^{-1} \\
\text { 4.1.2 } & \frac{9^{n-1} \cdot 27^{3-2 n}}{81^{2-n}} \\
\text { 4.1.3 } & 2 \log 2+\log 25 \\
\text { 4.1.4 } & \frac{\sqrt{98}-\sqrt{8}}{\sqrt{50}} \tag{4}
\end{array}
$$

4.2 Solve for $x$, without using a calculator:

$$
\begin{equation*}
\text { 4.2.1 } 2 x^{\frac{3}{4}}=16 \tag{4}
\end{equation*}
$$

$$
\begin{equation*}
\text { 4.2.2 } \quad 3^{x}-3^{x-2}=24 \tag{6}
\end{equation*}
$$

4.2.3 $\log x=\frac{\log 625}{\log 25}$

## QUESTION 5

5.1 The following arithmetic sequence is given: $-1 ; 6 ; 13 ; \ldots$

Determine:
5.1.1 The $49^{\text {th }}$ term
5.1.2 The sum of the first 87 terms
$5.2 \quad 20 ; 16 ; \ldots$ is a geometric sequence.
Calculate the sum of the first ten terms.
5.3 The following are three consecutive terms of a geometric sequence:
$3 x-2 ; 2 x+2 ; 4 x+1 \quad(x$ is a natural number $)$
Calculate the value of $x$.
5.4 Tiles are arranged as shown below. The first arrangement has 5 tiles, the second arrangement has 9 tiles, the third arrangement has 13 tiles and the fourth arrangement has 17 tiles. The arrangements continue in this pattern.


Derive, in terms of $n$, a formula for the number of tiles in the $n^{\text {th }}$ arrangement.

## QUESTION 6

Read the advertisement below carefully and then answer the question that follows. Round off your answer correct to TWO decimal places.


In 3 years' time Thembi needs R12 500 for a vacation. How much money does he need to deposit now into sOUJ\% AFFRJGeAN BoANKK in order to be able to withdraw that amount at the end of the 3 years?

## QUESTION 7

7.1 Use first principles to determine the derivative of $f(x)$ if $f(x)=4 x^{2}$
7.2 Use differentiation rules to determine the derivatives of the following functions:
7.2.1 $y=4 x^{3}+12 x^{2}+9 x$
7.2.2 $f(x)=-\frac{1}{x^{4}}+\sqrt{x}$
7.3 The graph below, not drawn to scale, represents the function given by:

$$
f(x)=x^{3}-3 x^{2}-9 x+25
$$


7.3.1 Determine the co-ordinates of the turning points A and C.
7.3.2 Use the graph to solve for $x$ if $f^{\prime}(x) \leq 0$.
7.3.3 Determine the gradient of the tangent to the graph of $f$ at the $y$-intercept B.

## QUESTION 8

A biologist states that when a certain type of antibacterium is introduced into a culture of bacteria, the number of bacteria present is given by the formula where $b(t)$, in millions, is the number of bacteria present at time $t$, measured in hours:
$b(t)=-4 t^{2}+60 t+1500$
8.1 How many bacteria were present at the beginning?
8.2 At what moment was the maximum number of bacteria present?

## Information Sheet (HG and SG)

## Inligtingsblad (HG en SG)

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

$$
\begin{aligned}
& T_{n}=a+(n-1) d \quad S_{n}=\frac{n}{2}(a+1) \quad S_{n}=\frac{n}{2}[2 a+(n-1) d] \\
& T_{n}=a . r^{n-1} \quad S_{n}=\frac{a\left(1-r^{n}\right)}{1-r} \quad S_{n}=\frac{a\left(r^{n}-1\right)}{r-1} \quad S_{\infty}=\frac{a}{1-r}
\end{aligned}
$$

$$
A=P\left(1+\frac{r}{100}\right)^{n} \quad A=P\left(1-\frac{r}{100}\right)^{n}
$$

$$
f^{\prime}(x)=\lim _{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}
$$

$$
d=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}
$$

$$
y=m x+c
$$

$$
y-y_{1}=m\left(x-x_{1}\right)
$$

$$
m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}
$$

$$
m=\tan \theta
$$

$$
\left(\frac{x_{1}+x_{2}}{2} ; \frac{y_{1}+y_{2}}{2}\right)
$$

$$
x^{2}+y^{2}=r^{2}
$$

$$
(x-p)^{2}+(y-q)^{2}=r^{2}
$$

$$
\text { In } \triangle A B C: \quad \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}
$$

$$
a^{2}=b^{2}+c^{2}-2 b c \cdot \cos A
$$

$$
\text { area } \triangle A B C=\frac{1}{2} a b \cdot \sin C
$$

