DEPARTMENT OF EDUCATION
REPUBLIC OF SOUTH AFRICA

DEPARTEMENT VAN ONDERWYS
REPUBLIEK VAN SUID-AFRIKA

SENIOR CERTIFICATE EXAMINATION - 2005
SENIORSERTIFIKAAT-EKSAMEN - 2005

MATHEMATICS P2 : GEOMETRY
WISKUNDE V2 : MEETKUNDE

STANDARD GRADE
STANDAARDGRAAD

FEBRUARY/MARCH 2005
FEBRUARIE/MAART 2005

301-2/2

MATHEMATICS SG: Paper 2
Geometry, etc

301 2 2 SG

Marks: 150
Punte : 150

3 Hours
3 Ure

This question paper consists of 12 pages, 1 formula sheet and 4 diagram sheets.
Hierdie vraestel bestaan uit 12 bladsye, 1 formuleblad en 4 diagramvelle.

X05

10 YEARS OF FREEDOM
South Africa 1994 - 2004

INSTRUKSIES

1. Hierdie vraestel bestaan uit **NEGE** vrae, 'n formuleblad en diagramvelle.
2. Gebruik die formuleblad om hierdie vraestel te beantwoord.
3. Maak die diagramvelle los van die vraestel en plaas dit in jou **ANTWOORDEBOEK**.
4. Die diagramme is nie volgens skaal geteken nie.
5. Beantwoord **AL** die vrae.
6. Nommer **AL** die antwoorde korrek en duidelik.
7. **AL** die nodige bewerkings moet getoon word.
8. Nie-programmeerbare sakrekenaars mag gebruik word, tensy anders vermeld.
9. Waar nodig, sal die aantal desimale syfers waartoe antwoorde afgerond moet word, in die vraag gemeld word.

INSTRUCTIONS

1. This question paper consists of **NINE** questions, a formula sheet and diagram sheets.
2. Use the formula sheet to answer this question paper.
3. Detach the diagram sheets from the question paper and place them inside your **ANSWER BOOK**.
4. The diagrams are not drawn to scale.
5. Answer **ALL** the questions.
6. Number **ALL** the answers correctly and clearly.
7. **ALL** the necessary calculations must be shown.
8. Non-programmable calculators may be used, unless otherwise stated.
9. The number of decimal digits to which answers must be rounded off will be stated in the question where necessary.

ANALITIESE MEETKUNDE

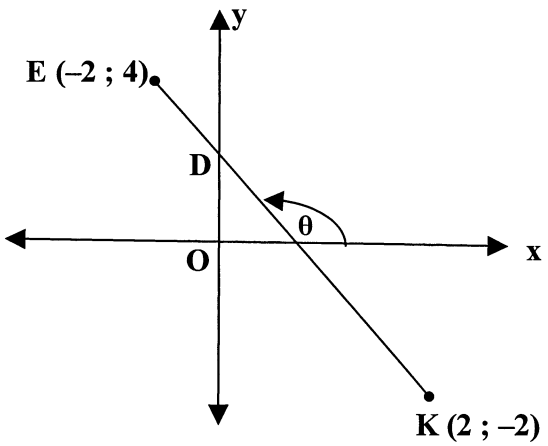
NOTE: – **GEBRUIK ANALITIESE METODES IN HIERDIE AFDELING.**

– **KONSTRUKSIE- EN METINGMETODES MAG NIE GEBRUIK WORD NIE.**

VRAAG 1

In die onderstaande diagram is E (–2 ; 4), K (2 ; –2) en N (p ; 7)

drie punte in 'n Cartesiese vlak.



Bepaal:

1.1 Die vergelyking van reguitlyn KE (5)

1.2 Die lengte van ED, as D die *y*-afsnit is van KE (laat die antwoord in wortelvorm) (4)

1.3 Die grootte van θ, die inklinasiehoek van KE (afgerond tot EEN desimale syfer) (3)

1.4 Die waarde van p as KE ⊥ KN (5)
[17]

ANALYTICAL GEOMETRY

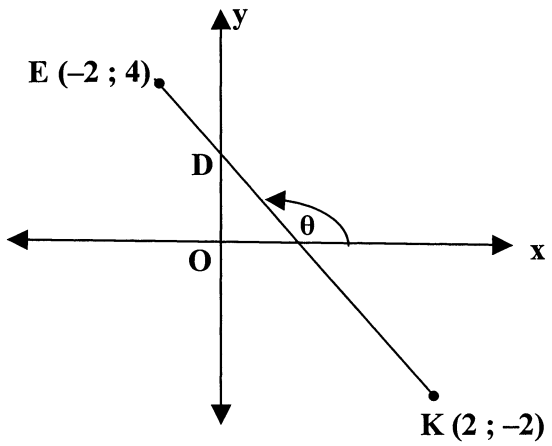
NOTE: – USE ANALYTICAL METHODS IN THIS SECTION.

– CONSTRUCTION AND MEASUREMENT METHODS MAY NOT BE USED.

QUESTION 1

In the diagram below E (–2 ; 4), K (2 ; –2) and N (p ; 7)

are three points in a Cartesian plane.



Determine:

1.1 The equation of straight line KE (5)

1.2 The length of ED, if D is the y-intercept of KE (leave the answer in surd form) (4)

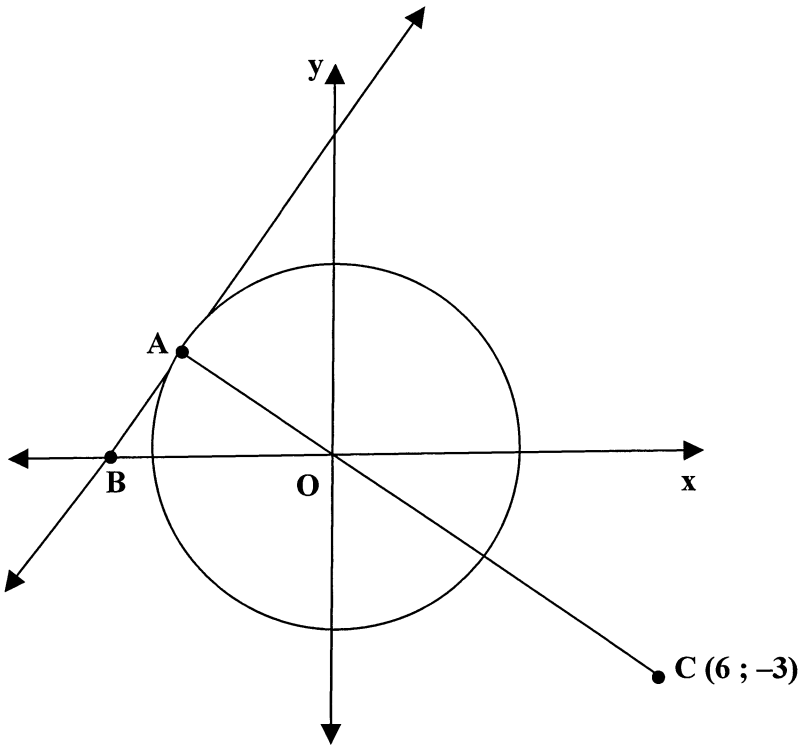
1.3 The size of θ, the angle of inclination of KE (rounded off to ONE decimal digit) (3)

1.4 The value of p if KE $\perp$ KN (5)
[17]

VRAAG 2

2.1 In die diagram langsaan, raak reguitlyn AB met vergelyking $y - 2x - 5 = 0$ die sirkel $x^2 + y^2 = 5$ by punt A.

AO word verleng tot punt C (6 ; –3).



Bepaal:

2.1.1 Die koördinate van A (7)

2.1.2 Die koördinate van middelpunt M van OC (2)

2.1.3 Vervolgens, die vergelyking van die reguitlyn ewewydig aan AB en wat deur M gaan (4)

2.2 In elk van die volgende bepaal die vergelyking van die lokus van punt P(x ; y) **en** skets ook in elke geval die lokus:

2.2.1 P is 3 eenhede vanaf die sirkel $x^2 + y^2 = 4$ (4)

2.2.2 P is twee eenhede links van die y-as (4)

[21]

QUESTION 2

2.1 In the diagram alongside,

straight line AB with equation

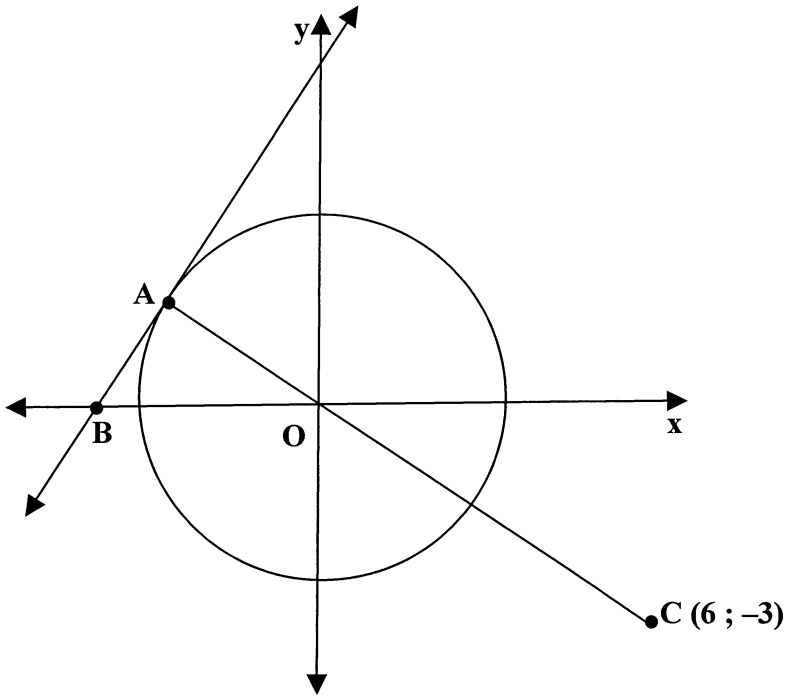
$y - 2x - 5 = 0$ touches

the circle $x^2 + y^2 = 5$

at point A.

AO is produced

to point C (6 ; –3).



Determine:

2.1.1 The coordinates of A (7)

2.1.2 The coordinates of midpoint M of OC (2)

2.1.3 Hence, the equation of the straight line parallel to AB and passing through M (4)

2.2 In each of the following determine the equation of the locus of point P(x ; y) **and** in each case also sketch the locus:

2.2.1 P is 3 units from the circle $x^2 + y^2 = 4$ (4)

2.2.2 P is two units to the left of the y-axis (4)
[21]

TRIGONOMETRIE

VRAAG 3

Beantwoord hierdie vraag sonder die gebruik van 'n sakrekenaar.

3.1 As $7 \sin\theta - 5 = 0$ en $\cos\theta < 0$, bereken, **met behulp van 'n diagram**, die waarde van $\cot\theta \,.\, \cos\theta$ (6)

3.2 Vereenvoudig: $$\frac{\sin(180° - x) \,.\, \sec(360° - x) \,.\, \cos(180° + x) \,.\, \tan 300°}{\cos(90° - x)}$$ (8)

[14]

VRAAG 4

4.1 Gegee: $f(x) = \cos 2x$ en $g(x) = \tan x$

4.1.1 Gebruik die assestelsel wat voorsien is op die diagramvel om die krommes van f en g te skets vir $x \in [0° ; 180°]$. Toon duidelik ALLE afsnitte met die asse aan, en ALLE draaipunte. Dui duidelik enige asimptote aan deur stippellyn(e) te gebruik. (8)

4.1.2 Gebruik jou grafieke in VRAAG 4.1.1 om die volgende te beantwoord, as $x \in [0° ; 180°]$:

(a) Vir watter waarde van x is $\tan x$ ongedefinieerd? (1)

(b) Wat is die periode van f? (1)

(c) Bepaal die waarde(s) van x waarvoor $f(x) - g(x) = 1$ (3)

(d) Vir watter waarde(s) van x is $f(x) \geq g(x)$

vir $x \in [45° ; 180°]$? (3)

4.2 Gegee die drie punte $(0° ; 0)$, $(90° ; -2)$ en $(180° ; 0°)$.

Op watter **EEN** van die volgende krommes sal AL drie bostaande punte lê:

$$f: \; y = -2\cos x$$
$$h: \; y = -\cos 2x$$
$$g: \; y = -2\sin x$$
$$k: \; y = -\sin 2x$$

(2)

[18]

TRIGONOMETRY

QUESTION 3

Answer this question without the use of a calculator.

3.1 If $7 \sin \theta - 5 = 0$ and $\cos \theta < 0$, calculate, **with the aid of a diagram,** the value of $\cot \theta \cdot \cos \theta$ (6)

3.2 Simplify: $\dfrac{\sin(180° - x) \cdot \sec(360° - x) \cdot \cos(180° + x) \cdot \tan 300°}{\cos(90° - x)}$ (8)
[14]

QUESTION 4

4.1 Given: $f(x) = \cos 2x$ and $g(x) = \tan x$

4.1.1 Use the system of axes provided on the diagram sheet to sketch the curves of f and g for $x \in [0° ; 180°]$. Clearly show ALL intercepts with the axes and ALL turning points. Clearly indicate any asymptotes using a dotted line(s). (8)

4.1.2 Use your graphs in QUESTION 4.1.1 to answer the following if $x \in [0° ; 180°]$:

(a) For which value of x is $\tan x$ undefined? (1)

(b) What is the period of f? (1)

(c) Determine the value(s) of x for which $f(x) - g(x) = 1$ (3)

(d) For which value(s) of x is $f(x) \geq g(x)$ for $x \in [45° ; 180°]$? (3)

4.2 Given the three points $(0° ; 0)$, $(90° ; -2)$ and $(180° ; 0)$.

On which **ONE** of the following curves would ALL of the above three points lie:

$f: y = -2 \cos x$
$h: y = -\cos 2x$
$g: y = -2 \sin x$
$k: y = -\sin 2x$

(2)
[18]

VRAAG 5

5.1 Gebruik fundamentele trigonometriese identiteite en nie 'n diagram nie, om die volgende identiteit te bewys:

$$(\cot x + \tan x)\cos x = \text{cosec } x \qquad (6)$$

5.2 Gegee: $3\cos x = 2{,}151$ vir $x \in [0^\circ ; 360^\circ]$

5.2.1 Los op vir x, afgerond tot TWEE desimale syfers. (3)

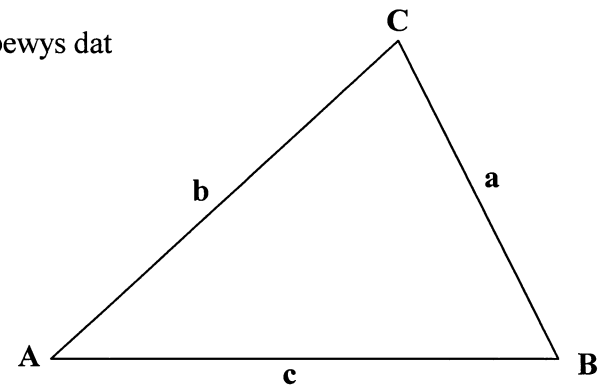
5.2.2 Bepaal vervolgens die waarde van $\cot\frac{1}{2}x$ as $x > 90^\circ$ (afgerond tot TWEE desimale syfers). (2)

[11]

VRAAG 6

6.1 Gebruik die diagram op die diagramvel of teken die diagram oor in die antwoordeboek om te bewys dat

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$



(4)

QUESTION 5

5.1 Use fundamental trigonometric identities and not a diagram to prove the following identity:

$$(\cot x + \tan x)\cos x = \operatorname{cosec} x \qquad (6)$$

5.2 Given: $3\cos x = 2{,}151$ for $x \in [0° ; 360°]$

5.2.1 Solve for x, rounded off to TWO decimal digits. (3)

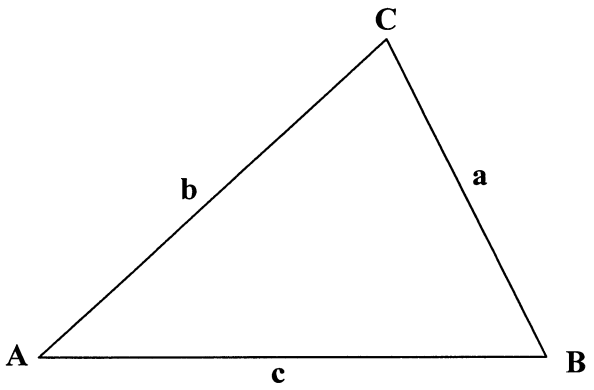
5.2.2 Hence determine the value of $\cot\frac{1}{2}x$ if $x > 90°$ (rounded off to TWO decimal digits). (2)

[11]

QUESTION 6

6.1 Use the diagram on the diagram sheet or redraw the diagram in your answer book to prove that

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$



(4)

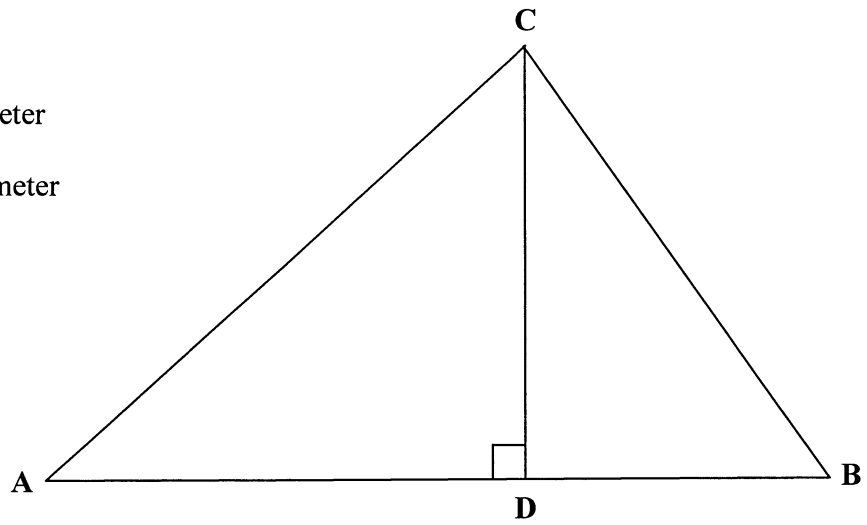
6.2 Die onderstaande diagram stel 'n driehoekige stuk grond ABC op Robbeneiland voor wat die Erfenisstigting wil gebruik vir 'n herdenkingsterrein.

$\hat{B} = 63°$

AB = 500 meter

BC = 300 meter

CD $\perp$ AB



Bepaal die volgende (afgerond tot TWEE desimale syfers):

6.2.1 Die afstand AC (4)

6.2.2 Die grootte van $A\hat{C}B$ (3)

6.2.3 Die area van ΔABC (3)

6.2.4 Vervolgens, die afstand DC (3)
[17]

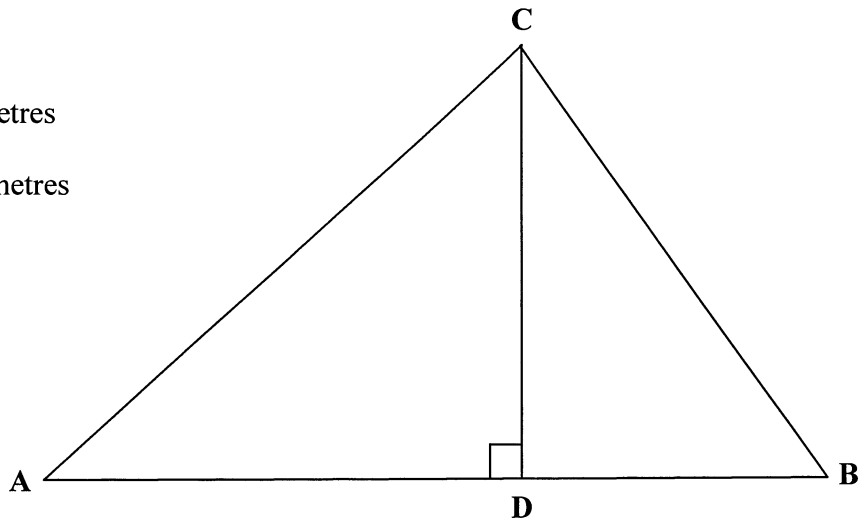
6.2 The diagram below represents a triangular piece of land ABC on Robben Island which the Heritage Foundation wants to use for a memorial site.

$\hat{B} = 63°$

AB = 500 metres

BC = 300 metres

CD $\perp$ AB



Determine the following (rounded off to TWO decimal digits):

6.2.1 The distance AC (4)

6.2.2 The size of $A\hat{C}B$ (3)

6.2.3 The area of ΔABC (3)

6.2.4 Hence, the distance DC (3)
[17]

EUKLIDIESE MEETKUNDE

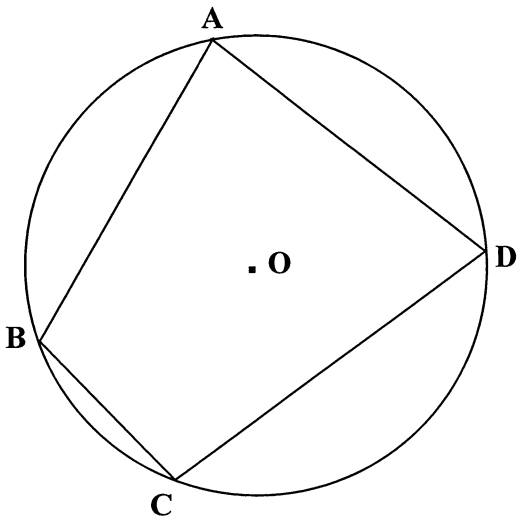
LET WEL :
- **DIAGRAMME VIR DIE BEWYS VAN TEORIE MAG OP DIE DIAGRAMVELLE GEBRUIK WORD OF IN DIE ANTWOORDEBOEK OORGETEKEN WORD.**
- **MAAK DIE DIAGRAMVELLE VAN DIE VRAESTEL LOS EN PLAAS DIT IN JOU ANTWOORDEBOEK.**
- **GEE 'N REDE VIR ELKE BEWERING.**

VRAAG 7

7.1 In die onderstaande diagram is ABCD 'n koordevierhoek van die sirkel met middelpunt O.

Gebruik die diagram op die diagramvel of teken die diagram oor in jou antwoordeboek om die stelling te bewys wat beweer dat

$\hat{B} + \hat{D} = 180°$



(5)

EUCLIDEAN GEOMETRY

NOTE :	**– DIAGRAMS FOR PROVING THEORY MAY BE USED ON THE DIAGRAM SHEETS, OR REDRAWN IN YOUR ANSWER BOOK.**
	– DETACH THE DIAGRAM SHEETS FROM THE QUESTION PAPER AND PLACE THEM IN YOUR ANSWER BOOK.
	– GIVE A REASON FOR EACH STATEMENT.

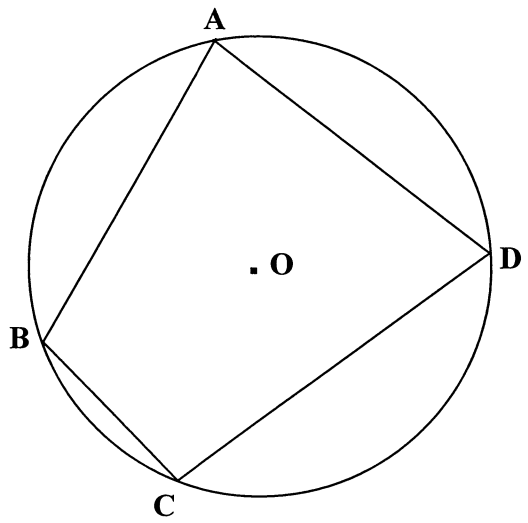
QUESTION 7

7.1 In the diagram below, ABCD is a cyclic quadrilateral of the circle with centre O.

Use the diagram on the diagram sheet or redraw the diagram in your answer book

to prove the theorem which states that

$\hat{B} + \hat{D} = 180°$

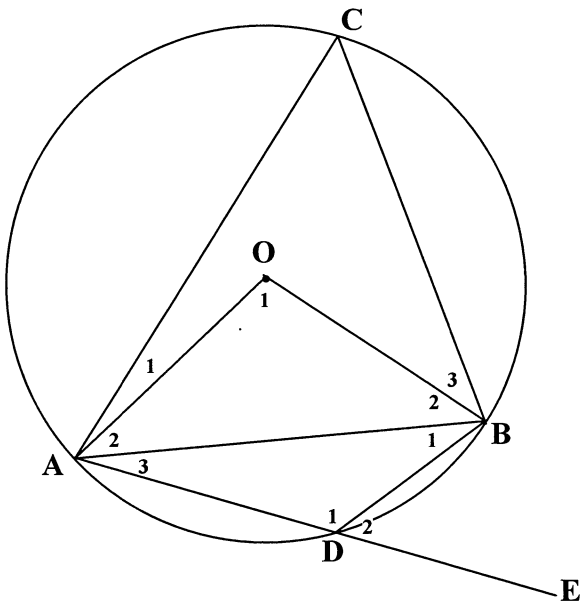


(5)

7.2 In die diagram langsaan is

O die middelpunt van sirkel ADBC

$\hat{B}_2 = 40°$



7.2.1 Bepaal, met verstrek van redes, die grootte van $\hat{D}_2$ (6)

7.2.2 As AO || DB , bepaal, met verstrek van redes, die grootte van $O\hat{A}D$. (1)

7.3 Voltooi die volgende bewerings deur slegs die toepaslike ontbrekende woord neer te skryf om die bewering WAAR te maak:

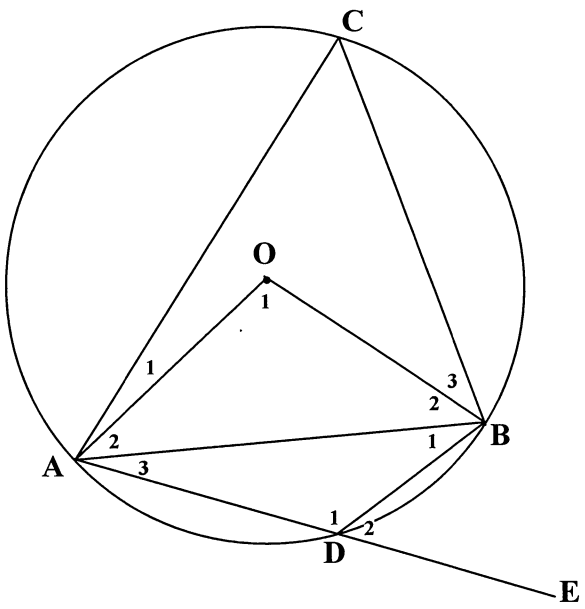
7.3.1 As 'n koord van 'n sirkel 'n regte hoek op die omtrek onderspan, dan is die koord 'n … (1)

7.3.2 As 'n lyn getrek word deur die eindpunt van 'n koord en 'n hoek vorm met die koord wat gelyk is aan 'n hoek in die teenoorstaande sirkelsegment, dan is die lyn 'n … aan die sirkel. (1)

7.2 In the diagram alongside,

O is the centre of circle ADBC

$\hat{B}_2 = 40°$



7.2.1 Determine, stating reasons, the size of $\hat{D}_2$ (6)

7.2.2 If AO || DB , determine, stating reasons, the size of $O\hat{A}D$. (1)

7.3 Complete the following statements, by only writing the appropriate missing word, to make the statement TRUE:

7.3.1 If a chord of a circle subtends a right angle on the circumference, then the chord is a … (1)

7.3.2 If a line is drawn through the end point of a chord making with the chord an angle equal to an angle in the alternate segment, then the line is a …to the circle. (1)

7.4 In die diagram langsaan is

O die middelpunt van die sirkel.

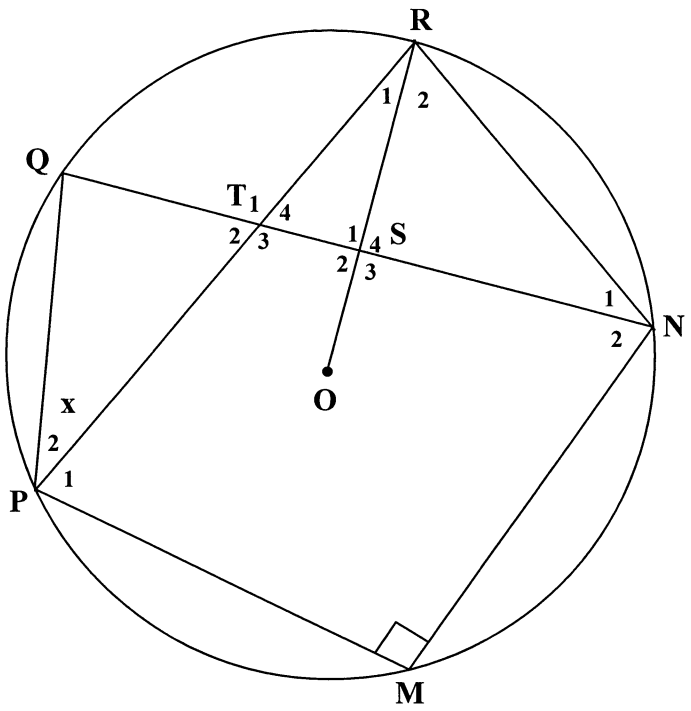
M, P, Q, R en N is

punte op die sirkel.

PM ⊥ MN

QN sny PR by T

en sny RO by S.



Laat $\hat{P}_2 = x$

7.4.1 Noem, met verstrek van 'n rede, EEN ander hoek gelyk aan x. (2)

7.4.2 Bewys dat PT 'n middellyn van sirkel PQT is. (3)

7.4.3 As dit verder gegee word dat S die middelpunt van koord QN is, bewys dat RN 'n middellyn van sirkel RSN is. (3)

7.4.4 Bewys vervolgens dat TR 'n raaklyn aan sirkel RSN is. (2)
[24]

7.4 In the diagram alongside,

O is the centre of the circle.

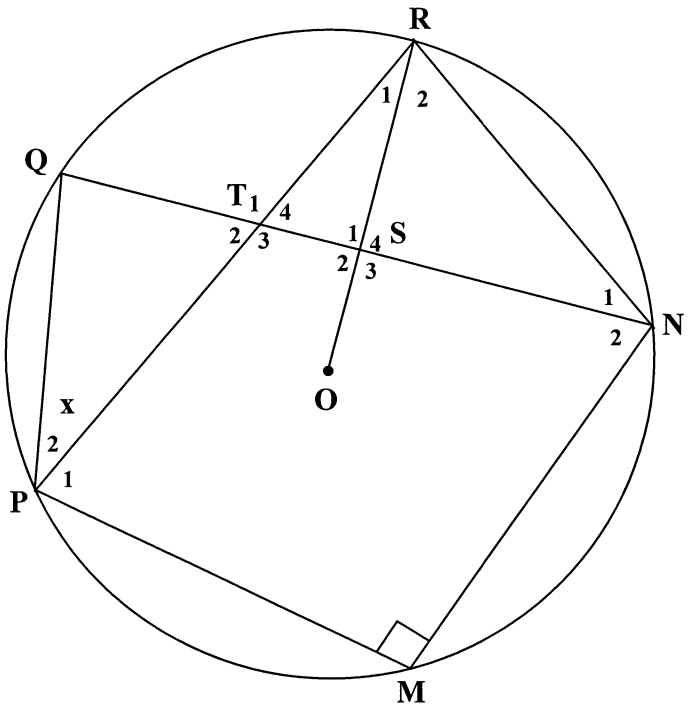
M, P, Q, R and N

are points on the circle.

PM ⊥ MN

QN intersects PR at T

and intersects RO at S.



Let $\hat{P}_2 = x$

7.4.1 Name, stating a reason, ONE other angle equal to x. (2)

7.4.2 Prove that PT is a diameter of circle PQT. (3)

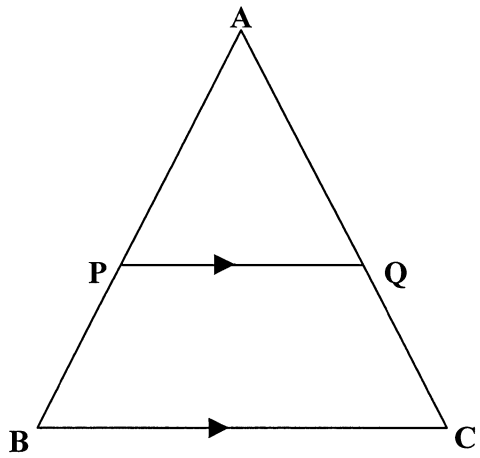
7.4.3 If it is further given that S is the midpoint of chord QN, prove that RN is a diameter of circle RSN. (3)

7.4.4 Hence, prove that TR is a tangent to circle RSN. (2)
[24]

VRAAG 8

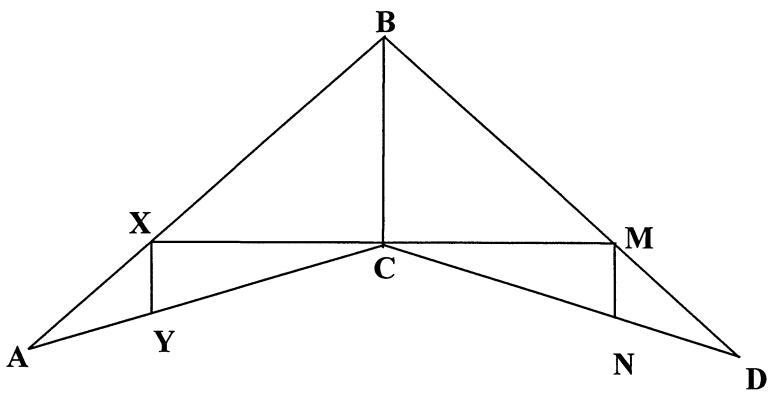
8.1 In die diagram langsaan is PQ || BC met P op AB en Q op AC.
Gebruik die diagram op die diagramvel of teken die diagram oor in jou antwoordeboek
om die stelling te bewys wat beweer dat

$$\frac{AP}{PB} = \frac{AQ}{QC}$$



(6)

8.2 Met die konstruksie van dakke ontwerp argitekte stutte om dakke te steun.
'n Simmetriese stut bekend as die 'skêr-stut' word in die diagram langsaan getoon.



AB = 7,5 meter

XB = 5 meter

AC = 6 meter

YC = 4 meter

8.2.1 Bepaal die numeriese waarde van die volgende:

(a) $\frac{AX}{XB}$ (1)

(b) $\frac{AY}{YC}$ (1)

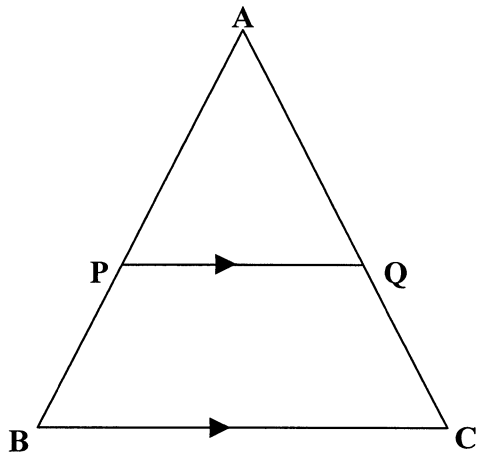
(c) $\frac{AX}{AB}$ (1)

8.2.2 Bewys dat XY || MN. (3)
[12]

QUESTION 8

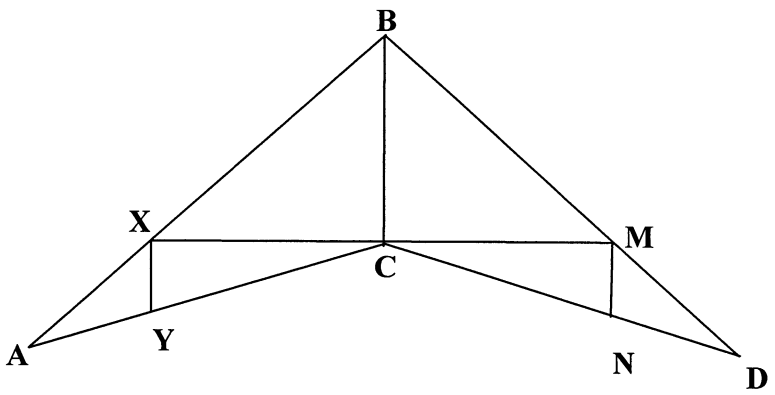
8.1 In the diagram alongside , PQ || BC with P on AB and Q on AC.
Use the diagram on the diagram sheet or redraw the diagram in your answer book to prove the theorem which states that

$$\frac{AP}{PB} = \frac{AQ}{QC}$$



(6)

8.2 In the construction of roofs, architects design trusses to support roofs.
A symmetrical truss known as the 'scissors truss' is shown in the diagram alongside.



AB = 7,5 metres

XB = 5 metres

AC = 6 metres

YC = 4 metres

8.2.1 Determine the numerical value of the following:

(a) $\frac{AX}{XB}$ (1)

(b) $\frac{AY}{YC}$ (1)

(c) $\frac{AX}{AB}$ (1)

8.2.2 Prove that XY || MN. (3)
[12]

VRAAG 9

In die diagram langsaan is AB 'n middellyn van die sirkel met oorsprong O.

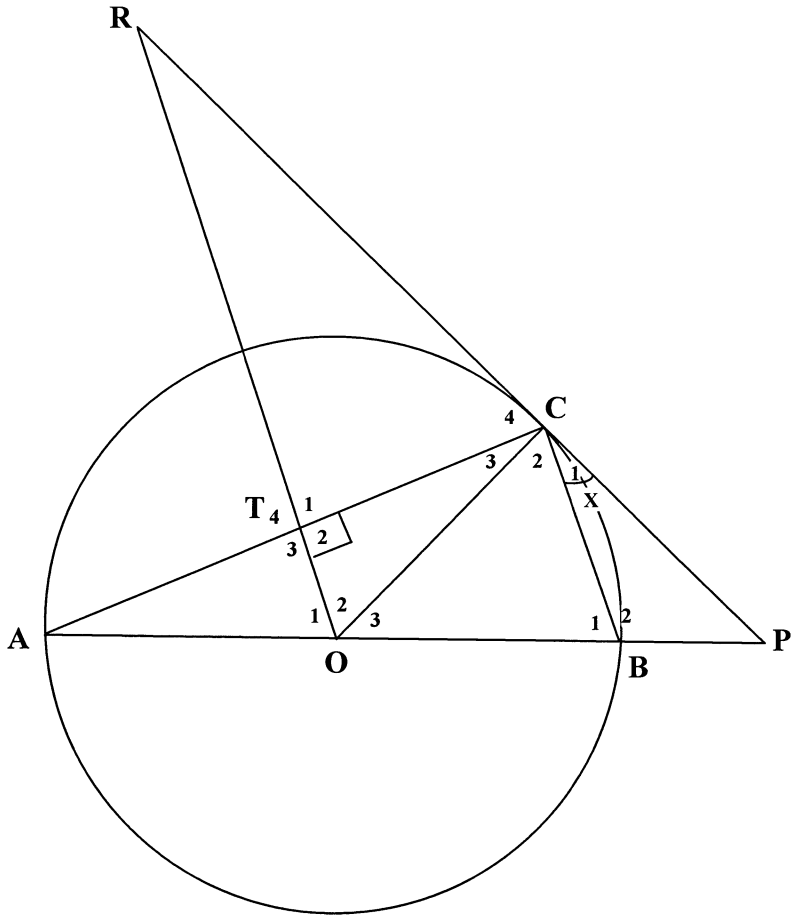
AB word verleng na P.

PR is 'n raaklyn aan die sirkel by C.

RO sny AC by T.

$RO \perp AC$

Laat $\hat{C}_1 = x$



9.1 Gee, met verstrek van redes, TWEE ander hoeke elk gelyk aan x. (3)

9.2 Bepaal die volgende in terme van x:

9.2.1 $P\hat{C}A$ (3)

9.2.2 $C\hat{B}P$ (3)

9.3 Bewys dat:

9.3.1 T is die middelpunt van AC (2)

9.3.2 Δ PCB ||| Δ PAC (2)

9.4 Bewys vervolgens dat 2 AT. PC = CB . AP (3)

[16]

TOTAAL: 150

QUESTION 9

In the diagram alongside, AB is a diameter of the circle with centre O.

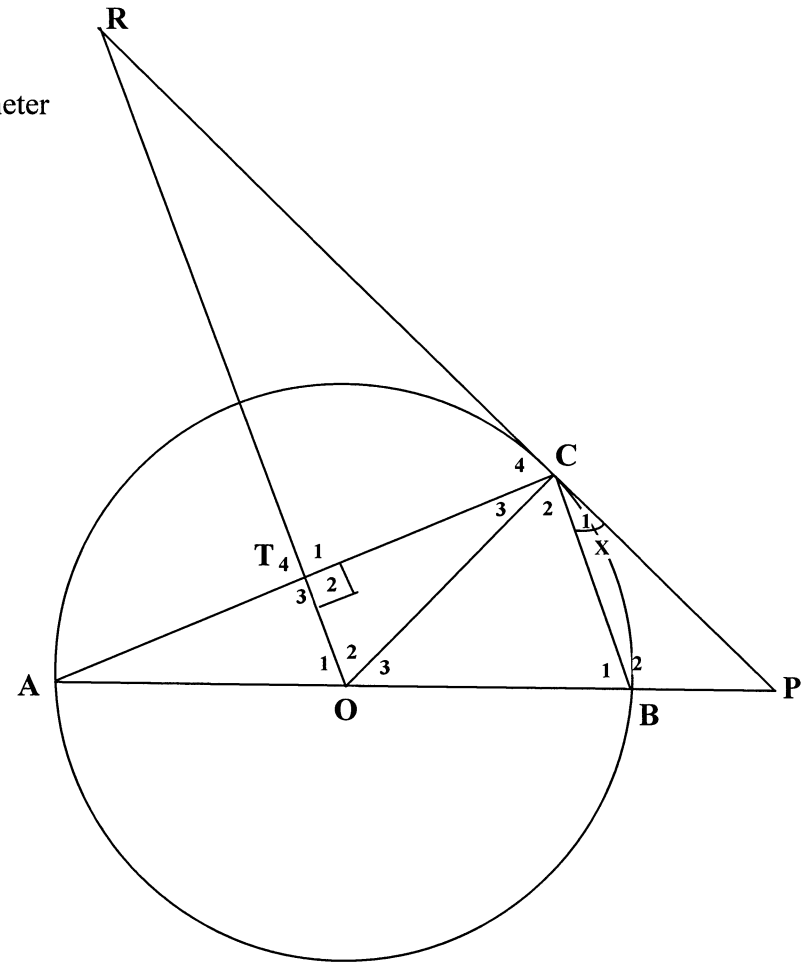
AB is produced to P.

PR is a tangent to the circle at C.

RO intersects AC at T.

$RO \perp AC$

Let $\hat{C}_1 = x$



9.1 Give, stating reasons, TWO other angles each equal to x. (3)

9.2 Determine the following in terms of x:

9.2.1 $P\hat{C}A$ (3)

9.2.2 $C\hat{B}P$ (3)

9.3 Prove that:

9.3.1 T is the midpoint of AC (2)

9.3.2 Δ PCB ||| Δ PAC (2)

9.4 Hence, prove that $2\ AT.\ PC = CB\ .\ AP$ (3)
[16]

TOTAL: 150

Mathematics Formula Sheet (HG and SG)
Wiskunde Formuleblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}(a+l) \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = a.r^{n-1} \qquad S_n = \frac{a(1-r^n)}{1-r}, r \neq 1 \qquad S_n = \frac{a(r^n - 1)}{r-1}, r \neq 1 \qquad S_\infty = \frac{a}{1-r}, r \neq 1$$

$$A = P\left(1 + \frac{r}{100}\right)^n \qquad A = P\left(1 - \frac{r}{100}\right)^n$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$x^2 + y^2 = r^2 \qquad (x-p)^2 + (y-q)^2 = r^2$$

In ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\,\Delta ABC = \frac{1}{2}ab.\sin C$$

Mathematics Formula Sheet (HG and SG)
Wiskunde Formuleblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}(a+l) \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = a.r^{n-1} \qquad S_n = \frac{a(1-r^n)}{1-r}, r \neq 1 \qquad S_n = \frac{a(r^n - 1)}{r-1}, r \neq 1 \qquad S_\infty = \frac{a}{1-r}, r \neq 1$$

$$A = P\left(1 + \frac{r}{100}\right)^n \qquad A = P\left(1 - \frac{r}{100}\right)^n$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$x^2 + y^2 = r^2 \qquad (x-p)^2 + (y-q)^2 = r^2$$

In ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

DEPARTMENT OF EDUCATION
DEPARTEMENT VAN ONDERWYS

REPUBLIC OF SOUTH AFRICA
REPUBLIEK VAN SUID-AFRIKA

SENIOR CERTIFICATE EXAMINATION/SENIORSERTIFIKAAT-EKSAMEN
MATHEMATICS SG/WISKUNDE SG
PAPER II/VRAESTEL II
MARCH/MAART 2005

DIAGRAM SHEET/DIAGRAMVEL

INSTRUCTION

This diagram sheet must be handed in with your answer book. Please ensure that your details are complete.

INSTRUKSIE

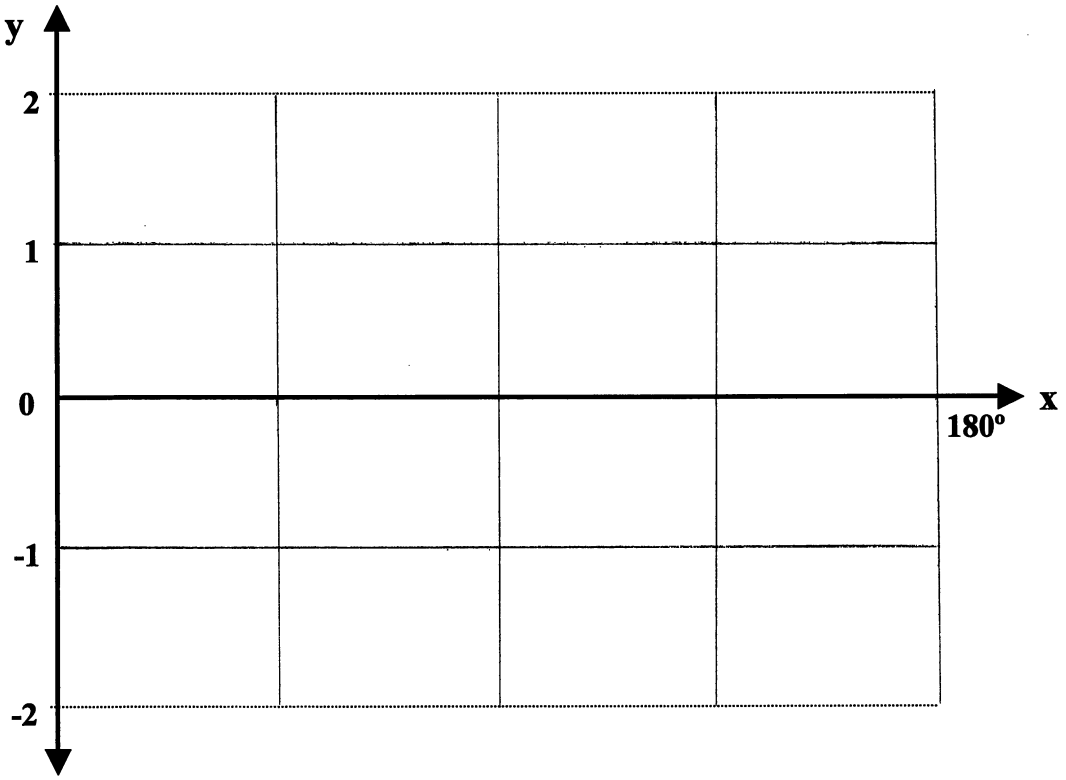
Hierdie diagramvel moet saam met jou antwoordeboek ingelewer word. Maak asseblief seker dat jou besonderhede volledig ingevul is.

EXAMINATION NUMBER
EKSAMENNOMMER

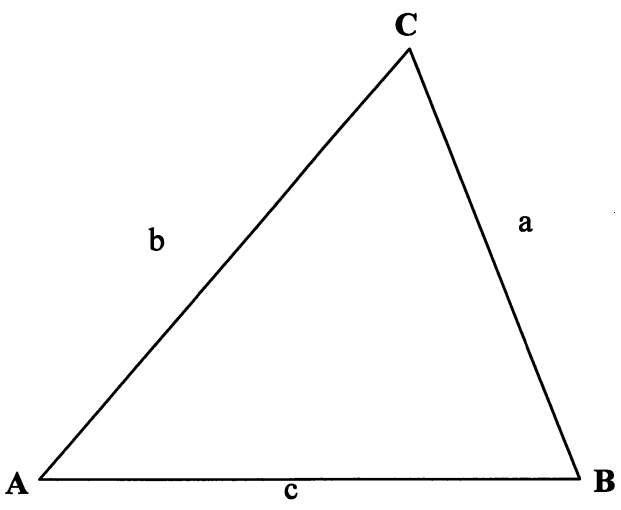
CENTRE NUMBER
SENTRUMNOMMER

EXAMINATION NUMBER
EKSAMENNOMMER

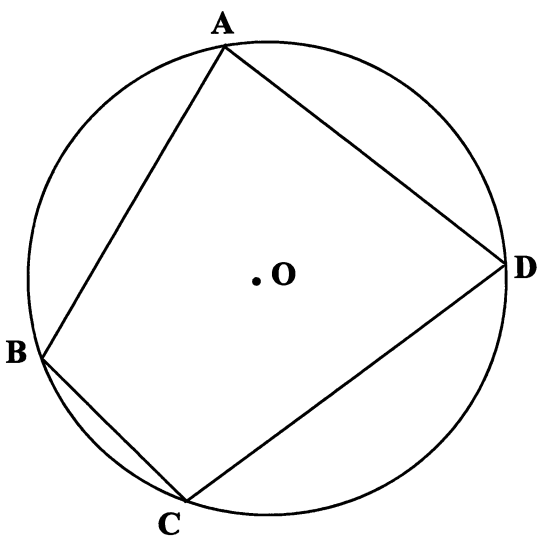
QUESTION 4.1/VRAAG 4.1



QUESTION 6.1/VRAAG 6.1

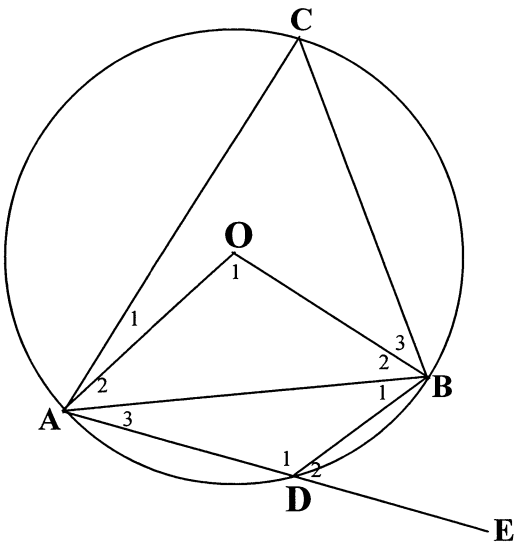


QUESTION 7.1/VRAAG 7.1

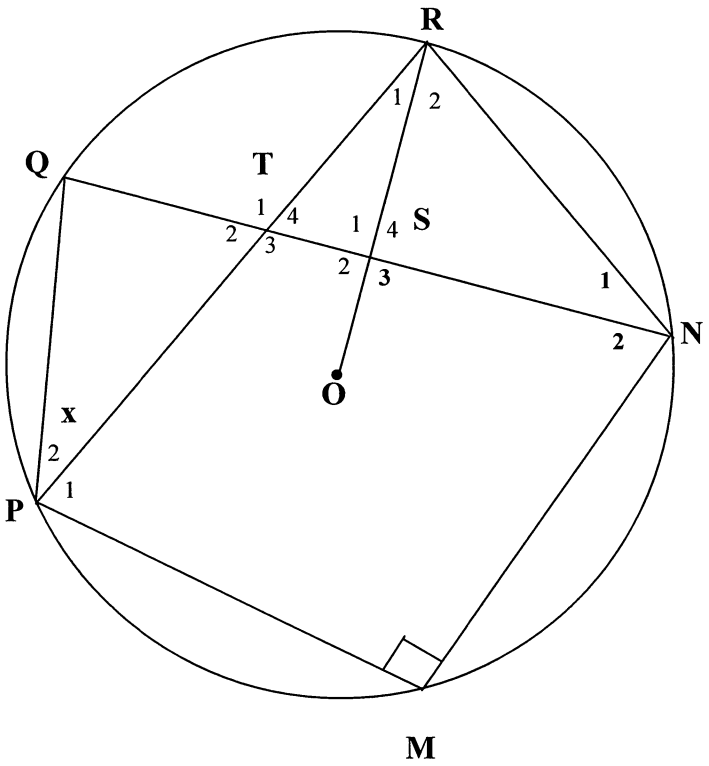


EXAMINATION NUMBER
EKSAMENNOMMER

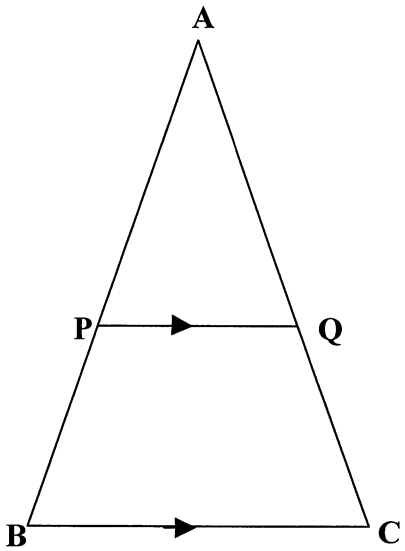
QUESTION 7.2/VRAAG 7.2



QUESTION 7.3/VRAAG 7.3

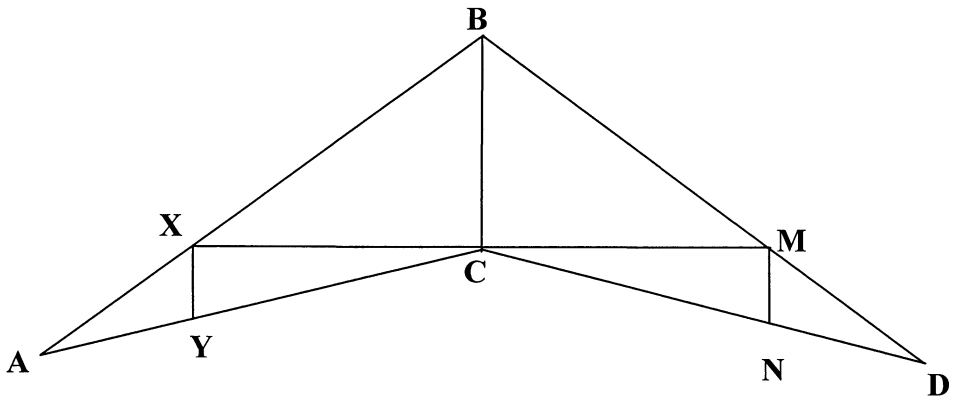


QUESTION 8.1/VRAAG 8.1



EXAMINATION NUMBER
EKSAMENNOMMER

QUESTION 8.2/VRAAG 8.2



QUESTION 9/VRAAG 9