education

Department:
Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATION - 2005

MATHEMATICS P2

HIGHER GRADE

OCTOBER/NOVEMBER 2005

Marks: 200

Time: 3 Hours

This question paper consists of 11 pages, 1 information sheet and 6 diagram sheets.

INSTRUCTIONS

1. This question paper consists of **9** questions, a formula sheet and diagram sheets.
2. Use the formula sheet to answer this question paper.
3. Detach the diagram sheets from the question paper and place them inside your **ANSWER BOOK.**
4. The diagrams are not drawn to scale.
5. Answer **ALL** the questions.
6. Number **ALL** the answers correctly and clearly.
7. **ALL** the necessary calculations must be shown.
8. Non-programmable calculators may be used, unless otherwise stated.
9. The number of decimal digits to which answers must be rounded off will be stated in the question where necessary.

ANALYTICAL GEOMETRY

NOTE :	– USE ANALYTICAL METHODS IN THIS SECTION. – CONSTRUCTION AND MEASUREMENT METHODS MAY NOT BE USED

QUESTION 1

1.1 A (– 2 ; 4) , B (– 6 ; 2) and C (3 ; p) are points in the Cartesian plane.
Calculate the value of p if $AB \perp AC$ (5)

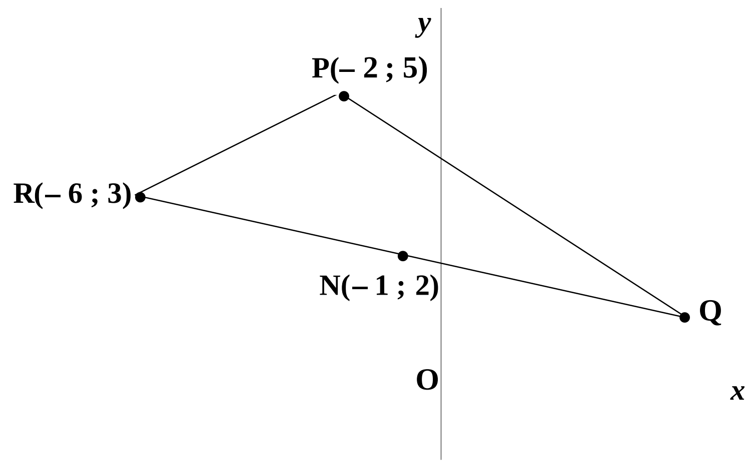
1.2 In the diagram alongside,

P (–2 ; 5) , R (– 6 ; 3) and

Q are the vertices of ΔPRQ.

N (–1 ; 2) is the midpoint of

RQ.



1.2.1 Calculate the size of $\hat{R}$, rounded off to TWO decimal digits. (6)

1.2.2 Determine the equation of PN. (4)

1.2.3 Determine the coordinates of L, the midpoint of PQ. (5)

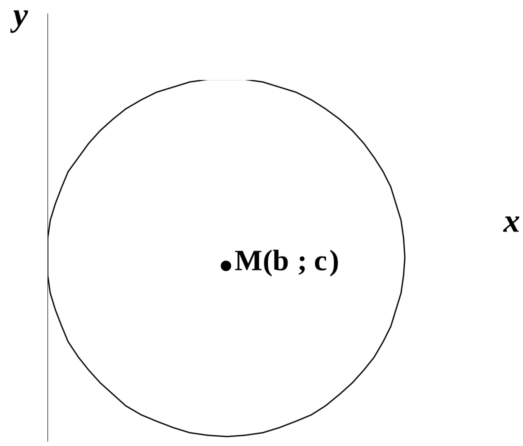
1.2.4 Determine the coordinates of A, the point of intersection of the medians of ΔPRQ. (6)

[26]

QUESTION 2

2.1 In the diagram alongside, a circle with centre M (b ; c) touches the y-axis at point P, where b and c are integers. Point Q (3 ; 1) lies on the circle. M lies on the straight line $2x + y = 4$

[P and Q are not shown on the diagram.]



2.1.1 Write down the coordinates of P in terms of b or c . (2)

2.1.2 Determine the equation of the circle in terms of b and c . (2)

2.1.3 Hence, determine the numerical values of b and c . (8)

2.1.4 Hence, determine the equation of the tangent to the circle at Q . (2)

2.2 A circle with equation $x^2 + y^2 + 2x - 6y - 6 = 0$ is given. P is a locus point in the Cartesian plane such that the length of the tangent line drawn from P to the circle is equal to the length of the radius of the circle.

2.2.1 Determine:

(a) the length of the radius of the circle. (3)

(b) the equation of the locus of P . (4)

2.2.2 Describe fully the locus obtained in QUESTION 2.2.1(b). (3)

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TRIGONOMETRY

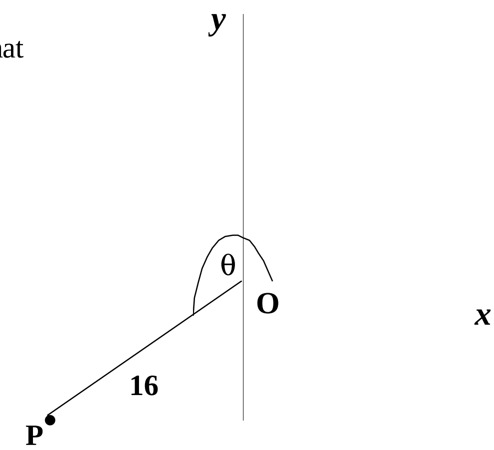
QUESTION 3

3.1 In the diagram alongside,

P is a point in the Cartesian plane such that

OP =16 units and the size of

reflex angle θ is equal to 200°.



Determine the coordinates of P, rounded off to TWO decimal digits. (4)

3.2 If $\sin 54^\circ = p$, express the following in terms of p, **without using a calculator**:

3.2.1 $\sin 594^\circ$ (2)

3.2.2 $\cos 18^\circ$ (6)

3.2.3 $\dfrac{\tan 27^\circ + \cot 63^\circ}{1 + (\tan 207^\circ)(\cot 117^\circ)}$ (6)

[18]

QUESTION 4

Given : $f(x) = \cos x - \frac{1}{2}$ and $g(x) = \sin(x + 30^\circ)$

4.1 Use the set of axes provided on the diagram sheet to draw sketch graphs of the curves of f and g for $x \in [-120^\circ ; 60^\circ]$. Show clearly all intercepts with the axes, coordinates of all turning points and coordinates of all end points of both curves. (9)

4.2 Use the graphs drawn in QUESTION 4.1 to determine for which value(s) of $x \in [-120^{\circ} ; 60^{\circ}]$ is:

4.2.1 $\cos(60° - x) < 0$ (3)

4.2.2 $f(x) - g(x) > 0$ (2)

4.2.3 $\dfrac{g(x)}{f(x)}$ undefined? (2)

[16]

QUESTION 5

5.1 5.1.1 Give an expression for $\cos(A + B)$ in terms of the sines and the cosines of A and B. (1)

5.1.2 Hence or otherwise, prove **without using a calculator** that

$$\sin 15° = \frac{\sqrt{6} - \sqrt{2}}{4}$$ (6)

5.2 Given the identity: $\cot 2x + \operatorname{cosec} 2x = \cot x$

5.2.1 Prove the identity. (7)

5.2.2 Determine the values of x for which the identity is undefined. Give the answer as a general solution. (5)

5. 3 Solve the equation :

$$\sec^2 A - 3\tan A - 5 = 0, \quad \text{for} \quad A \in [-90°; 0°].$$ (6)

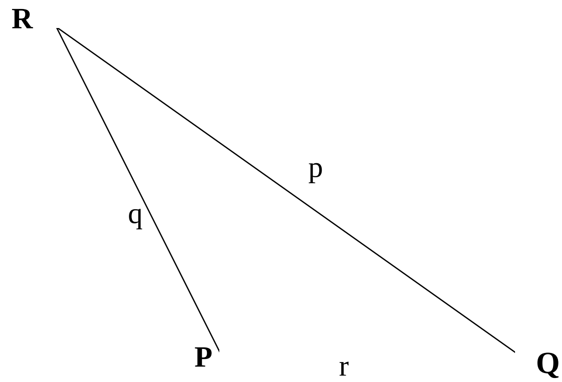
[25]

QUESTION 6

6.1 Given ΔPQR with $\hat{P}$ obtuse.

Use the diagram on the diagram sheet, or redraw the diagram in your answer book to prove that :

Area of $\Delta PQR = \frac{1}{2}(q)(r)\sin P$



(4)

6.2 In ΔLMN, LM = 5 units, LN = $2x$ units and MN = $3x$ units.

6.2.1 Prove that $\cos L = \dfrac{5 - x^2}{4x}$ (4)

6.2.2 Give the restrictions for $\cos L$ if $\hat{L}$ is obtuse. (2)

6.2.3 Is it possible for x to be equal to 6 ? (1)

6.2.4 If $x = 3$, calculate the area of ΔLMN, rounded off to ONE decimal digit. (5)

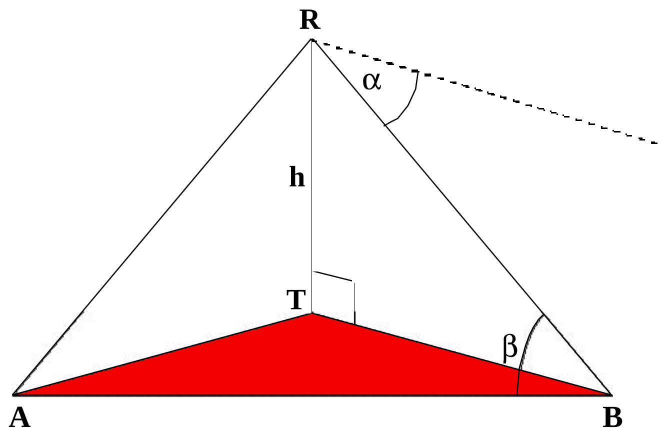
6.3 In the diagram alongside, RT represents the height of a vertical tower, with T the foot of the tower.

A and B represent two points equidistant from T and which lie in the same horizontal plane as T.

The height of the tower is h.

The angle of depression of B from R is α.

$R\hat{B}A = \beta$



6.3.1 Give the size of $A\hat{R}B$ in terms of β. (1)

6.3.2 Prove that $AB = 2h.\cos\beta.\text{cosec}\,\alpha$ (5)

6.3.3 Calculate the height of the tower, rounded off to the nearest unit, if $AB = 5{,}4$ units, $\alpha = 51°$ and $\beta = 65°$ (3)

[25]

EUCLIDEAN GEOMETRY

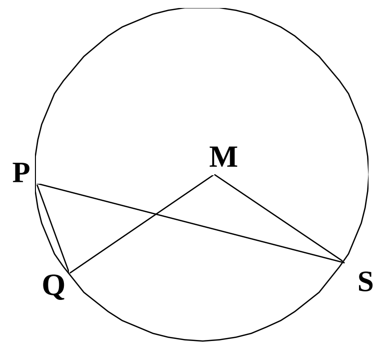
NOTE:	**– DIAGRAMS FOR PROVING THEORY MAY BE USED ON THE DIAGRAM SHEETS OR REDRAWN IN THE ANSWER BOOK.** **– DETACH THE DIAGRAM SHEETS FROM THE QUESTION PAPER AND PLACE THEM IN YOUR ANSWER BOOK.** **– GIVE A REASON FOR EACH STATEMENT, UNLESS OTHERWISE STATED**

QUESTION 7

7.1 In the diagram alongside, circle PQS is drawn.

Use the diagram on the diagram sheet or redraw the diagram in your answer book to prove the theorem which states that:

If M is the centre of circle PQS, then $\hat{M} = 2\hat{P}$ (6)



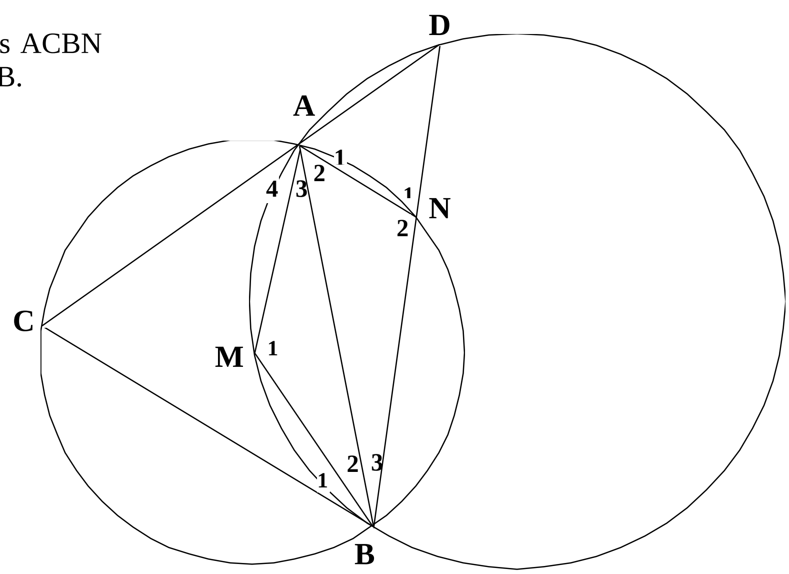
7.2 In the diagram alongside, circles ACBN and AMBD intersect at A and B.

CB is a tangent to the larger circle at B.

M is the centre of the smaller circle.

CAD and BND are straight lines.

Let $\hat{A}_3 = x$



7.2.1 Determine the size of $\hat{D}$ in terms of x. (4)

7.2.2 Prove that:

(a) $CB \parallel AN$ (7)

(b) AB is a tangent to circle ADN. (4)

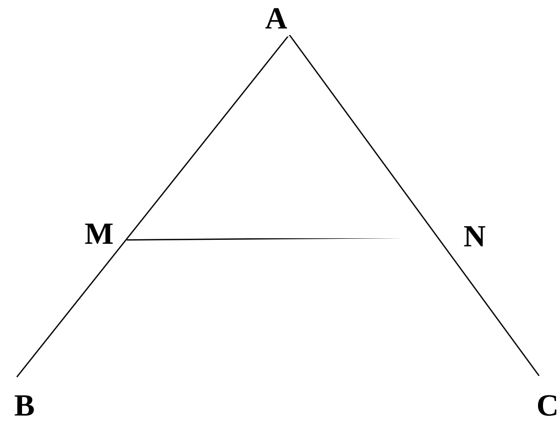
[21]

QUESTION 8

8.1 In the diagram alongside, M and N as two points on AB and AC respectively of ? ABC .

Use the diagram on the diagram sheet or redraw the diagram in your answer book to prove the theorem which states that :

If $\frac{AM}{MB} = \frac{AN}{NC}$, then MN || BC. (7)

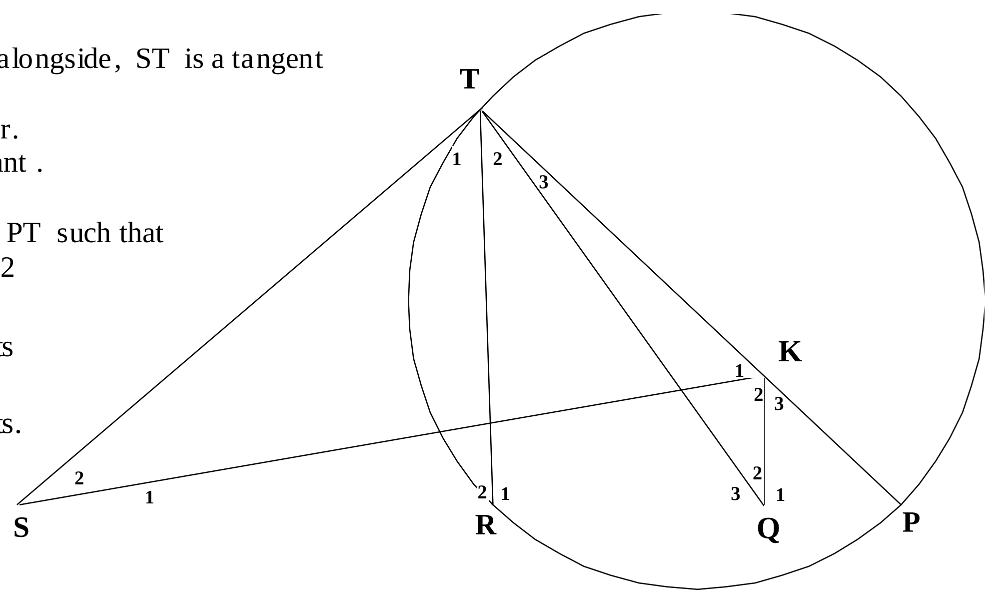


8.2 In the diagram alongside, ST is a tangent to circle TRP.
PT is a diameter.
SRQP is a secant .

K is a point on PT such that
PK : KT = 1 : 2

PR = $\sqrt{18}$ units

PQ = $\sqrt{2}$ units.



8.2.1 Prove that:

(a) RT || QK (4)

(b) TKQS is a cyclic quadrilateral. (5)

(c) Δ QRT ||| Δ KTS (4)

8.2.2 If PS = $\sqrt{32}$ units, calculate stating reasons and **without using a calculator**, the length of :

(a) ST (6)

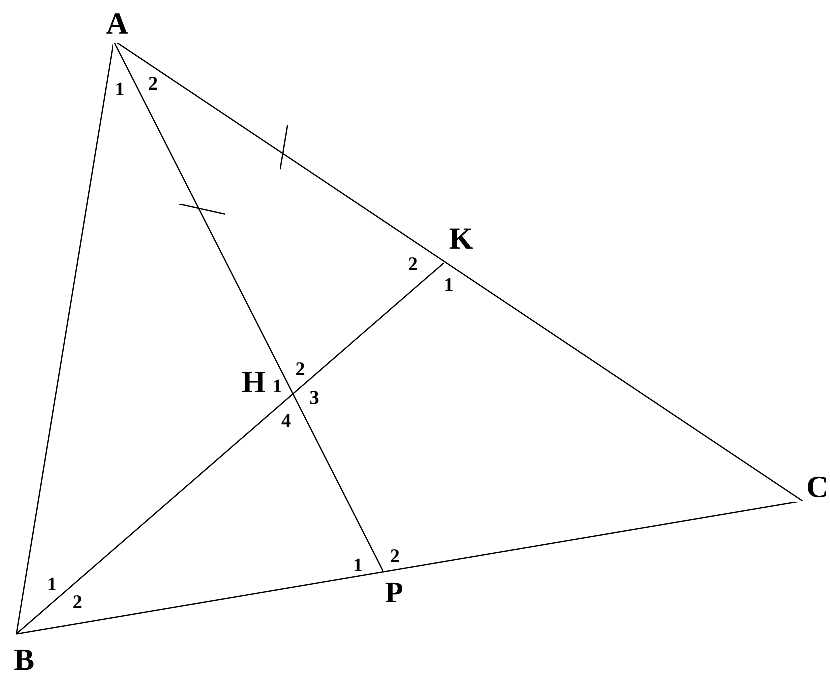
(b) KT (5)

[31]

QUESTION 9

In the diagram below, $A\hat{B}C$ is bisected by BK with K on AC.

AP and BK intersect at H with P on BC so that AH = AK



9.1 Prove that $\frac{AB}{BC} = \frac{AK}{KC}$ (7)

9.2 If it is further given that AKPB is a cyclic quadrilateral and that H is the centre of the inscribed circle of ΔABC, calculate the size of $B\hat{A}C$, stating reasons. (7)

[14]

TOTAL 200

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Mathematics Formula Sheet (HG and SG)
Wiskunde Formuleblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}(a + T_n) \qquad S_n = \frac{n}{2}(a + l) \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = a.r^{n-1} \qquad S_n = \frac{a(1-r^n)}{1-r} \ (r \neq 1) \qquad S_n = \frac{a(r^n - 1)}{r-1} \ (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

$$A = P\left(1 + \frac{r}{100}\right)^n \quad \text{OR / OF} \quad A = P\left(1 - \frac{r}{100}\right)^n$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$(x_3 \ ; \ y_3) = \left(\frac{x_1 + x_2}{2} \ ; \ \frac{y_1 + y_2}{2}\right)$$

$$x^2 + y^2 = r^2$$

$$(x-p)^2 + (y-q)^2 = r^2$$

In Δ ABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta ABC = \tfrac{1}{2} ab.\sin C$$