

SENIOR CERTIFICATE EXAMINATION

MATHEMATICS P1 HG

QUESTION 1			
1.1	$k + 5 = \frac{14}{k}$		
	1.1.1	$k^2 + 5k - 14 = 0$ $(k - 2)(k + 7) = 0$ $k = 2$ or $k = -7$	(3) ✓ multiplying ✓ factorising ✓ both solutions
	1.1.2	$\sqrt{x+5} = 2$ or $\sqrt{x+5} = -7$ $x+5 = 4$ or invalid/ no solution $x = -1$	(3) ✓ substitution ✓ identifying invalid ✓ value of x
1.2	1.2.1	$ 5 - x = 9$ $5 - x = 9$ or $-(5 - x) = 9$ $x = -4$ or $x = 14$	(3) ✓ interpretation/ use definition ✓✓ each value one
	1.2.2	$\frac{2x-2}{x-3} > 3$ $\frac{2x-2}{x-3} - 3 > 0$ $\frac{2x-2-3(x-3)}{x-3} > 0$ $\frac{2x-2-3x+9}{x-3} > 0$ $\frac{7-x}{x-3} > 0$ OR $\frac{x-7}{x-3} < 0$ $3 < x < 7$	(5) ✓ transfer of 3 ✓ common denominator ✓ simplification ✓✓ answer
1.3	$\frac{1}{x} + \frac{1}{y} = 3$ (1) $x - y = \frac{1}{2}$ (2) From (2): $x = y + \frac{1}{2}$ (3) Substituting (3) in (2) yields: $\frac{1}{y + \frac{1}{2}} + \frac{1}{y} = 3$ $y + y + \frac{1}{2} = 3y \left(y + \frac{1}{2} \right)$ $4y + 1 = 3y(2y + 1) = 6y^2 + 3y$ $6y^2 - y - 1 = 0$ $(2y - 1)(3y + 1) = 0$ $y = \frac{1}{2}$ or $y = -\frac{1}{3}$ $x = \frac{1}{2} + \frac{1}{2}$ or $x = -\frac{1}{3} + \frac{1}{2}$ $= 1$ or $= \frac{1}{6}$ i.e. $\left(\frac{1}{2}; 1\right)$ or $\left(-\frac{1}{3}; \frac{1}{6}\right)$		(8) [22] ✓ equation (3) ✓ substitution ✓ simplification ✓ standard form ✓ factors ✓ both y values ✓✓ each x-value gets 1

	<p>Alternative solution: From (1): $y + x = 3xy$ (4) Substitute (3) in (4) : $y + y + \frac{1}{2} = 3y\left(y + \frac{1}{2}\right)$ $2y + \frac{1}{2} = 3y^2 + \frac{3}{2}y$ $4y + 1 = 6y^2 + 3y$ $\therefore 6y^2 - y - 1 = 0$ $(2y - 1)(3y + 1) = 0$ $y = \frac{1}{2} \text{ or } y = -\frac{1}{3}$ $\therefore x = 1 \text{ or } x = \frac{1}{6}$</p>		<ul style="list-style-type: none"> ✓ equation (4) ✓ substitution ✓ standard form ✓ factors ✓ both y values ✓✓ each x-value gets 1
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QUESTION 2

2.1	$4x - 5 = k(x^2 - 1)$ $kx^2 - 4x + 5 - k = 0$ $\Delta = b^2 - 4ac$ $= (-4)^2 - 4(k)(5 - k)$ $= 16 - 20k + 4k^2$ <p>For equal roots : $\Delta = 0$ i.e. $4k^2 - 20k + 16 = 0$ $4(k - 4)(k - 1) = 0$ $k = 4 \text{ or } k = 1$</p>		<ul style="list-style-type: none"> ✓ standard form ✓ substitution ✓ simplifying ✓ equating Δ to 0 ✓ factors ✓ both values of k <p>(6)</p>	
2.2	$x^2 + p = (p + 2)x$ $x^2 - (p + 1)x + p = 0$ $\Delta = [-(p + 1)]^2 - 4(1)(p)$ $= p^2 + 2p + 1 - 4p$ $= p^2 - 2p + 1$ $= (p - 1)^2$ <p>Δ is a perfect square; \therefore roots are rational for all rational p</p>	$x^2 - px + p - x = 0$ $x(x - p) - (x - p) = 0$ $(x - 1)(x - p) = 0$ $x = 1 \text{ or } x = p$ <p>both of which are rational</p>		<ul style="list-style-type: none"> ✓ standard form ✓ use of Δ ✓ substitution ✓ simplifying ✓ expressing as perfect square and conclusion <p>(5) [11]</p>

QUESTION 3

3.1	$(-1)^3 + p(-1) + r = -9$ $\therefore r - p = -8 \text{ (1)}$ $(1)^3 + p(1) + r = -1$ $\therefore r + p = -2 \text{ (2)}$ <p>(1) + (2): $2r = -10$ $r = -5$ $\therefore p = 3$</p>		<ul style="list-style-type: none"> ✓ substitution ✓ equating to 9 ✓ substitution ✓ equating to -1 ✓ value of r ✓ value of p <p>(6)</p>
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3.2	<p>Let $f(x) = 3 - 7x + 5x^2 - x^3$</p> <p>$f(1) = 3 - 7(1) + 5(1)^2 - (1)^3$</p> <p>$= 3 - 7 + 5 - 1 = 0$</p> <p>$\therefore x - 1$ is a factor of $f(x)$</p> <p>$f(x) = (x - 1)(-x^2 + 4x - 3)$</p> <p>$= (x - 1)(x - 1)(3 - x)$</p> <p>$f(x) = 0$</p> <p>$\therefore x = 3$ or $x = 1$</p>	(6) [12]	<p>✓ ✓ finding a factor</p> <p>✓ ✓ quadratic factor</p> <p>✓ linear factors</p> <p>✓ both values of x</p>
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QUESTION 4

4.1	<p>$f(x) = ax^2 + bx + c$ and $g(x) = x - 2$</p>		
4.1.1	<p>C (0 ; 2) and D (4 ; 2):</p> <p>$g(0) = 0 - 2 = 2$</p>	(3)	<p>✓ C D: ✓ $x = 4$</p> <p>✓ $y = 2$</p>
4.1.2	<p>$y = a(x - p)^2 + q$</p> <p>At A (2 ; 4): $y = a(x - 2)^2 + 4$</p> <p>Subst C(0 ; 2): $2 = a(0 - 2)^2 + 4$</p> <p>$-2 = 4a$</p> <p>$\therefore a = -\frac{1}{2}$</p> <p>$\therefore y = -\frac{1}{2}(x - 2)^2 + 4$</p> <p>$= -\frac{1}{2}(x^2 - 4x + 4) + 4$</p> <p>$y = -\frac{1}{2}x^2 + 2x + 2$</p> <p>$\therefore b = 2; c = 2$</p>	(5)	<p>✓ subst. Turning pt.</p> <p>✓ subst C or D</p> <p>✓ value of a</p> <p>✓ simplification</p> <p>✓ equation</p>
4.1.3	<p>$-\frac{1}{2}x^2 + 2x + 2 = 0$</p> <p>$x^2 - 4x - 4 = 0$</p> <p>$x = \frac{4 \pm \sqrt{16 + 16}}{2}$ or $x = \frac{-2 \pm \sqrt{4 + 4}}{-1}$</p> <p>$= 2 \pm 2\sqrt{2}$</p> <p>$= 4,83$ or $-0,83$</p> <p>E (-0,83 ; 0) & H (4,83 ; 0)</p>	(4)	<p>✓ standard form</p> <p>✓ subst in formula</p> <p>✓ x-value of E</p> <p>✓ x-value of H</p>
4.1.4	<p>(a) $x \leq 0$ or $x \geq 4$</p> <p>(b) $x \leq -0,83$ or $x \geq 4,83$</p>	(2) (4)	<p>✓ ✓ one mark per inequality</p> <p>✓ ✓ ✓ ✓ two marks per inequality</p>
4.2	<p>$h(x) = \log_a x$ & A(8 ; 3)</p>		
4.2.1	<p>$y = \log_a x$</p> <p>$3 = \log_a 8$</p> <p>$3 = 3 \log_a 2$ OR $8 = a^3$</p> <p>$1 = \log_a 2$ $2^3 = a^3$</p> <p>$\therefore a = 2$</p>	(3)	<p>✓ substitution</p> <p>✓ exp. form or $8 = 2^3$</p> <p>✓ answer</p>
4.2.2	<p>B (1 ; 0)</p>	(1)	<p>✓ answer</p>

4.2.3	$x = \log_2 y$ OR $y = 2^x$ $\therefore h^{-1}(x) = 2^x$ or a^x	$y = \log_2 x$ $x = 2^y$ $y = 2^x$	(2)	✓ either expression ✓ correct form
4.2.4	$h^{-1}(3) = 2^3 = 8$ OR	$h(8) = 3 \therefore h^{-1}(3) = 8$	(1)	✓ answer
4.2.5	$x > 0$		(1)	✓ answer
			[26]	

QUESTION 5

5.1	5.1.1	$\sqrt{\frac{8}{3}} + 5\sqrt{\frac{2}{3}} - \sqrt{\frac{1}{6}} = \frac{13}{2}\sqrt{\frac{2}{3}}$ $\text{LHS} = 2\sqrt{\frac{2}{3}} + 5\sqrt{\frac{2}{3}} - \sqrt{\frac{2}{12}}$ $= 7\sqrt{\frac{2}{3}} - \frac{1}{2}\sqrt{\frac{2}{3}}$ $= \left(6\frac{1}{2}\right)\sqrt{\frac{2}{3}}$ $= \frac{13}{2}\sqrt{\frac{2}{3}} = \text{RHS}$	$\frac{1}{6} = \frac{2}{12} = \frac{1}{4} \cdot \frac{2}{3}$	(4)	✓ $2\sqrt{\frac{2}{3}}$ ✓ $\frac{1}{6} = \frac{1}{4} \cdot \frac{2}{3}$ ✓ $\frac{1}{2}\sqrt{\frac{2}{3}}$ ✓ addition
	5.1.2	$\log \frac{75}{16} - 2\log \frac{5}{9} + \log \frac{32}{243} = \log 2$ $\text{LHS} = \log \left(\frac{75}{16} \times \frac{32}{243}\right) - \log \frac{25}{81}$ $= \log \left(\frac{50}{81} \div \frac{25}{81}\right)$ $= \log \left(\frac{50}{81} \times \frac{81}{25}\right)$ $= \log 2 = \text{RHS}$	$\log \frac{\left(\frac{75}{16} \times \frac{32}{243}\right)}{\frac{25}{81}}$ $= \log \frac{75}{16} \times \frac{32}{243} \times \frac{81}{25}$ $= \log \frac{3 \cdot 5^2}{2^4} \times \frac{2^5}{3^5} \times \frac{3^4}{5^2}$ $= \log 2$	(4)	✓ application of "product" rule ✓ $-\log \frac{25}{81}$ ✓ application of "quotient" rule ✓ simplification
5.2	5.2.1	$2^{2x+1} - 3 \cdot 2^x + 1 = 0$ $2 \cdot 2^{2x} - 3 \cdot 2^x + 1 = 0$ $(2 \cdot 2^x - 1)(2^x - 1) = 0$ $2^x = \frac{1}{2}$ or $2^x = 1$ $2^x = 2^{-1}$ or $2^x = 2^0$ $x = -1$ or $x = 0$		(5)	✓ $2^{2x+1} = 2 \cdot 2^x$ ✓ factors ✓ split equations ✓ one value of x ✓ other value of x
	5.2.2	$\log_2 9x + \log_4 x^2 = 2$ $\frac{\log 9x}{\log 2} + \frac{\log x^2}{\log 4} = 2$ $\frac{\log 9x}{\log 2} + \frac{\log x}{\log 2} = 2$ $\log 9x + \log x = 2 \log 2$ $\log 9x^2 = \log 4$ $9x^2 = 4$ $x^2 = \left(\frac{2}{3}\right)^2$ $x = \frac{2}{3}$		(6)	✓ change of base ✓ simplification ✓ multiplying by LCD ✓ simplifying LHS ✓ simplifying RHS ✓ answer ($x > 0$)

	5.2.3	$\log_{\frac{1}{2}} 3x > 2$ $\log_{\frac{1}{2}} 3x > 2 \log_{\frac{1}{2}} \frac{1}{2} \quad \text{OR}$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin: 5px;"> $3x < \left(\frac{1}{2}\right)^2 \quad \checkmark\checkmark$ </div> $3x < \frac{1}{4}$ $x < \frac{1}{12} \text{ but } x > 0 \text{ by definition}$ $\therefore 0 < x < \frac{1}{12}$	(5)	<ul style="list-style-type: none"> ✓ exponential form ✓ with correct < ✓ $x < \frac{1}{12}$ ✓ $x > 0$ definition ✓ answer
5.3	$2^x \cdot 3^{x+1} = 10$ $2^x \cdot 3^x \cdot 3 = 10$ $6^x = \frac{10}{3}$ $x = \log_6 \frac{10}{3} \quad \text{OR}$ $= \frac{\log 10 - \log 3}{\log 6}$ $= 0,67$	$\log 6^x = \log \frac{10}{3}$ $x = \frac{\log \frac{10}{3}}{\log 6}$ $= 0,67$	(5) [29]	<ul style="list-style-type: none"> ✓ taking out 3 ✓ 6^x form ✓ taking logs ✓ x as subject ✓ answer

QUESTION 6				
6.1	$S_n = \frac{5}{2}n^2 + \frac{7}{2}n$			
	6.1.1	$T_1 = S_1$ $= \frac{5}{2}(1)^2 + \frac{7}{2}(1)$ $= \frac{12}{2} = 6$	(3)	<ul style="list-style-type: none"> ✓ statement ✓ substitution ✓ answer
	6.1.2	$S_2 = \frac{5}{2}(2)^2 + \frac{7}{2}(2)$ $= 10 + 7 = 17$ $T_2 = S_2 - S_1$ $= 17 - 6 = 11$ $\therefore d = T_2 - T_1$ $= 11 - 6 = 5$	(5)	<ul style="list-style-type: none"> ✓ T_2 equation ✓ value of S_2 ✓ value of T_2 ✓ d equation ✓ value of d
	6.1.3	$T_{10} = a + 9d$ $= 6 + 9(5)$ $= 51$ <p>OR</p> $T_{10} = S_{10} - S_9$ $= \frac{5}{2}(10)^2 + \frac{7}{2}(10) - \frac{5}{2}(9)^2 - \frac{7}{2}(9)$ $= \frac{1}{2}(500 + 70 - 405 - 63)$ $= \frac{1}{2}(102)$ $= 51$	(2)	<ul style="list-style-type: none"> ✓ formula ✓ answer ✓ substitution ✓ answer

6.2	$1 + 2 + 3 + \dots = 630$ $\underbrace{\hspace{10em}}_n$		
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6.2.1	$S_n = \frac{n}{2}[2a + (n-1)d]$ $630 = \frac{n}{2}(2 + n - 1)$ $1260 = n^2 + n$ $n^2 + n - 1260 = 0$ $(n + 36)(n - 35) = 0$ $n \neq -36$ or $n = 35$ \therefore there are 35 rows in the stack	$n = \frac{-1 \pm \sqrt{5041}}{2}$ $= \frac{-1 + 71}{2} \text{ or } \frac{-1 - 71}{2}$ $= 35$	<ul style="list-style-type: none"> ✓ choice of formula ✓ correct substitution ✓ standard form ✓ factors ✓ answer (correct conclusion)
6.2.2	$T_n = a + (n-1)d$ $T_{35} = 1 + (35-1)(1)$ $= 35$ OR By inspection $T_{35} = 35$		<ul style="list-style-type: none"> ✓ answer
6.3	$\sum_{k=1}^x 3^k = 1\ 092$ $3 + 9 + 27 + \dots + 3^x = 1\ 092$ $S_n = \frac{a(r^n - 1)}{r - 1}$ $1\ 092 = \frac{3(3^x - 1)}{3 - 1}$ $728 = 3^x - 1$ $3^x = 729 = 3^6$ $x = 6$		<ul style="list-style-type: none"> ✓ expansion ✓ choice of formula ✓ substitution ✓ 729 ✓ 3^6 ✓ answer

6.4	6.4.1	$r = \frac{T_{n+1}}{T_n} = \frac{3(m-1)^{n+2}}{3(m-1)^{n+1}}$ $= m - 1$ is a constant. \therefore sequence is geometric	<ul style="list-style-type: none"> ✓ r in terms of T & n ✓ $m - 1$ ✓ a constant
	6.4.2	$ m - 1 < 1$ $\therefore -1 < m - 1 < 1$ $0 < m < 2$ $S_\infty = \frac{a}{1 - r}$ $= \frac{3(m-1)^2}{1 - (m-1)}$ $= \frac{3(m-1)^2}{2 - m}$ OR $S_\infty = \frac{3m^2 - 6m + 3}{2 - m}$	<ul style="list-style-type: none"> ✓ statement ✓ without absolute valu ✓✓ value of m ✓ formula ✓ answer
			(6) [31]

QUESTION 7			
7.1		$f(x) = x^2 - 6x$ $f(x+h) = (x+h)^2 - 6(x+h) = x^2 + 2xh + h^2 - 6x - 6h$ $\frac{f(x+h) - f(x)}{h} = \frac{2xh - 6h + h^2}{h}$ $= 2x - 6 + h$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} (2x - 6 + h)$ $= 2x - 6$	<p>✓ subst ✓ simplification</p> <p>✓ difference</p> <p>✓ dividing by h</p> <p>✓ use of formula</p> <p>✓ answer</p>
7.2	7.2.1	$y = (2x)^2 - \frac{1}{3x}$ $= 4x^2 - \frac{1}{3}x^{-1}$ $\frac{dy}{dx} = 8x + \frac{1}{3}x^{-2}$	<p>✓ simplifying</p> <p>✓✓ derivative of each term</p>
	7.2.2	$y = \frac{2\sqrt{x} - 5}{\sqrt{x}}$ $= 2 - 5x^{-\frac{1}{2}}$ $\frac{dy}{dx} = +\frac{5}{2}x^{-\frac{3}{2}}$	<p>✓✓ simplifying each term</p> <p>✓✓ derivative</p>
7.3	7.3.1	<p>For <u>turning points</u>: $f'(x) = 0$</p> $6x^2 - 10x - 4 = 0$ $2(3x+1)(x-2) = 0$ $x = -\frac{1}{3} \text{ or } x = 2$ $f\left(-\frac{1}{3}\right) = 2\left(-\frac{1}{27}\right) - 5\left(\frac{1}{9}\right) - 4\left(-\frac{1}{3}\right) + 3$ $= 3\frac{19}{27}$ $f(2) = 2(8) - 5(4) - 4(2) + 3 = -9$	<p>✓ derivative</p> <p>✓ = 0</p> <p>✓ factors</p> <p>✓ x-values</p> <p>✓ one y-value</p> <p>✓ other y-value</p>

7.3.2		(5)	<ul style="list-style-type: none"> ✓ one turning pt ✓ other turning pt ✓ x-intercepts ✓ y-intercept ✓ shape
7.3.3	$k = 9$ or $k = -3\frac{19}{27}$	(4)	✓✓ per solution
7.3.4	<p>Gradient of tangent $m_T = f'(x)$ i.e. $m_T = 6x^2 - 10x - 4$ at $x = 1$: $m_T = 6 - 10 - 4 = -8$ $f(1) = 2 - 5 - 4 + 3 = -4$ \therefore point of contact is $(1; -4)$ $y = mx + c$ subst m_T and point: $-4 = (-8)(1) + c$ $\therefore c = 4$ $\therefore y = -8x + 4$</p>	(6) [34]	<ul style="list-style-type: none"> ✓ know that $m_T = f'(x)$ ✓ $m_T = -8$ ✓ $f(1) = -4$ ✓ formula ✓ substitution ✓ equation

QUESTION 8			
8.1	$P(x) = \frac{55}{2x} + \frac{x}{200}$		
8.1.1	<p>Petrol consumption rate: $P(x) = \frac{55}{2x} + \frac{x}{200}$ For 2 000 km, the amount of petrol consumed is: $2000\left(\frac{55}{2x} + \frac{x}{200}\right) = \frac{55000}{x} + 10x$ litres Petrol costs = $4\left(\frac{55000}{x} + 10x\right) = \frac{220000}{x} + 40x$ rands Hours traveled = $\frac{2000}{x}$ Driver's earnings = $\frac{2000}{x} \cdot 18 = \frac{36000}{x}$ rands $C(x) = \text{petrol} + \text{driver}$ $= \frac{220000}{x} + 40x + \frac{36000}{x}$ $= \frac{256000}{x} + 40x$</p>	(6)	<ul style="list-style-type: none"> ✓ petrol used ✓✓ petrol cost ✓ hours traveled ✓ driver's earnings ✓ cost equation

8.1.2	For minimum costs: $C'(x) = 0$ $-\frac{256000}{x^2} + 40 = 0$ $40x^2 - 256000 = 0$ $x^2 = 6400$ $x = 80 \text{ km/h}$	(5)	✓✓ derivative (each term) ✓ = 0 ✓ multiplying/ std form ✓ answer
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8.2	$T(t) = 30 + 4t - \frac{1}{2}t^2$		
8.2.1	$T'(t) = 4 - t$ OR $\frac{dT}{dt} = 4 - t$	(2)	✓✓ derivative (each term)
8.2.2	$T'(t) \leq 0$ $4 - t \leq 0$ $t \geq 4$ $\therefore 4 \leq t \leq 10$	(4) [17]	✓✓ statement ✓ t - subject ✓ answer

QUESTION 9			
9.1	$x \leq 150$ (1) $y \leq 120$ (2) $x + y \leq 200$ (3) $x \geq 40$ (4) $y \geq 10$(5)	(4)	✓ for...(2) ✓ for... (3) ✓ for....(4) ✓ for ... (5)
9.2	See graph	(6)	✓✓✓✓✓ lines ✓ shading
9.3	$P = 5x + 10y$	(2)	✓✓ equation
9.4	Search line on graph : $y = -\frac{1}{2}x + \frac{P}{10}$	(2)	✓✓ search line
9.5	Maximum at A Coordinates of A (50 ; 80) $P = 5(80) + 10(120)$ $= R 1 600$	(4)	✓ statement ✓ coordinates of A ✓ substitution ✓ answer
		[18]	
T O T A L			200

QUESTION 9

EXAMINATION NUMBER																			
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