education

Department:
Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATION - 2005

MATHEMATICS P1

HIGHER GRADE

OCTOBER/NOVEMBER 2005

Marks: 200

Time: 3 Hours

This question paper consists of 9 pages, 1 page of graph paper and 1 information sheet.

INSTRUCTIONS

Read the following instructions carefully before answering the questions:

1. This paper consists of **8** questions. Answer **ALL** the questions.
2. Clearly show **ALL** calculations, diagrams, graphs, etc. you have used in determining the answers.
3. An approved calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
4. If necessary, answers should be rounded off to **TWO** decimal places, unless stated otherwise.
5. The attached graph paper must be used only for **QUESTION 8**.
6. Number the answers **EXACTLY** as the questions are numbered.
7. Diagrams are not necessarily drawn to scale.
8. It is in your own interest to write legibly and present the work neatly.
9. **An information sheet with formulae is included at the end of the question paper.**

QUESTION 1

1.1 Solve for x:

1.1.1 $6 - x = 2\sqrt{x+2}$ (6)

1.1.2 $\dfrac{-3}{(x+1)(x-3)} < 0$ (3)

1.2 Given: $(x-3)(5x^2 - x - 2) = 0$

1.2.1 Write down a rational value of x which satisfies the equation. (1)

1.2.2 Calculate the irrational roots of the equation (rounded off to TWO decimal places). (6)

1.3 Given: $A(x) = \dfrac{2x^2+1}{x^2-2x}$

1.3.1 For which values of x is $A(x)$ undefined? (3)

1.3.2 Calculate the values of k for which $\dfrac{2x^2+1}{x^2-2x} = k$ has real roots. (9)

1.3.3 What is the range of the function A? (1)

1.4 Solve for x and y:

1.4.1 $|x-2| \leq 5$ (4)

1.4.2 $|y-2| = 1$ (3)

1.5 Determine the maximum value of $|x-y|$, if x and y are solutions to both QUESTION 1.4.1 and QUESTION 1.4.2. (2)

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QUESTION 2

2.1 Given: $f(x) = -2(x-3)^2 + 8$

2.1.1 Write down the co-ordinates of the turning point of the graph of f. (2)

2.1.2 Draw a sketch graph of f. Clearly indicate the co-ordinates of the turning point as well as the intercepts with the axes. (7)

2.1.3 **Without any further calculations,** sketch the graph of $y = f(x-2)$ on a different set of axes. Indicate only the co-ordinates of the turning point. (2)

2.2 Given: $h(x) = \dfrac{4}{x}$

2.2.1 Draw a sketch graph of h. (2)

2.2.2 Explain why h and h^{-1} are the same function. (3)

2.2.3 Determine the values of m for which the graph of $y = mx$ intersects the graph of h. (3)

2.2.4 Write down the coordinates of the points of intersection of the line $y = x$ with the graph of h. (2)

[21]

QUESTION 3

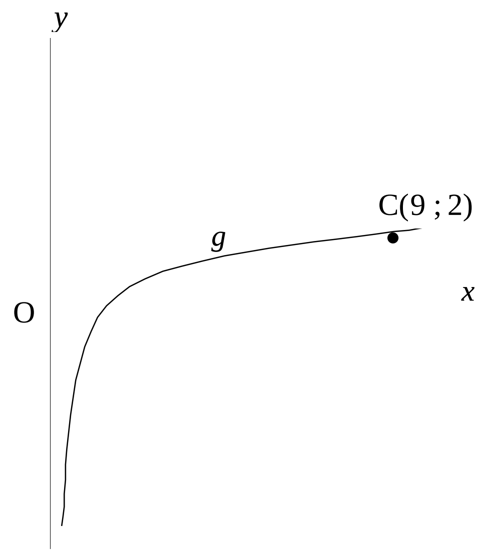
3.1 Prove the Remainder Theorem which states that if a polynomial $p(x)$ is divided by $(x - a)$ until the remainder is independent of x, then the remainder equals $p(a)$. (4)

3.2 Solve for x: $4x^3 - 4x^2 - 7x - 2 = 0$ (8)

[12]

QUESTION 4

4.1 The sketch graph below shows the curve of $g(x) = \log_a x$.
The curve passes through C(9 ; 2).



4.1.1 Calculate the value of a. (3)

4.1.2 Give the equation of the function h which is symmetrical to g with respect to the x-axis. (1)

4.1.3 For which value of x is $g(x) = h(x)$? (2)

4.1.4 Use the graph to solve for x: $\log_a x \leq 2$ (3)

4.2 Simplify to a single number, **without using a calculator:**

$$\left(\frac{5}{4^{-1} - 9^{-1}} \right)^{\frac{1}{2}} + \log_3 9^{2,12}$$ (6)

4.3 Solve for x **without using a calculator:**

4.3.1 $x\sqrt{3} = \frac{20}{\sqrt{6}}$ (**Note that:** $\sqrt{2} = 1{,}414$). (4)

4.3.2 $10^{2x} = 5(2^{2x}) + 120(4^x)$ (5)

4.3.3 $2\log_4 x + \log_x 64 = 5$ (8)

4.4 The strength x of the earthquake that caused the recent tsunami disaster was measured as $\log x = 8{,}5$ using the Richter scale. Three months later another earthquake struck and its strength was measured as $\log y = 5{,}1$. Compare the strengths of the earthquakes by calculating the ratio $\frac{x}{y}$. Round the answer off to TWO decimal places. (4)

[36]

QUESTION 5

5.1 The common difference of an arithmetic series is 3. Calculate the values of n for which the n^{th} term of the series is 93 and the sum of the first n terms is 975. (9)

5.2 Given the geometric series:

$$54 + 18 + 6 + \ldots + 54\left(\frac{1}{3}\right)^{n-1}$$

5.2.1 Show that the sum of the first n terms is given by $81 - 81\left(\frac{1}{3}\right)^{n}$. (3)

5.2.2 Calculate the smallest value of n for which the sum of the first n terms is greater than 80,99. (7)

5.2.3 What does $\left(\frac{1}{3}\right)^{n}$ approach as $n \to \infty$? (1)

5.2.4 Determine the value of $\sum_{k=1}^{\infty} 54\left(\frac{1}{3}\right)^{k-1}$, using QUESTIONS 5.2.1 and 5.2.3 or in another way. (2)

5.3 A new soccer competition is planned to stimulate interest in the 2010 World Cup.

This competition will require each of 8 teams to play every other team once.

5.3.1 Calculate the number of matches to be played in the competition. (3)

5.3.2 Determine a formula in terms of n for the number of matches that would be needed if each of n teams played each other once. (4)

[29]

QUESTION 6

6.1 Calculate: $\lim_{x \to 1} \frac{1-x^3}{1-x}$ (2)

6.2 Given: $f(x) = 2x^2 - x$

6.2.1 Use **the definition of the derivative** to calculate $f'(x)$. (6)

6.2.2 Hence, calculate the co-ordinates of the point at which the gradient of the tangent to the graph of f is 7. (4)

6.3 If $xy - 5 = \sqrt{x^3}$, determine $\frac{dy}{dx}$. (5)

6.4 Given : $g(x) = (x^{-2} + x^2)^2$. Calculate $g'(2)$. (4)

[21]

QUESTION 7

7.1 The graph of a cubic function f has turning points at A$(-1\,;p)$ and B$(2\,;q)$.
The function f has the following properties:

$f'(x) > 0$ for $x < -1$ or $x > 2$
$f'(x) < 0$ for $-1 < x < 2$
$f(2) > 0$

7.1.1 Draw a neat sketch graph of f. Clearly label points A and B on the sketch. (It is not necessary to show the x- and y- intercepts). (4)

7.1.2 Use the graph to deduce how many roots $f(x) = 0$ has. (1)

7.1.3 If $f(x) = x^3 + bx^2 + cx + d$, calculate the values of b and c. (7)

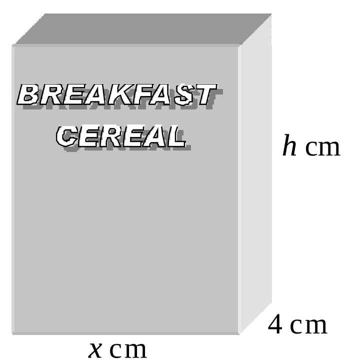
7.2 In order to reduce the temperature in a room from 28 °C, a cooling system is allowed to operate for 10 minutes. The room temperature, T after t minutes is given in °C by the formula:

$$T = 28 - 0{,}008t^3 - 0{,}16t \quad \text{where} \quad t \in [0\ ;\ 10]$$

7.2.1 At what rate (rounded off to TWO decimal places) is the temperature falling when $t = 4$ minutes? (4)

7.2.2 Calculate the lowest room temperature reached during the 10 minutes for which the cooling system operates. (4)

7.3 A cereal box has the shape of a rectangular prism as shown in the diagram below. The box has a volume of $480\ \text{cm}^3$, a breadth of 4 cm and a length of x cm.



7.3.1 Show that the total surface area of the box (in cm^2) is given by:
$A = 8x + 960x^{-1} + 240$ (5)

7.3.2 Determine the value of x for which the total surface area is a minimum. Round the answer off to the nearest cm. (5)

[30]

QUESTION 8

To meet the requirements of a specialised diet a meal is prepared by mixing two types of cereal, *Vuka* and *Molo*. The mixture must contain x packets of *Vuka* cereal and y packets of *Molo* cereal. The meal requires at least 15 g of protein and at least 72 g of carbohydrates. Each packet of *Vuka* cereal contains 4 g of protein and 16 g of carbohydrates. Each packet of *Molo* cereal contains 3 g of protein and 24 g of carbohydrates. There are at most 5 packets of cereal available. The feasible region is shaded on the attached graph paper.

8.1 Write down the constraint inequalities. (6)

8.2 If *Vuka* cereal costs R6 per packet and *Molo* cereal also costs R6 per packet, use the graph to determine how many packets of each cereal must be used for the mixture to satisfy the above constraints in each of the following cases:

8.2.1 The total cost is a minimum. (4)

8.2.2 The total cost is a maximum (give all possibilities). (3)
[13]

TOTAL: 200

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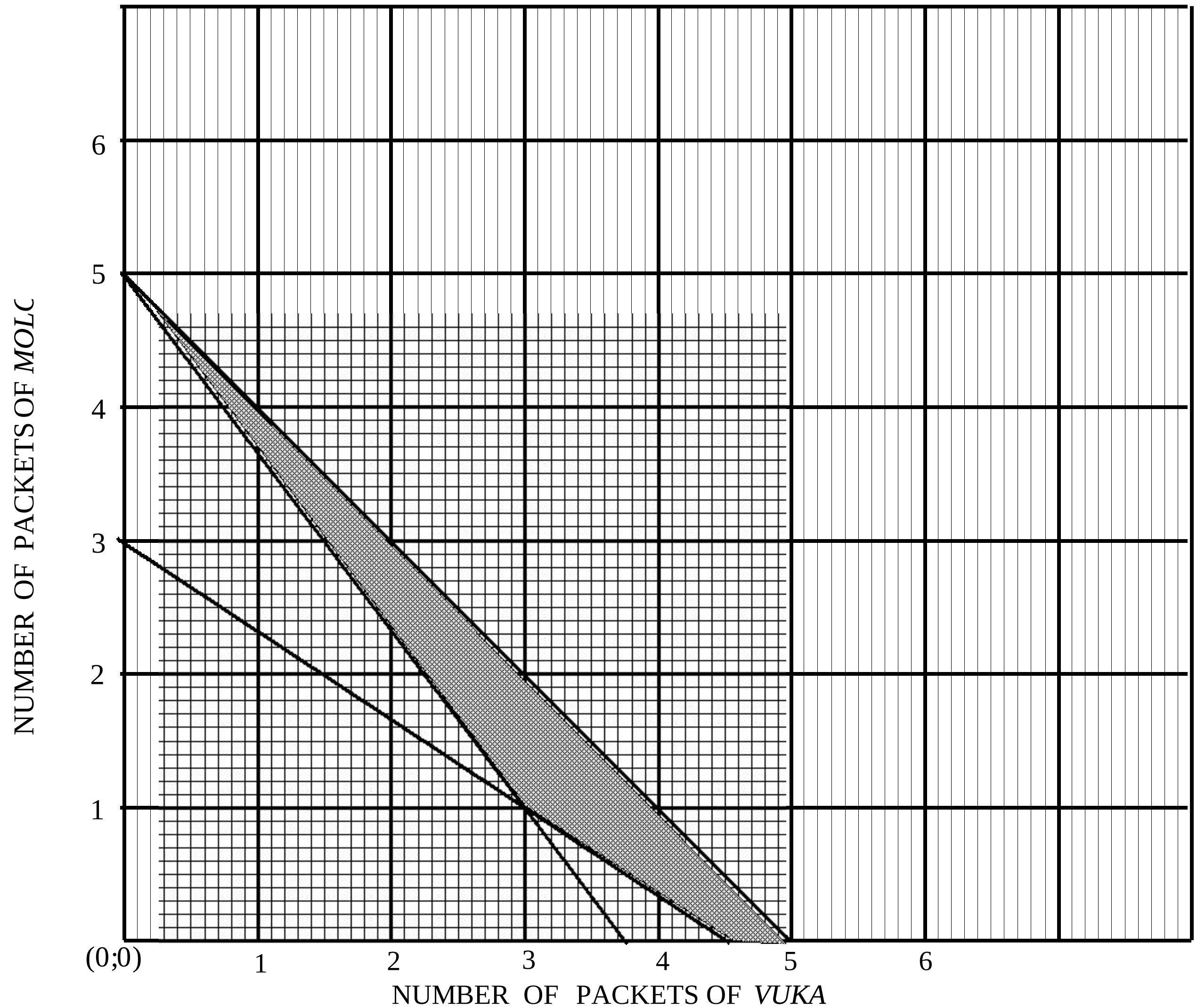
GRAPH PAPER FOR QUESTION 8



Mathematics Formula Sheet (HG and SG)
Wiskunde Formuleblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}(a + T_n)$ $\quad$ or / of $\quad S_n = \frac{n}{2}(a + l)$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(1 - r^n)}{1 - r} \ (r \neq 1)$ $\quad S_n = \frac{a(r^n - 1)}{r - 1} \ (r \neq 1)$

$S_\infty = \frac{a}{1 - r} \quad (|r| < 1)$

$A = P\left(1 + \frac{r}{100}\right)^n$ $\quad$ or / of $\quad A = P\left(1 - \frac{r}{100}\right)^n$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$y = mx + c$

$y - y_1 = m(x - x_1)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \tan\theta$

$(x_3; y_3) = \left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$x^2 + y^2 = r^2$

$(x - p)^2 + (y - q)^2 = r^2$

In ΔABC: $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

area ? ABC $= \frac{1}{2} ab.\sin C$