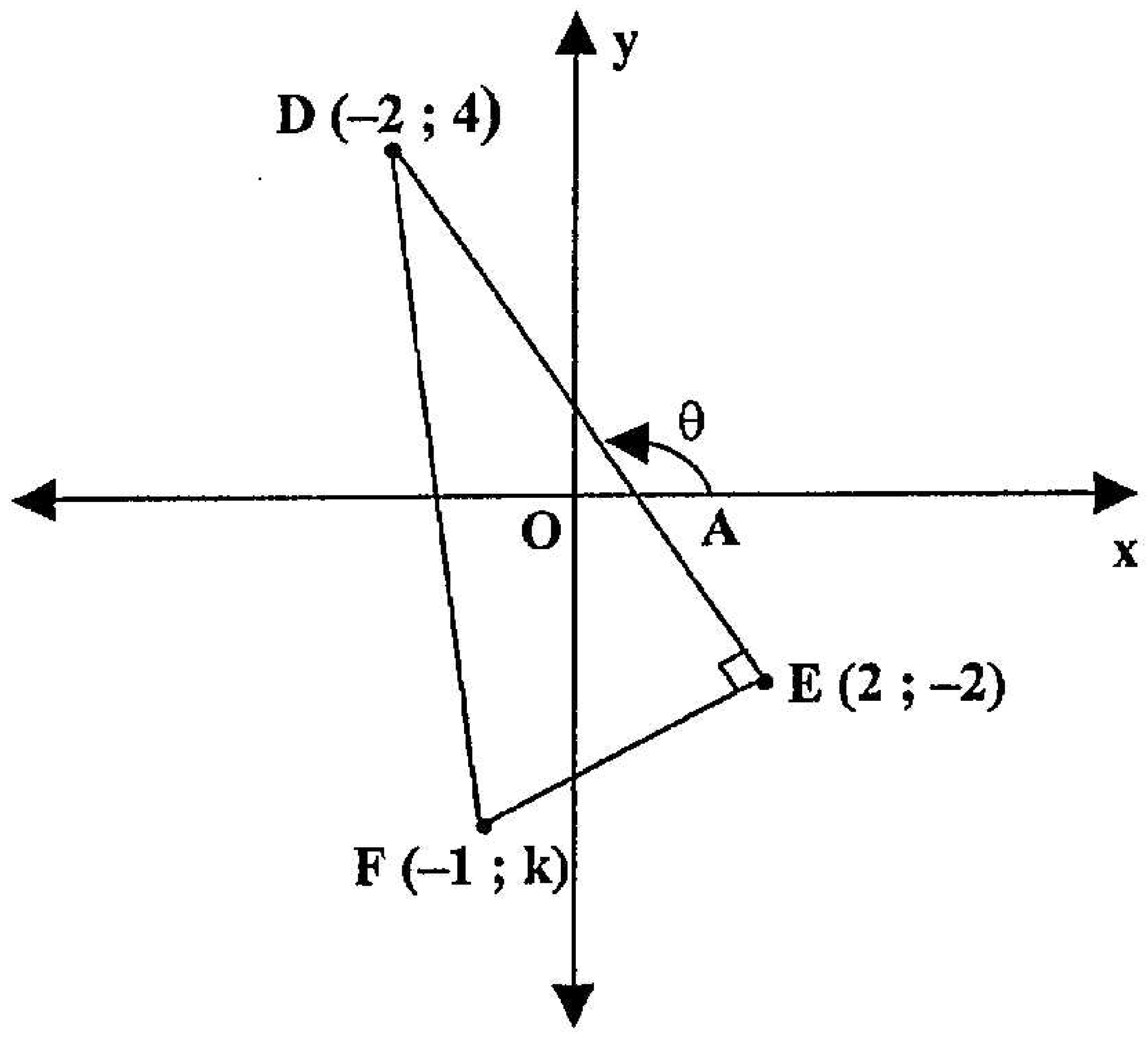
ANALYTICAL GEOMETRY

QUESTION 1 **[21]**

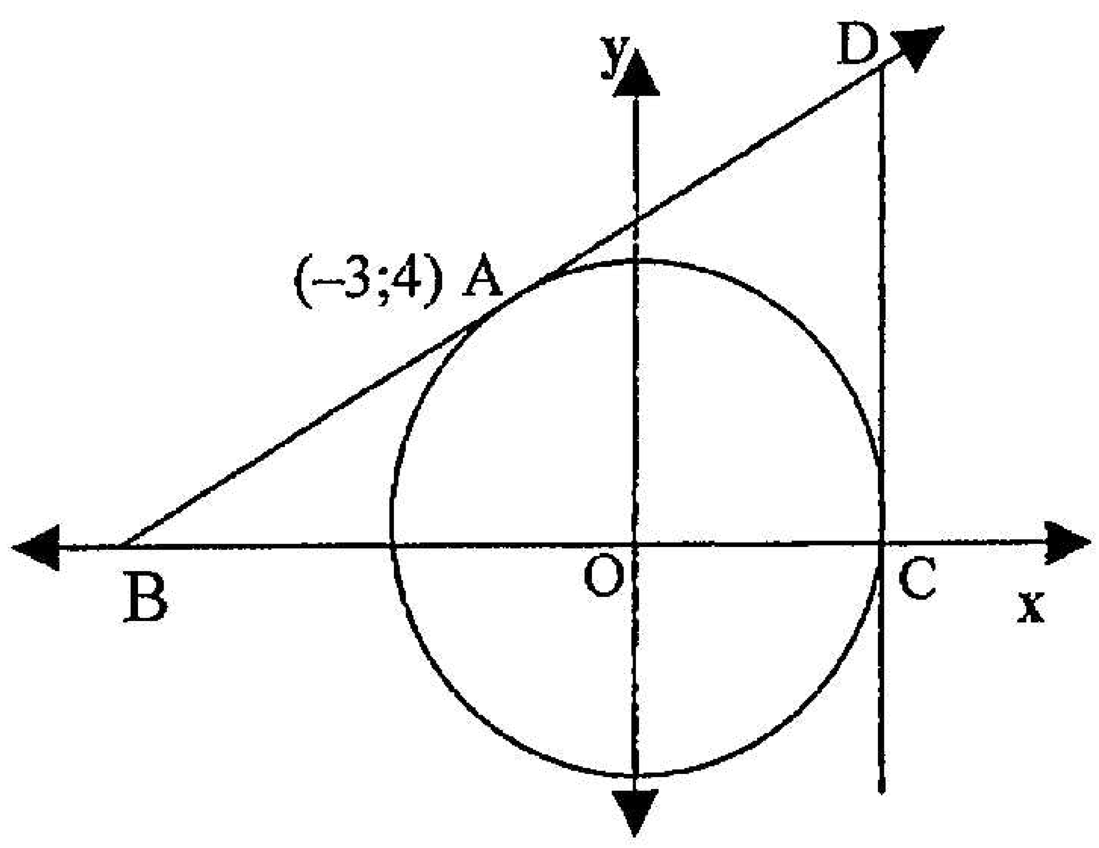


1.1	$m_{DE} = \frac{y_2 - y_1}{x_2 - x_1}$ ✓M $= \frac{-2-4}{2+2}$ $= \frac{-6}{4}$ $= \frac{-3}{2}$ ✓A (2)	correct formula correct answer Answer only 2 marks
1.2	$\tan\theta = m$ ✓M $= \frac{-3}{2}$ ✓CA Ref angle $= 56{,}31°$ $\theta = 123{,}69°$ ✓CA (3)	correct formula / substitution correct θ incorrect rounding off – penalty 1 mark **IN THIS QUESTION ONLY**

1.3	DE ⊥ EF $m_{DE} \times m_{EF} = -1$ ✓M $\frac{-3}{2} \times \frac{k+2}{-1-2} = -1$ ✓A $\frac{k+2}{-3} = \frac{2}{3}$ ✓CA $k+2 = -2$ $k = -4$	concept of perpendicular lines substitution simplification cannot use 1.4 to answer 1.3
1.3	**OR** $m_{DE} = \frac{-3}{2}$ $m_{EF} = \frac{k+2}{-1-2}$ ✓M $= \frac{-4+2}{-3} = \frac{2}{3}$ ✓A $m_{DE} \times m_{EF} = \frac{-3}{2} \times \frac{2}{3} = -1$ ✓CA $\therefore\ k = -4$ (3)	gradient substitution of k = –4 answer –1 answer only k = -4 - no marks
1.4	Midpt of FE $= M = \left(\frac{-1+2}{2}; \frac{-4-2}{2}\right)$ ✓M $= (\frac{1}{2}; -3)$ ✓A (2)	correct sub. in correct formula calc. x and y correctly Answer only FULL marks
1.5	$M(\frac{1}{2}; -3)$; $m_{DE} = \frac{-3}{2}$ Equation is $y + 3 = \frac{-3}{2}(x - \frac{1}{2})$ ✓CA ✓M $2y + 6 = -3(x - \frac{1}{2})$ ✓CA $2y + 6 = -3x + \frac{3}{2}$ $4y + 12 = -6x + 3$ $4y + 6x + 9 = 0$ or $y = \frac{-3}{2}x - \frac{9}{4}$ ✓CA **OR**	correct formula Subst. gradient and point M simplification (or any other acceptable form of the equation)

	OR $y = mx + c$ ✓M $-3 = \frac{-3}{2}(\frac{1}{2}) + c$ ✓CA $\frac{-12+3}{4} = c$ $\frac{-9}{4} = c$ ✓CA $y = \frac{-3}{2}x - \frac{9}{4}$ ✓CA (4)	correct formula correct substitution correct c substitution in correct formula (any other version of the equation)
1.6	$DE = \sqrt{(-2-2)^2+(4+2)^2}$ ✓M $= \sqrt{16+36}$ $= \sqrt{52}$ or 7,2 ✓A $EF = \sqrt{(2+1)^2+(-2+4)^2}$ ✓M $= \sqrt{9+4}$ $= \sqrt{13}$ or 3,6 ✓A Area of $\Delta DEF = \frac{1}{2} b \times h$ ✓M $= \frac{1}{2} DE.EF$ Area of $\Delta DEF = \frac{1}{2}\sqrt{52}\sqrt{13} = 13$ units^2 ✓CA ✓CA or 12,96 (7)	correct formula correct answer subst. correct answer correct area formula (or trig. area formula) subst. and simplification do not penalise for units or rounding off if calculator is used , no penalisation

QUESTION 2 **[18]**

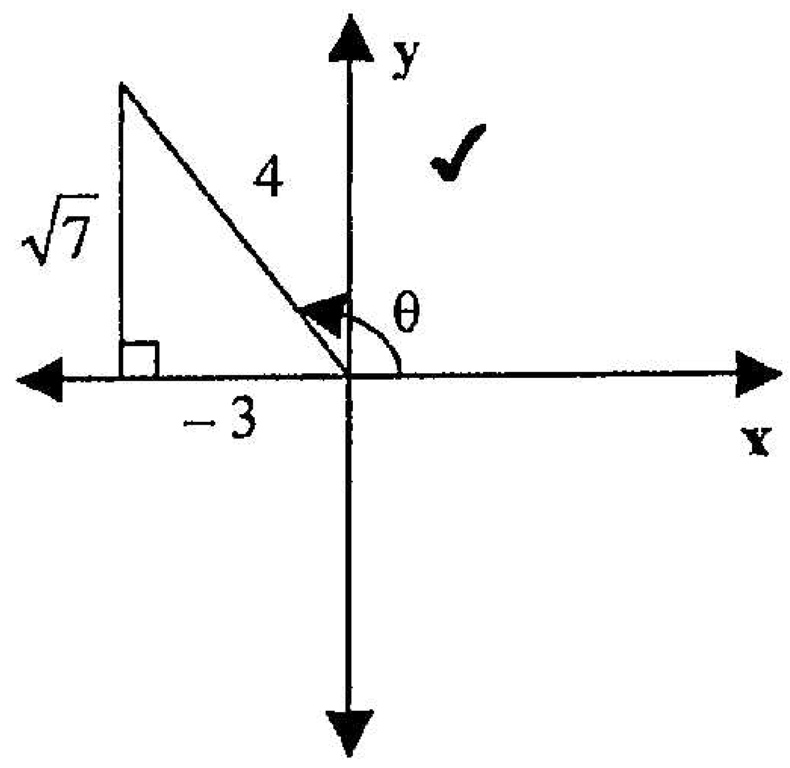


2.1.1	$x^2 + y^2 = r^2$ ✓M $= (-3)^2 + 4^2$ ✓A $= 9 + 16$ $= 25$ ✓CA **OR** Equation: $x^2 + y^2 = 25$ ✓M ✓A✓A (3)	formula Subst. correct answer if $x^2 + y^2 = 5$ - 2 marks
2.1.2	C (5 ; 0) ✓CA ✓A (2)	
2.1.3	$m_{OA} = \frac{4-0}{-3-0}$ ✓M $= -\frac{4}{3}$ ✓A $m_{BD} = \frac{3}{4}$ ✓CA (3)	subst. in correct formula simplification perpendicular gradient answer only – full marks
	OR $x_1x + y_1y = r^2$ ✓M $(-3)x + 4y = 25$ ✓A $4y = 3x + 25$ $y = \frac{3}{4} + \frac{25}{4}$ $\therefore m_{BD} = \frac{3}{4}$ ✓CA	subst. in correct formula simplification gradient answer only – full marks

2.1.4	Equation of BD: $y - 4 = \frac{3}{4}(x + 3)$ ✓M ✓CA $y - 4 = \frac{3}{4}x + \frac{9}{4}$ $y = \frac{3}{4}x + \frac{25}{4}$ ✓CA OR $4y = 3x + 25$	correct formula and subst simplification
	OR $y - 4 = \frac{3}{4}(x + 3)$ ✓M ✓CA $4y - 16 = 3x + 9$ $4y = 3x + 25$ ✓CA or $y = \frac{3}{4}x + \frac{25}{4}$ OR $4y = 3x + 25$ **OR** $y = mx + c$ $4 = \frac{3}{4}(-3) + c$ ✓M ✓CA $16 + 9 = 4c$ $\frac{25}{4} = c$ $y = \frac{3}{4}x + \frac{25}{4}$ ✓CA OR $4y = 3x + 25$ **OR** $x_1x + y_1y = r^2$ ✓M $(-3)x + 4y = 25$ ✓A $-3x + 4y = 25$ ✓A (3)	correct formula and subst. simplification correct formula and subst. substitution of c and m correct formula substitution equation

2.1.5	CD: $q = \frac{3}{4}(5) + \frac{25}{4}$ ✓CA D(5 ; q) $= \frac{40}{4} = 10$ ✓CA **OR** DA = DC $(5+3)^2 + (y-4)^2 = (5-5)^2 + (y-0)^2$ ✓CA $64 + y^2 - 8y + 16 = y^2$ $-8y = -80$ $y = 10$ ✓CA (2)	subst. simplification Answer only FULL marks tangent to circle substitution simplification
2.2	$m_{PS} = 2(m_{RQ})$ ✓M ✓A $\frac{y-4}{x-1} = 2\left[\frac{1+4}{-1-1}\right]$ ✓A $\frac{y-4}{x-1} = 2\left[\frac{5}{-2}\right]$ ✓CA $y-4 = -5(x-1)$ $y = -5x + 9$ ✓CA (5)	correct relationship correct subst. simplification any equivalent form if lengths are used – breakdown no mark

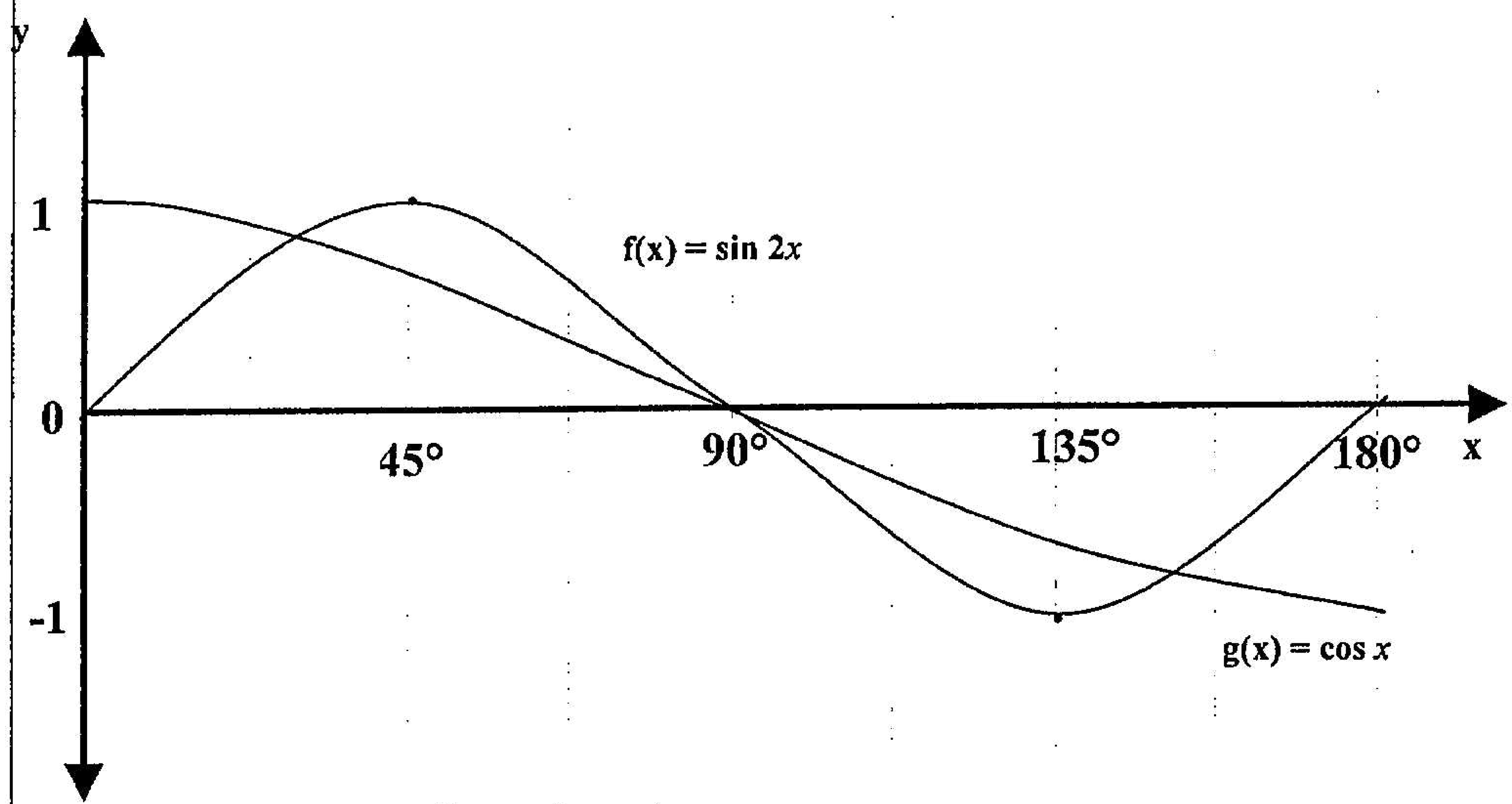
QUESTION 3 **[16]**



3.1	$\cos\theta = -\frac{3}{4}$ ✓A $y^2 = r^2 - x^2$ $= 16 - 9$ $= 7$ $y = \sqrt{7}$ ✓A ✓CA ✓CA $12\sin\theta \cdot \tan\theta = 12 \cdot \frac{\sqrt{7}}{4} \cdot \frac{\sqrt{7}}{-3}$ $= -7$ ✓CA (6)	correct value cosθ correct y value or on the sketch substitution of values simplification
3.2.1	$\frac{\cot(90° - \alpha) \cdot \sin(180° + \alpha)}{\tan(360° - \alpha)}$ ✓A ✓A ✓A $= \frac{(\tan\alpha)(-\sin\alpha)}{-\tan\alpha}$ $= \sin\alpha$ ✓CA (4)	correct reduction with signs (this step could be ommitted) simplification if angle left out – penalty 1 mark
3.2.2	$\sec 120° \ (\operatorname{cosec} 315° + \cot 210°)$ ✓A ✓A ✓A $= -\sec 60° \ (-\operatorname{cosec} 45° + \cot 30°)$ ✓CA ✓CA ✓CA $= -2(-\frac{\sqrt{2}}{1} + \sqrt{3})$ OR $-2(-\sqrt{2} + \sqrt{3})$ OR $2\sqrt{2} - 2\sqrt{3}$ (6)	correct reduction with signs correct subst. of special angle ratios if calculator used – no marks full mark if answer given in surd form

QUESTION 4 **[15]**

4.1



For each graph

✓✓ shape

✓✓ y-intercept

✓✓ turning points (8)

✓✓ x-int

Turning points need not be in coordinate form

if graph out of domain – penalise 1 mark

4.2.1	✓A ✓CA $y \in [-1;1]$ or $-1 \leq y \leq 1$ (2)	notation and endpoints
4.2.2	1 ✓CA (1)	reading from graph or answer only
4.2.3	✓CA ✓CA ✓notation $x \in (90°; 180°)$ or $90° < x < 180°$ (3)	one mark for each correct endpoint of interval and one mark for notation
4.3	180° ✓A (1)	Answer

QUESTION 5		[10]
5.1	L.H.S = $\frac{\sin\theta}{\cos\theta}\tan\theta + \frac{\cos\theta}{\cos\theta}$ ✓A ✓A	for splitting correctly
	$= \tan\theta\tan\theta + 1$ ✓CA ✓CA	if in first step cancellation of cos θ - max. 1 mark
	$= \tan^2\theta + 1$ ✓A	correct simplification
	$= \sec^2\theta$	simplification
	= R.H.S	Penalty of 1 if working with LHS and RHS simultaneously
		use of x, y and r - no marks
	OR	
	LHS = $\frac{\sin.\tan\theta + \cos\theta}{\cos\theta}$	
	$= \frac{\sin\theta\frac{\sin\theta}{\cos\theta} + \cos\theta}{\cos\theta}$ ✓A	correct identity
	$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta} \times \frac{1}{\cos\theta}$ ✓A ✓M	simplification and multiplication
	$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta}$ ✓CA	
	$= \frac{1}{\cos^2\theta}$ ✓CA	simplification
	$= \sec^2\theta$ = RHS (5)	
5.2.1	$\sin\theta = \tan 323°$	
	$\sin\theta = -0{,}75....$ ✓A	correct value
	Reference angle $= 48{,}90°$	
	$\theta = 180° + 48{,}90°$ ✓CA	angle in third quadrant
	$= 228{,}90°$ ✓CA (3)	correct ∠
		Penalty if $\cos\theta = 228{,}9$
		Answer only full marks
5.2.2	$\sec(\theta + 10°) = \sec(228{,}9° + 10°)$	
	$= \sec 238{,}9°$ ✓CA	substitution from 5.2.1
	$= -1{,}94$ ✓CA (2)	answer
		Answer only full marks

QUESTION 6 [22]

6.1

Draw AD ⊥ BC produced. ✔M or shown on the diagram

$AD^2 = b^2 - CD^2$ (pythagoras)

$AD^2 = c^2 - DB^2$ ✔M

$b^2 - CD^2 = c^2 - DB^2$ ✔A

$c^2 = b^2 + DB^2 - CD^2$

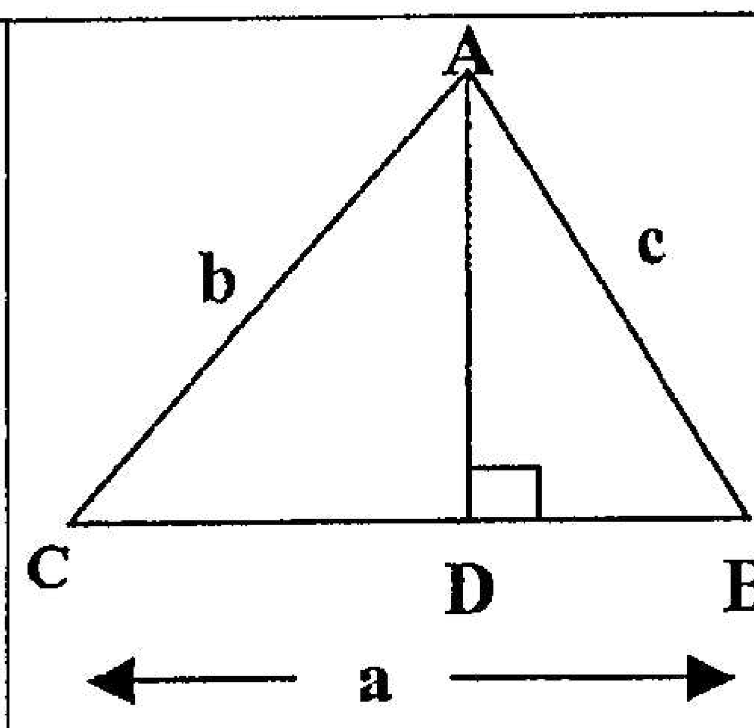
$= b^2 + (a - CD)^2 - CD^2$

$= b^2 + a^2 - 2a.CD$ ✔A

But $\frac{CD}{b} = \cos C$ ✔A

$\therefore CD = b\cos C$ ✔A

$\therefore c^2 = a^2 + b^2 - 2ab\cos C$



penalty of 1 mark – if right angle not shown or written in words

OR Draw AD ⊥ BC produced. or shown on the diagram ✔M

$c^2 = AD^2 + DB^2$ ✔M

$= AD^2 + (a - CD)^2$

$= AD^2 + a^2 - 2.a.CD + CD^2$

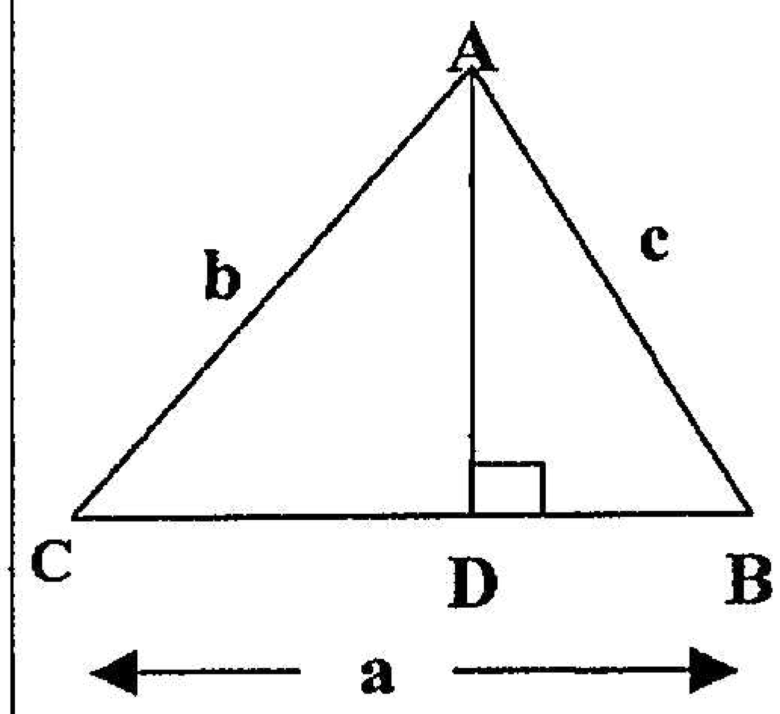
$= a^2 + (AD^2 + CD^2) - 2.a.CD$ ✔A

but $AD^2 + CD^2 = b^2$ ✔A

and $\frac{CD}{b} = \cos C$ ✔A

$\therefore CD = b\cos C$ ✔A

$\therefore c^2 = a^2 + b^2 - 2.a.b\cos C$



penalty of 1 mark – if right angle not shown or written in words

OR ✔A

Place C in standard position. A(b cos C; b sin C) and B(a ; 0)

$c^2 = (x_A - x_B)^2 + (y_A - y_B)^2$ ✔M

✔A

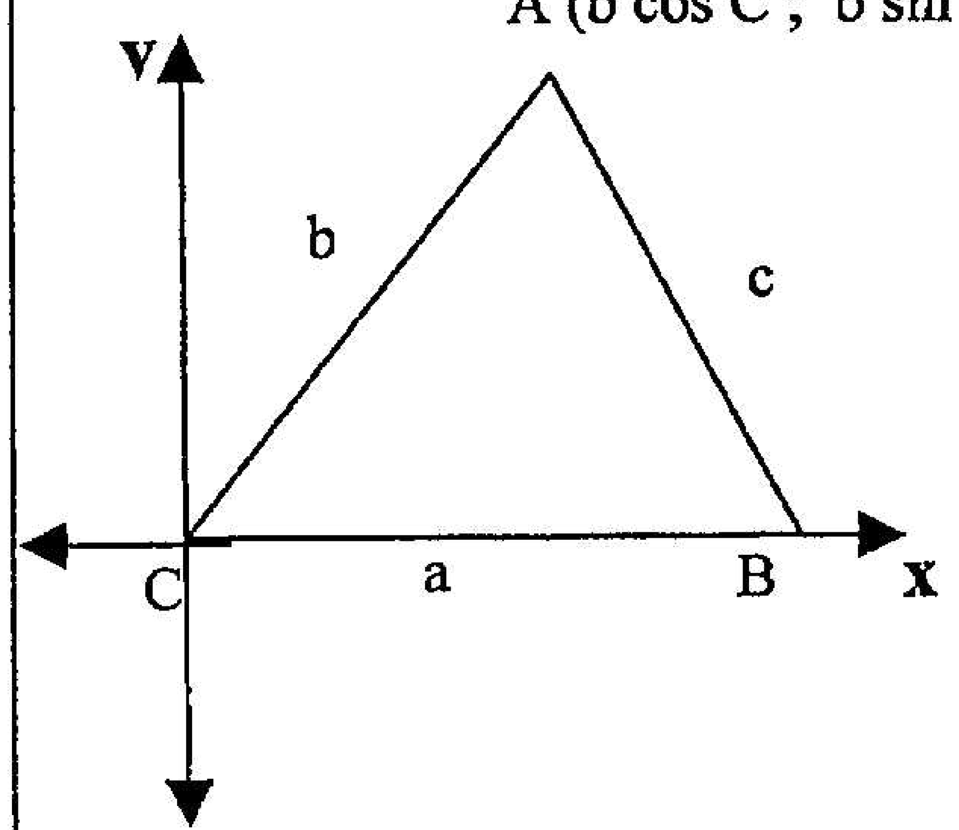
$c^2 = (b\cos C - a)^2 + (b\sin C - 0)^2$ ✔A

$= b^2\cos^2 C - 2ab\cos C + a^2 + b^2\sin^2 C$ ✔A

$= b^2(\cos^2 C + \sin^2 C) - 2ab\cos C + a^2$ ✔A

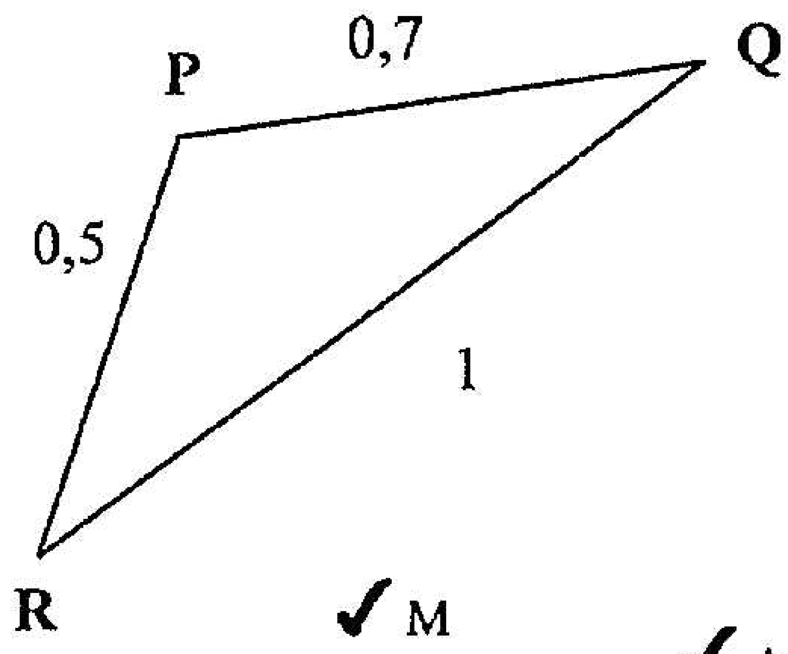
$= a^2 + b^2 - 2ab\cos C$ (6)

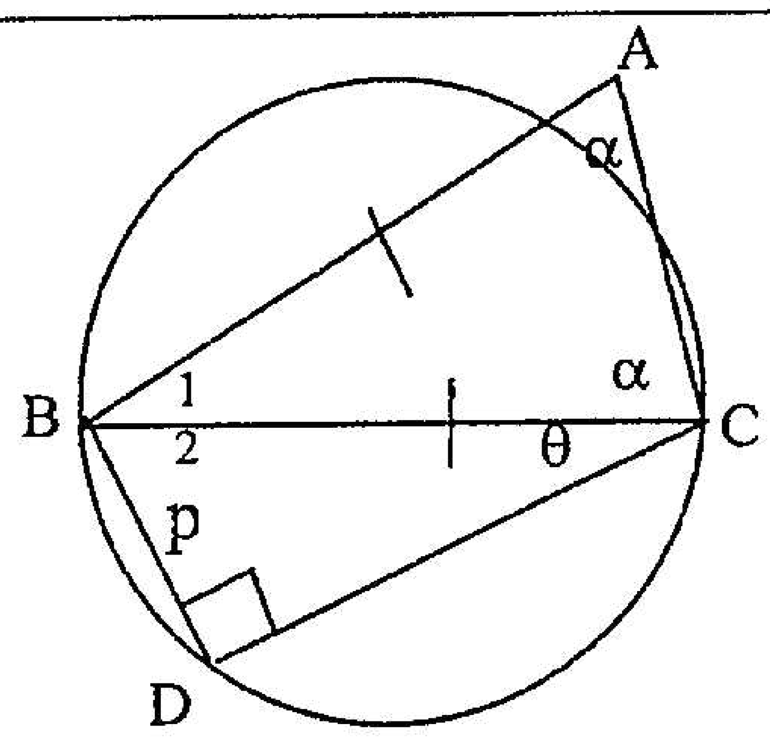
A (b cos C ; b sin C)

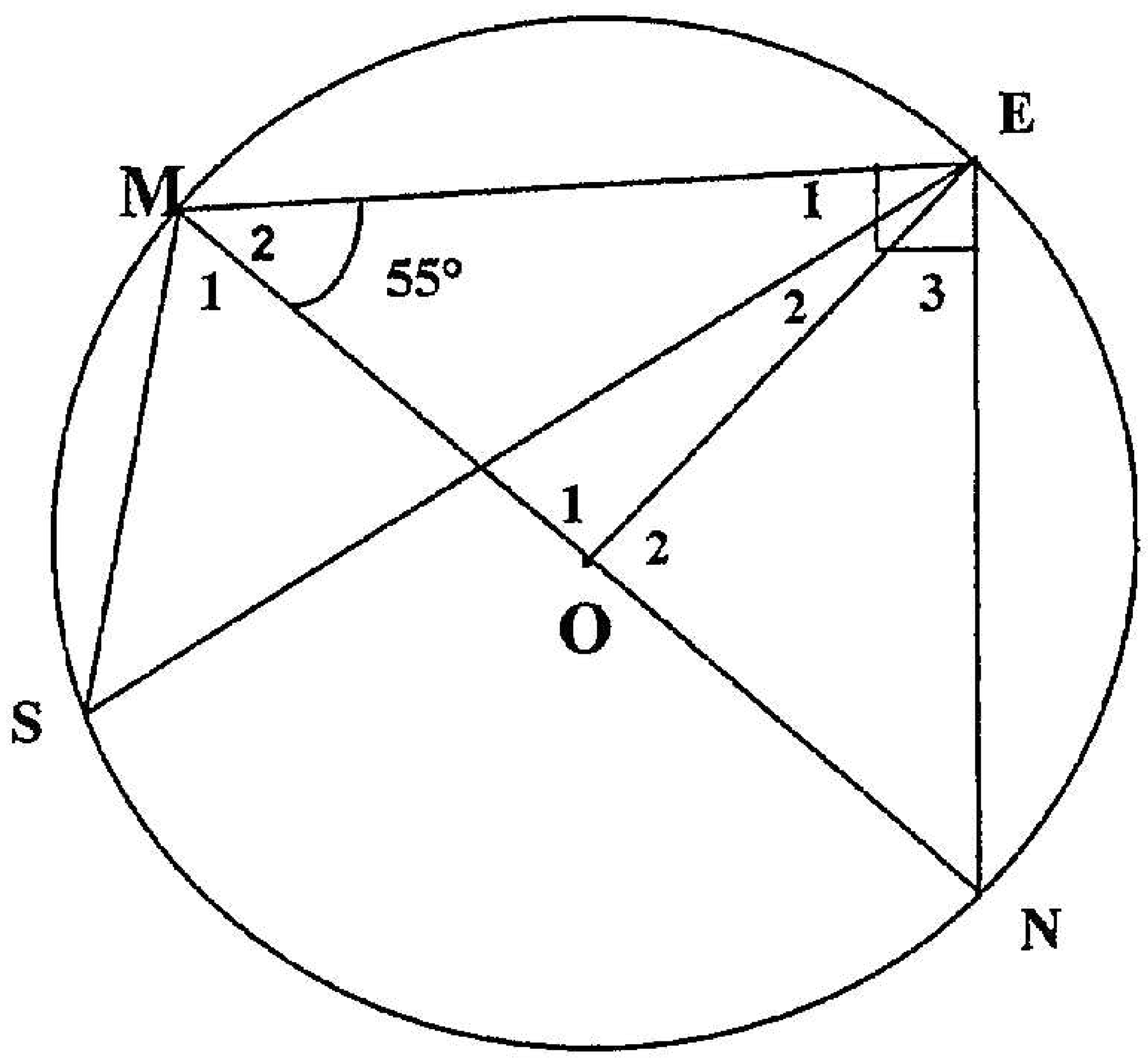


first mark could be given on sketch if only first step was done

no penalty if axis not drawn

6.2.1	 $1^2 = (0,5)^2 + (0,7)^2 - 2(0,5)(0,7)\cos P$ ✓M ✓A $1 = 0,74 - 0,7\cos P$ $\cos P = \frac{-0,26}{0,7} = -0,3714285$ ✓CA $\hat{P} = 180° - 68,20°$ ✓A $= 111,80°$ ✓CA (5) **OR** $\cos P = \frac{(0,5)^2 + (0,7)^2 - 1^2}{2(0,5)(0,7)}$ ✓M ✓A $= \frac{-0,26}{0,7} = -0,3714285$ ✓CA $\hat{P} = 180° - 68,20°$ ✓A $= 111,80°$ ✓CA	using cosine rule subtitution correct value of cos P correct ref angle correct $\hat{P}$ no penalty for rounding off using cosine rule subtitution correct value of cos P correct ref angle correct value of p
6.2.2	Area of Δ PQR= $\frac{1}{2}$ q r sin P ✓M $= \frac{1}{2}(0,5)(0,7)\sin 111,8°$ ✓CA $= 0,16\,km^2$ ✓CA **OR** Area of Δ PQR= $\frac{1}{2}$ q r sinP ✓M $= \frac{1}{2}(500)(700)\sin 111,8°$ ✓CA $= 162485.02\,m^2$ ✓CA (3)	Area rule correct substitution. answer - no penalty for incorrect rounding off Area rule correct substitution answer answer only full marks No penalty if units ommitted

6.3.1	 $\hat{D} = 90°$ ✓A (1)	Correct value
6.3.2	$\sin\theta = \frac{BD}{BC}$ ✓M $BC = \frac{p}{\sin\theta}$ ✓CA or $p \operatorname{cosec}\theta$ **OR** $\frac{BC}{\sin D} = \frac{BD}{\sin C}$ ✓M $BC = \frac{p.\sin 90°}{\sin\theta} = \frac{p}{\sin\theta}$ ✓CA (2)	applying sine function correctly correct BC sine rule correct BC
6.3.3	In Δ ABC: $\hat{B}_1 = 180° - 2\alpha$ ✓A (1)	correct ans.
6.3.4	$\frac{AC}{\sin B_1} = \frac{BC}{\sin A}$ ✓M $AC = \frac{BC.\sin B_1}{\sin A}$ ✓A ✓CA ✓CA $AC = \frac{p}{\sin\theta}.\frac{\sin[180-2\alpha]}{\sin\alpha}$ $= \frac{p.\sin 2\alpha}{\sin\theta.\sin\alpha}$ **OR** $\frac{AC}{\sin(180°-2\alpha)} = \frac{BC}{\sin\alpha}$ ✓M ✓CA $AC = BC.\frac{\sin(180°-2\alpha)}{\sin\alpha}$ ✓A ✓CA $= \frac{p}{\sin\theta}.\frac{\sin 2\alpha}{\sin\alpha}$ (4)	using sine rule finding AC correct subst. for BC & $\hat{B}_1$ using sine rule subst. finding AC correct subst. for BC

QUESTION 7		[7]
		
7.1	$\hat{O}_2 = 110°$ ✓S ($\angle$ at the centre =2x $\angle$ at the circum) ✓R **OR** $M\hat{E}O = 55°$ ✓S/R (MO = OE) $\hat{O}_2 = 55° + 55°$ ✓S/R (ext. angle of Δ) $= 110°$ (2)	different reasons could be found here (ext angle of triangle) Sum of angles of triangle
7.2	$M\hat{E}N = 90°$ ✓S (angle in semi-circle) ✓R $\hat{S} = \hat{N}$ ✓S $\angle$s in the same segment) ✓R $= 35°$ ✓S ($\angle$ s in a Δ) **OR** In Δ OEN , $\hat{N} + \hat{E}_3 = 70°$ ($\angle$ s in a Δ) ✓S/R $\hat{E}_3 = \hat{N}$ (ON = OE) ✓S/R $= \frac{180° - 110°}{2}$ $= 35°$ ✓S $\hat{S} = \hat{N} = 35°$ ✓S ($\angle$s in the same segment) ✓R (5)	**OR** $\hat{S} = \hat{N}$ ✓S ($\angle$s in same segment) ✓R $= \frac{1}{2}\hat{O}_1$ ✓S ($\angle$ at the centre =2 $\angle$ at the circum) ✓R $= \frac{1}{2} 70°$ $= 35°$ ✓S **OR** $\hat{O}_1 = 70°$ ✓S (adj. suppl. $\angle$ s) ✓R $\hat{S} = \frac{1}{2}\hat{O}_1$ ✓S ($\angle$ at the centre =2 $\angle$ at the circum) ✓R $= 35°$ ✓S

QUESTION 8 **[20]**

8.1

Constr. Draw diameter AOE, with E on the circle.
Join EC . or shown in the diagram ✓ M

Proof:

$\hat{A}_1 + \hat{A}_2 = 90°$ ✓ S (radius ⊥ tangent) ✓ R

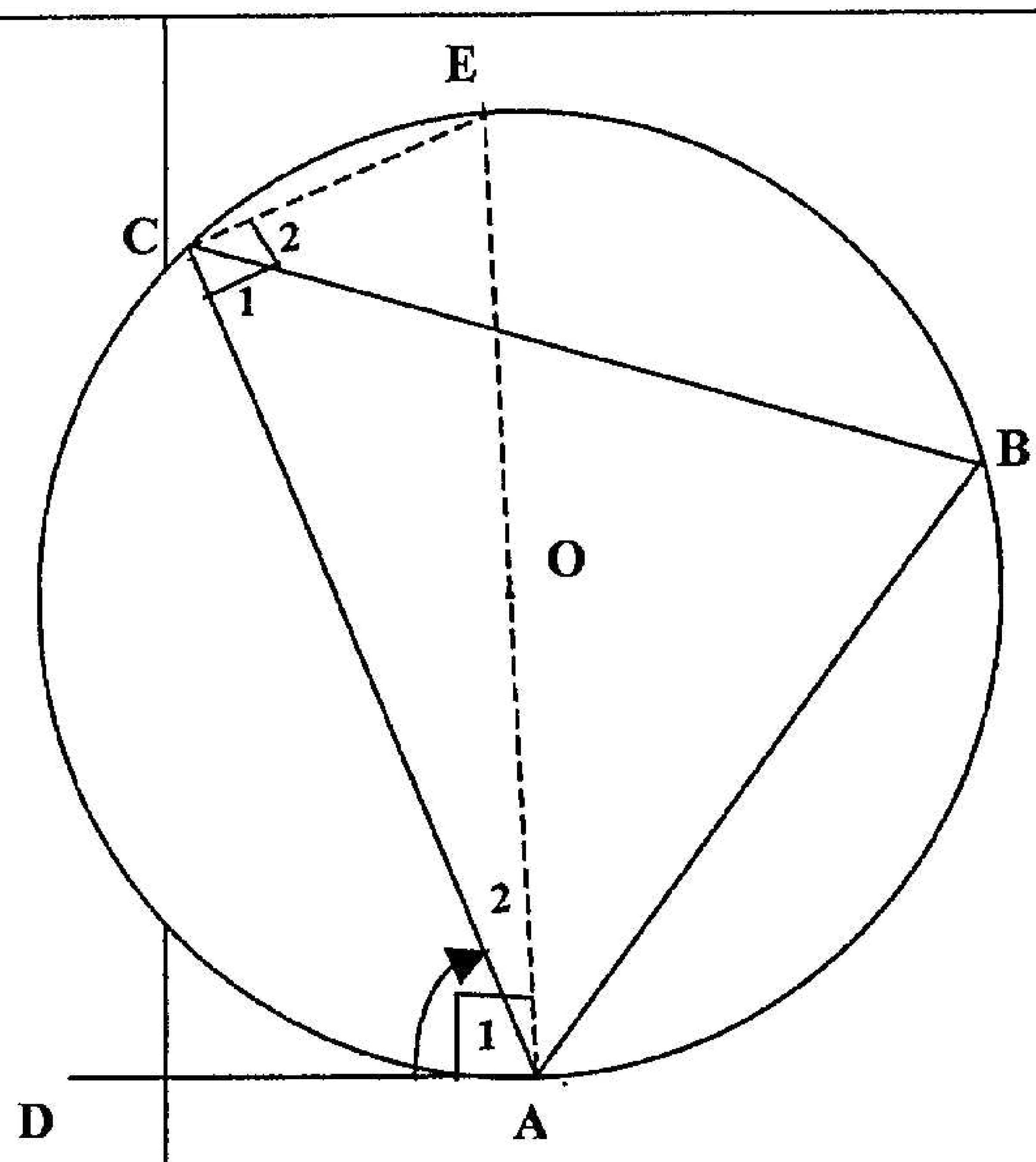
$\hat{C}_1 + \hat{C}_2 = 90°$ (∠ in semi-circle) ✓ S/R

$\hat{A}_2 + \hat{E} = 90°$ (sum ∠ s in a triangle) ✓ S/R

$\hat{A}_1 = \hat{E}$

$= \hat{B}$ (∠s in same segment) ✓ S/R

$\therefore D\hat{A}C = \hat{B}$



if line not drawn through O, but used as such, penalty of 1 mark

OR

Constr. Draw diameter AOE, with E on the circle. ✓M
Join EB .or shown in the diagram

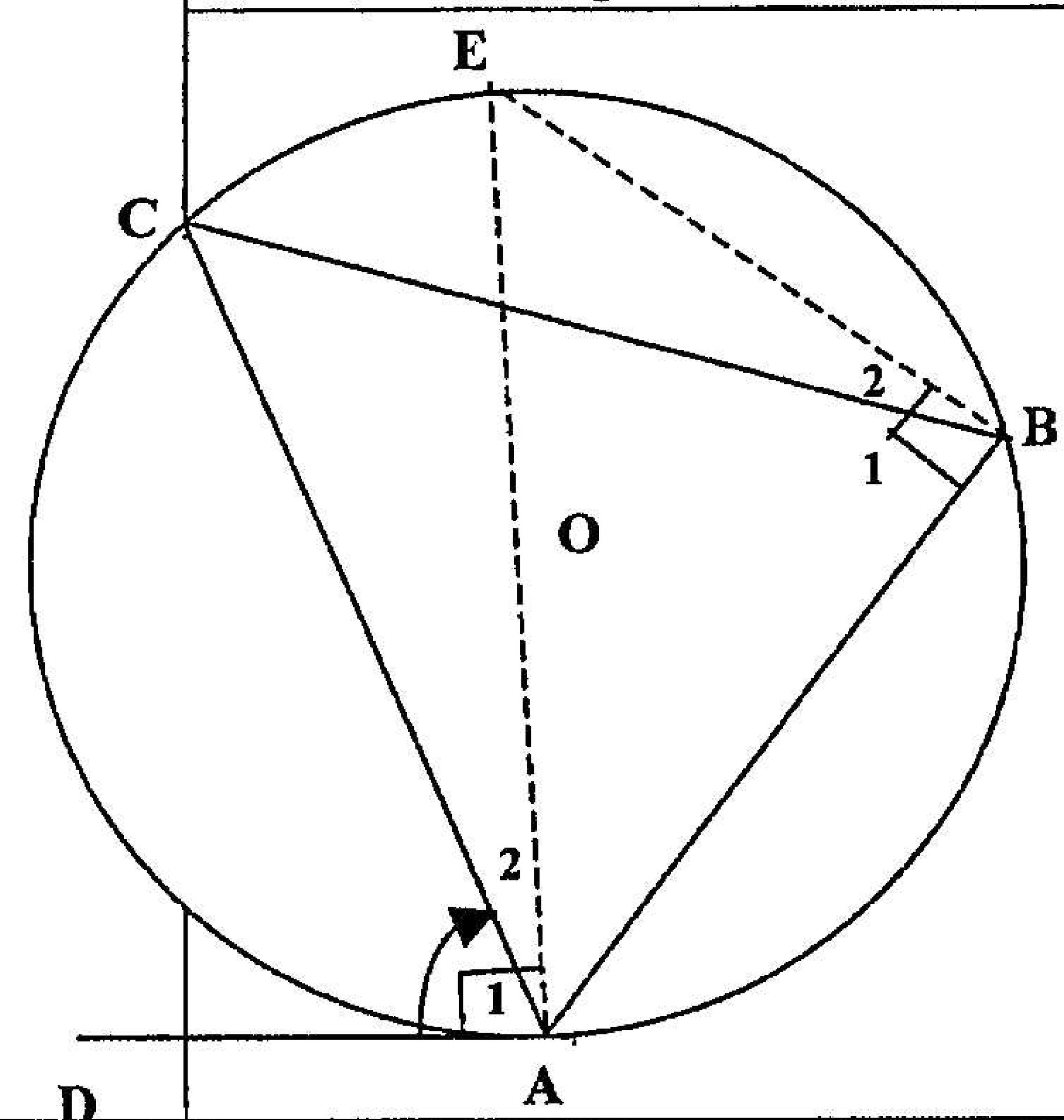
$\hat{A}_1 + \hat{A}_2 = 90°$ ✓ S (radius ⊥ tangent) ✓R

$\hat{B}_1 + \hat{B}_2 = 90°$ (∠ in semi-circle) ✓ S/R

but $\hat{A}_2 = \hat{B}_2$ (∠s in the same segment) ✓ S/R

$\therefore \hat{A}_1 = \hat{B}_1$ ✓ S

$\therefore D\hat{A}C = \hat{B}$



OR

const: Join OA and OC or shown in the diagram ✓M

Let $\hat{B} = x$

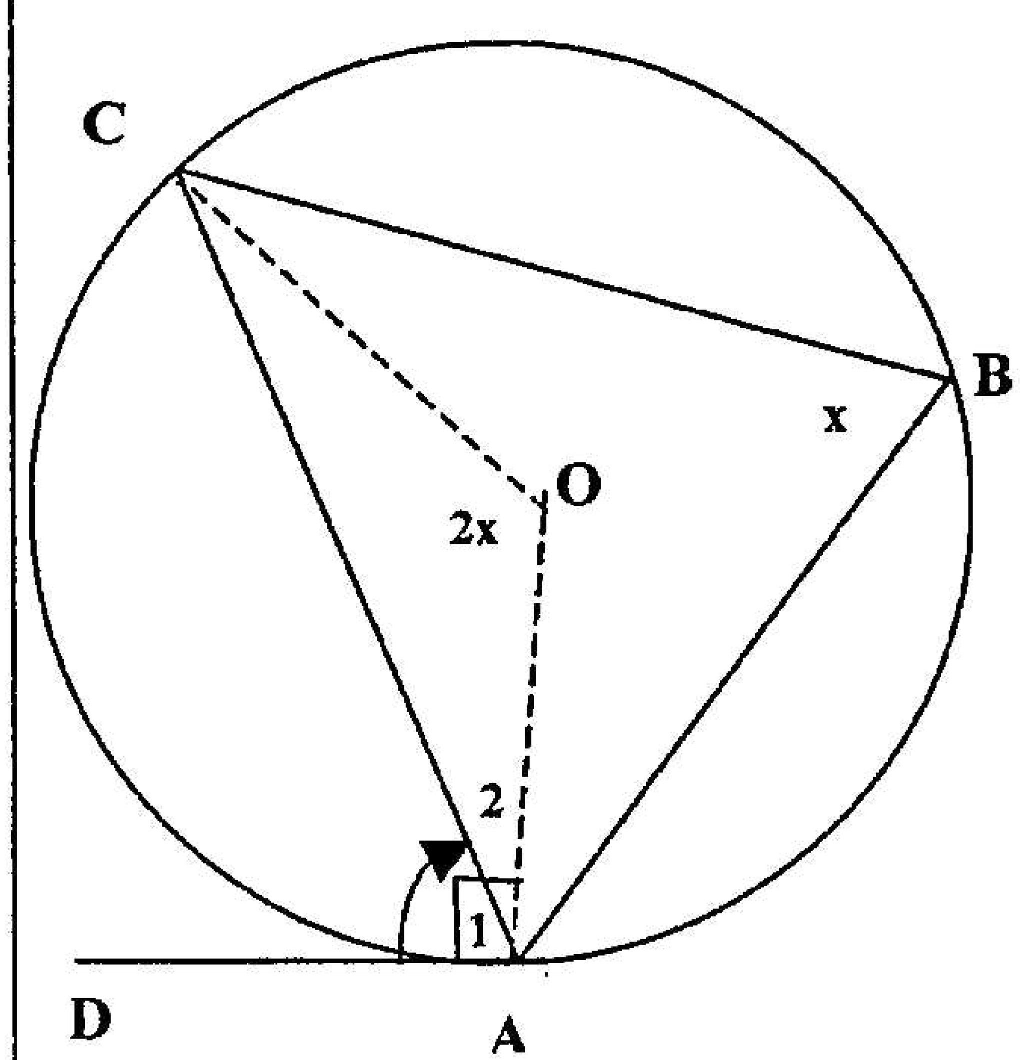
$C\hat{O}A = 2x$ ✓ S/R (∠at the centre = 2 ∠at the circum.)

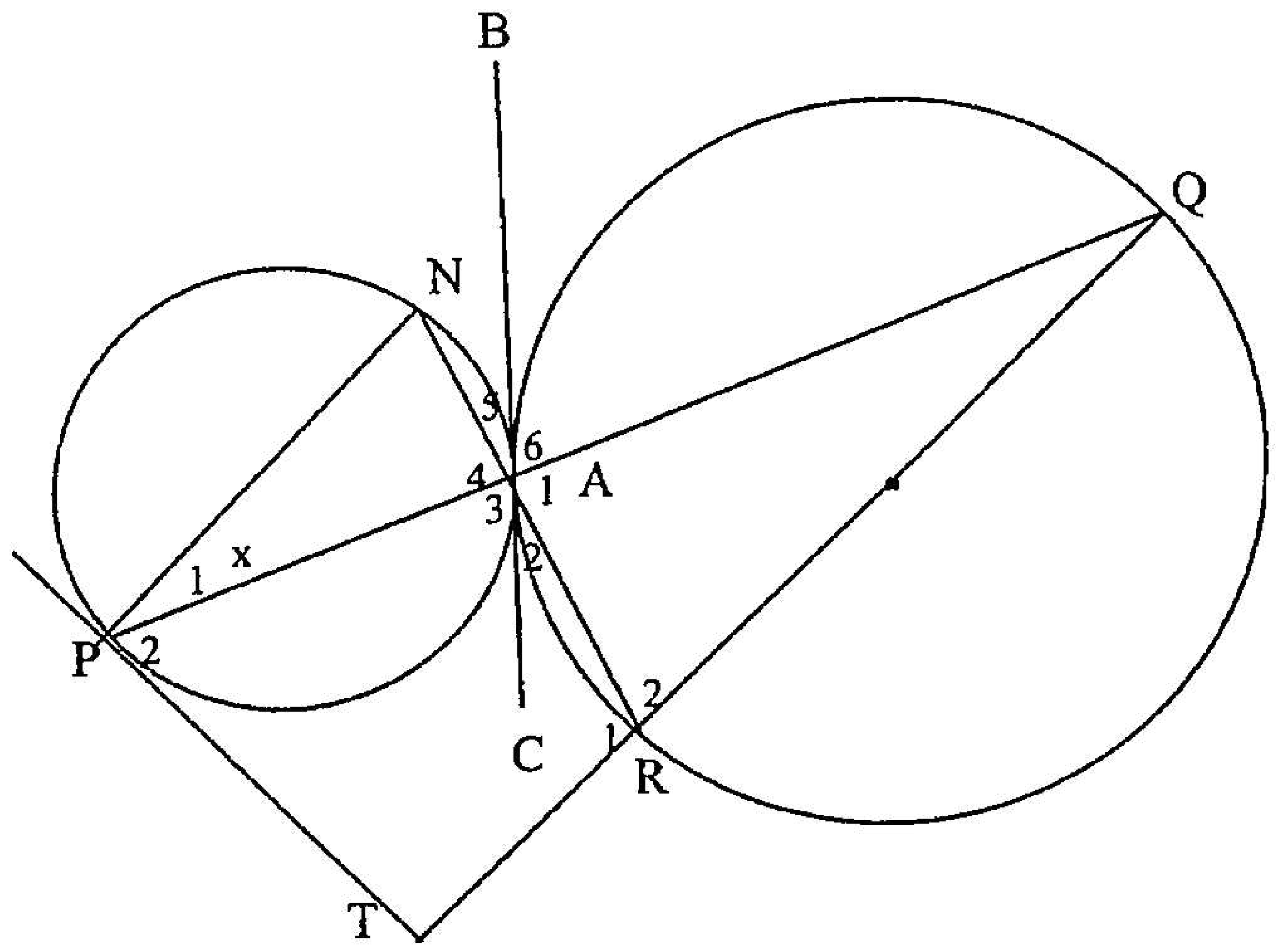
$\hat{A}_2 = 90° - x$ ✓ S/R (∠s in a Δ; OA = OC)

but $\hat{A}_1 + \hat{A}_2 = 90°$ ✓ S (radius ⊥ tangent) ✓ R

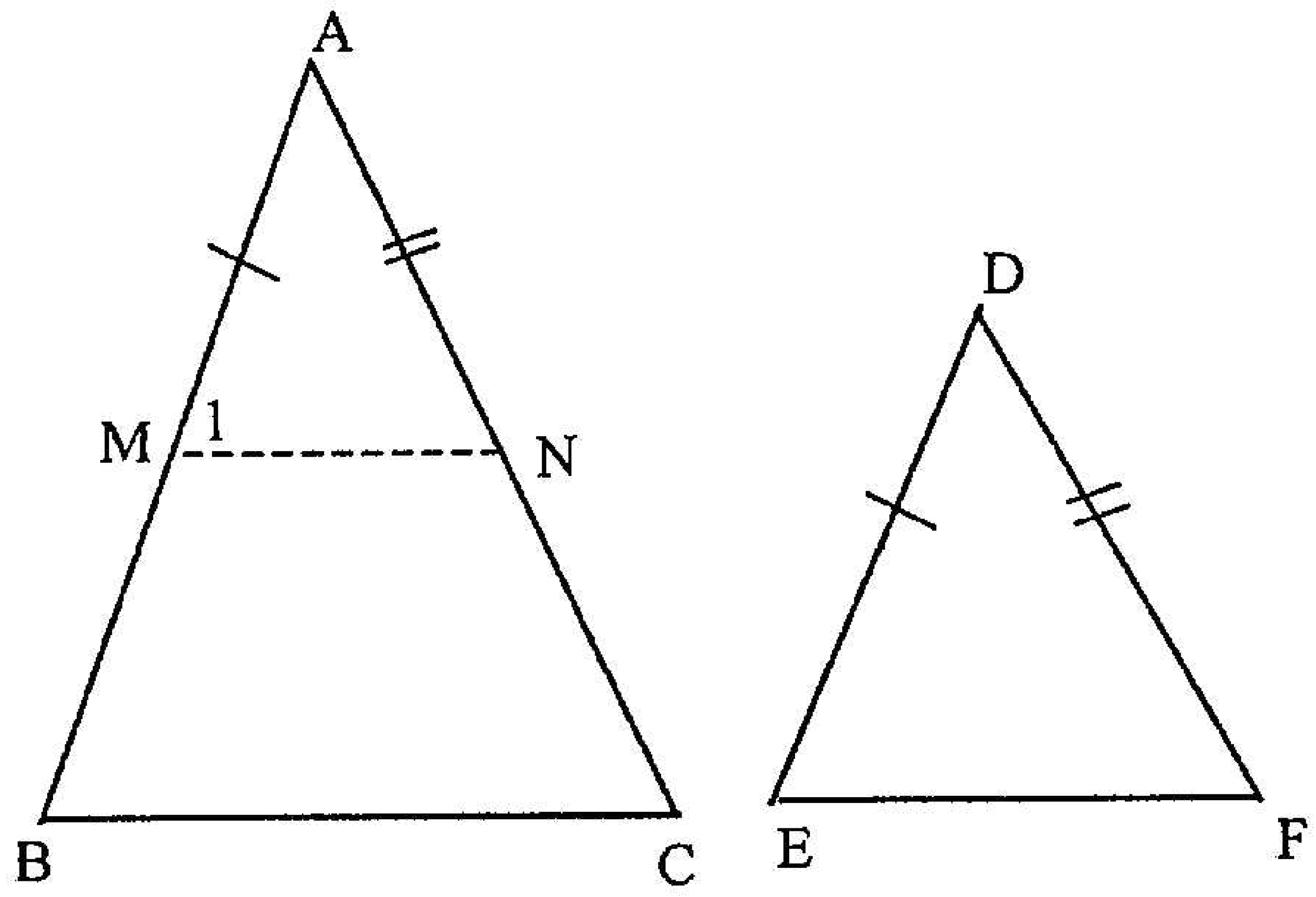
$\therefore \hat{A}_1 = x$ ✓ S

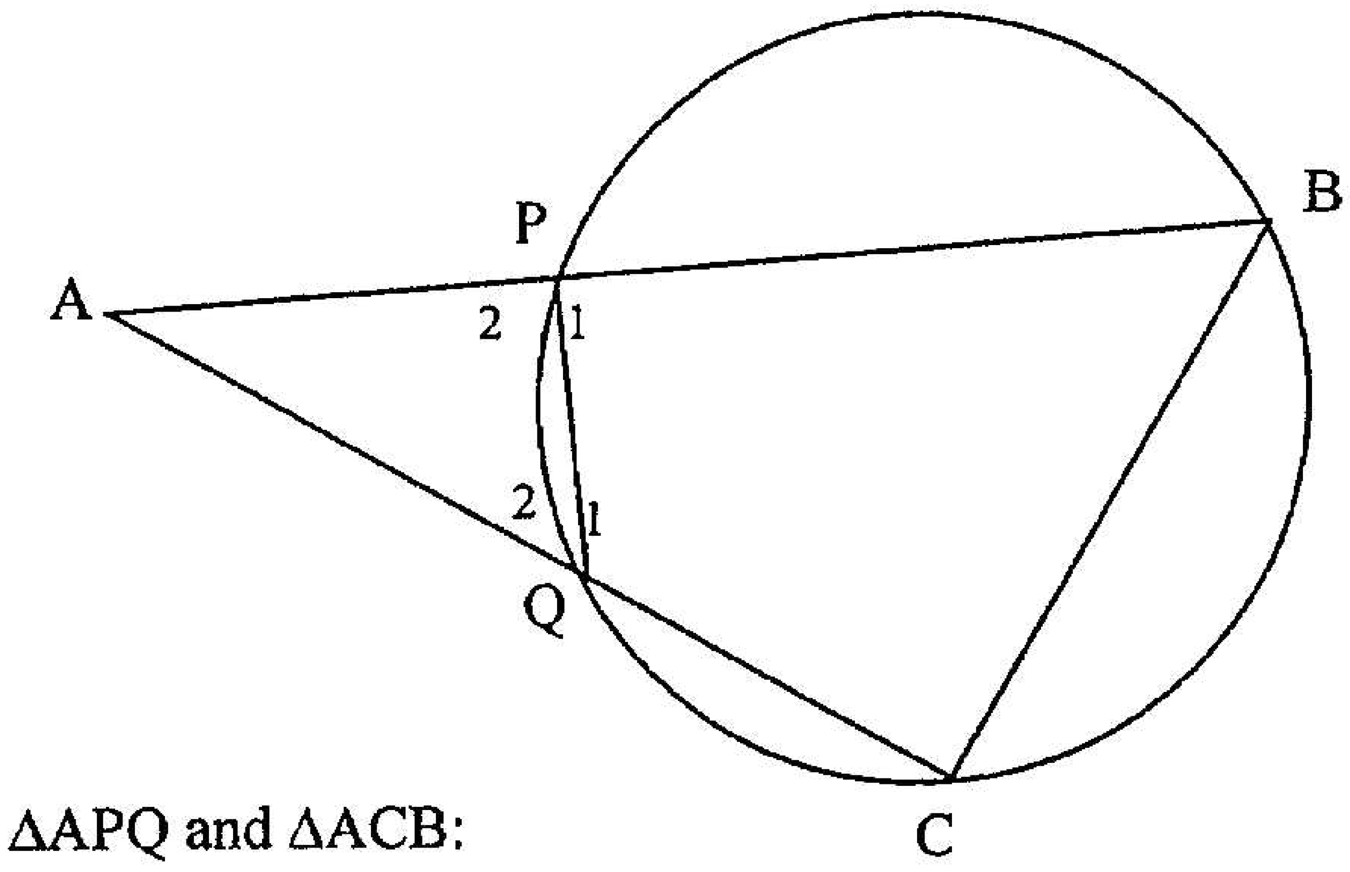
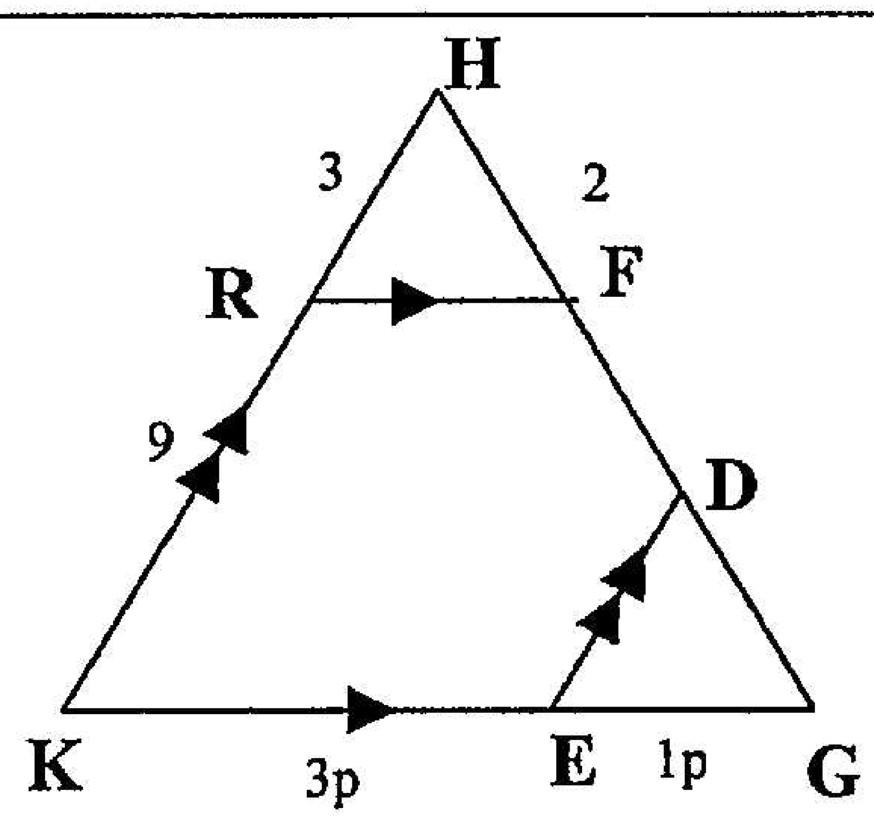
$\therefore D\hat{A}C = \hat{B}$ (6)



8.2.1	 $\hat{P}_1 = \hat{A}_5 = x$ ✓S (tan–chord) ✓R $= \hat{A}_2 = x$ (vertically opp. ∠s) ✓S/R $= \hat{Q} = x$ ✓S (tan–chord) ✓R (5)	
8.2.2	$\hat{P}_1 = \hat{Q} = x$ or alt. ∠s equal ✓R (1)	
8.2.3 (a)	$\hat{A}_1 = 90°$ ✓S (∠ in semi-circle) ✓R $\hat{A}_4 = \hat{A}_1 = 90°$ (vertically opp. ∠s) ✓S/R ∴PN is a diameter of the smaller circle. (subtends 90° at circum.) ✓R	
8.2.3 (b)	$\hat{P}_1 + \hat{P}_2 = 90°$ (diam⊥tan) ✓S/R $\hat{T} + \hat{P}_1 + \hat{P}_2 = 180°$ (//) ✓S/R $\therefore \hat{A}_1 = \hat{T} = 90°$ ✓S ∴APTR is cyclic (ext. ∠ = int. opp. ∠) ✓R **OR**	**OR** $\hat{P}_2 = \hat{N}$ ✓S (tan-chord) ✓R but $\hat{N} = \hat{R}_2$ (alt. ∠s, //) ✓S/R $\therefore \hat{P}_2 = \hat{R}_2$ ✓R APTR is a cyclic quadr. (ext. ∠ = int. opp. ∠)

	OR $\hat{A}_2 + \hat{A}_3 = 90°$ ✓S ($\hat{A}_1 = 90°$ & ∠s in st.line) $\hat{P}_1 + \hat{P}_2 = 90°$ (diam⊥tan) $\hat{T} + \hat{P}_1 + \hat{P}_2 = 180°$ (//) ✓S/R $\hat{T} = 90°$ ✓S $\therefore \hat{A}_2 + \hat{A}_3 + \hat{T} = 180°$ APTR is cyclic (opp. ∠ s supp.) ✓R **OR** $\hat{R}_2 = 90° - x$ ($\hat{A} = 90°$from sum of ∠s of Δ) ✓S/R $\hat{P}_1 + \hat{P}_2 = 90°$ (diam⊥tan) ✓S $\hat{P}_2 = 90° - x$ ✓S $\therefore \hat{P}_2 = \hat{R}_2$ ∴APTR is cyclic (ext. ∠ = int. opp. ∠) ✓R (4)	

QUESTION 9		[21]
9.1	 Proof: Δ AMN ≡ ΔDEF ✓S (S,∠,S) ✓R $\therefore \hat{M}_1 = \hat{E}$ ✓S (congruency) but $\hat{B} = \hat{E}$ (given) and $\hat{M}_1 = \hat{B}$ ✓S ∴ MN ∥ BC (corr. ∠s equal) ✓S/R $\therefore \frac{AB}{AM} = \frac{AC}{AN}$ (line ∥ one side of a Δ) ✓S/R $\therefore \frac{AB}{DE} = \frac{AC}{DF}$ (6)	reason or proved

9.2.1	 In ΔAPQ and ΔACB: $\hat{A} = \hat{A}$ (common) ✓S/R $\hat{P}_2 = \hat{C}$ ✓S (ext. ∠ of cyclic quad.)✓R $\hat{Q}_2 = \hat{B}$ (sum ∠s of Δ) or (ext. ∠ of cyclic quad) ∴ΔAPQ ||| ΔACB (∠,∠,∠) ✓R (4)	Any of the 2 angles ∠∠ or equiangular or third angle
9.2.2	$\frac{AQ}{AB} = \frac{PQ}{BC}$ ✓S (Δ APQ ||| ΔACB) ✓R $\frac{AQ}{AB} = \frac{PQ}{AQ}$ ✓S (AQ = BC) $AQ^2 = AB.PQ$ (3)	if candidates start with ∴, no reason necessary – follows from 9.2.1 substitution if first step is omitted – no marks
9.3.1	In ΔHKG: $\frac{FG}{2} = \frac{9}{3}$ ✓S ✓R (line || one side of a Δ) or (proportionality theorem) or (RF // KG) ∴ FG = 6 units ✓CA answer only – 2 marks (3)	
9.3.2	but $\frac{GD}{GH} = \frac{EG}{GK} = \frac{1}{4}$ ✓S ✓R (line || one side of a Δ) or propotional theorem or ED// KH ∴ GD = $\frac{1}{4}$.GH $= \frac{1}{4}(8)$ ✓CA = 2 ✓CA ∴ FD = 6 – 2 = 4 units ✓CA (5)	**OR** In ΔHKG, HK || DE $\frac{GD}{DH} = \frac{GE}{EK} = \frac{1}{3}$ ✓S ✓R (line || one side of a Δ) propotional theorem or ED// KH ✓CA $\frac{6-FD}{2+FD} = \frac{1}{3}$ 18 – 3FD = 2 + FD ✓CA ∴FD = 4 units ✓CA