DEPARTMENT OF EDUCATION
REPUBLIC OF SOUTH AFRICA

DEPARTEMENT VAN ONDERWYS
REPUBLIEK VAN SUID-AFRIKA

SENIOR CERTIFICATE EXAMINATION - 2004
SENIORSERTIFIKAAT-EKSAMEN - 2004

MATHEMATICS P2 : GEOMETRY
WISKUNDE V2 : MEETKUNDE

STANDARD GRADE
STANDAARDGRAAD

OCTOBER/NOVEMBER 2004
OKTOBER/NOVEMBER 2004

301-2/2

Marks: 150
Punte : 150

3 Hours
3 Ure

This question paper consists of 11 pages, 1 formula sheet and 5 diagram sheets.
Hierdie vraestel bestaan uit 11 bladsye, 1 formuleblad en 5 diagramvelle.

MATHEMATICS SG: Paper 2
Geometry

301 2 2   SG

INSTRUKSIES

1. Hierdie vraestel bestaan uit **9** vrae, 'n formuleblad en diagramvelle.

2. Gebruik die formuleblad om hierdie vraestel te beantwoord.

3. Maak die diagramvelle los van die vraestel en plaas dit in jou **ANTWOORDEBOEK**.

4. Die diagramme is nie volgens skaal geteken nie.

5. Beantwoord **AL** die vrae.

6. Nommer **AL** die antwoorde korrek en duidelik.

7. **AL** die nodige bewerkings moet getoon word.

8. Nie-programmeerbare sakrekenaars mag gebruik word, tensy anders vermeld.

9. Waar nodig, sal die aantal desimale syfers waartoe antwoorde afgerond moet word in die vraag gemeld word.

INSTRUCTIONS

1. This question paper consists of **9** questions, a formula sheet and diagram sheets.

2. Use the formula sheet to answer this question paper.

3. Detach the diagram sheets from the question paper and place them inside your **ANSWER BOOK**.

4. The diagrams are not drawn to scale.

5. Answer **ALL** the questions.

6. Number **ALL** the answers correctly and clearly.

7. **ALL** the necessary calculations must be shown.

8. Non-programmable calculators may be used, unless otherwise stated.

9. The number of decimal digits to which answers must be rounded off will be stated in the question where necessary.

ANALITIESE MEETKUNDE

| **LET WEL:** – GEBRUIK ANALITIESE METODES IN HIERDIE AFDELING. |
| --- |
| – KONSTRUKSIE- EN METINGMETODES MAG NIE GEBRUIK WORD NIE. |

VRAAG 1

In die diagram langsaan is

$D(-2; 4)$, $E(2; -2)$ en $F(-1; k)$

drie punte in 'n Cartesiese vlak.

$\hat{E} = 90°$

$D\hat{A}X = \theta$
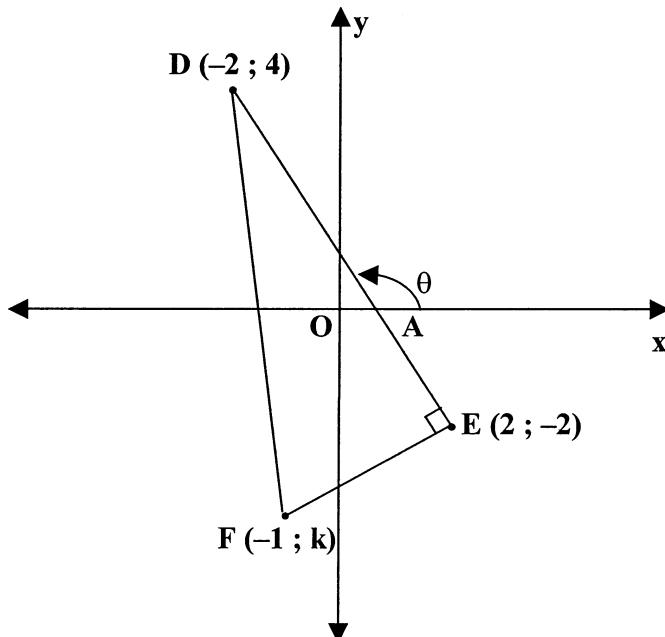
1.1	Bereken die gradiënt van DE.	(2)
1.2	Bereken die grootte van θ, afgerond tot TWEE desimale syfers.	(3)
1.3	Toon dat $k = -4$.	(3)
1.4	Bereken vervolgens die koördinate van M, die middelpunt van FE.	(2)
1.5	Bepaal die vergelyking van die reguitlyn ewewydig aan DE wat deur punt M gaan.	(4)
1.6	Bereken die area van $\triangle$ DEF.	(7)

[21]

ANALYTICAL GEOMETRY

NOTE:	– USE ANALYTICAL METHODS IN THIS SECTION.
	– CONSTRUCTION AND MEASUREMENT METHODS MAY NOT BE USED.

QUESTION 1

In the diagram alongside,

$D(-2; 4)$, $E(2; -2)$ and $F(-1; k)$

are three points in a Cartesian plane.

$\hat{E} = 90°$

$D\hat{A}X = \theta$
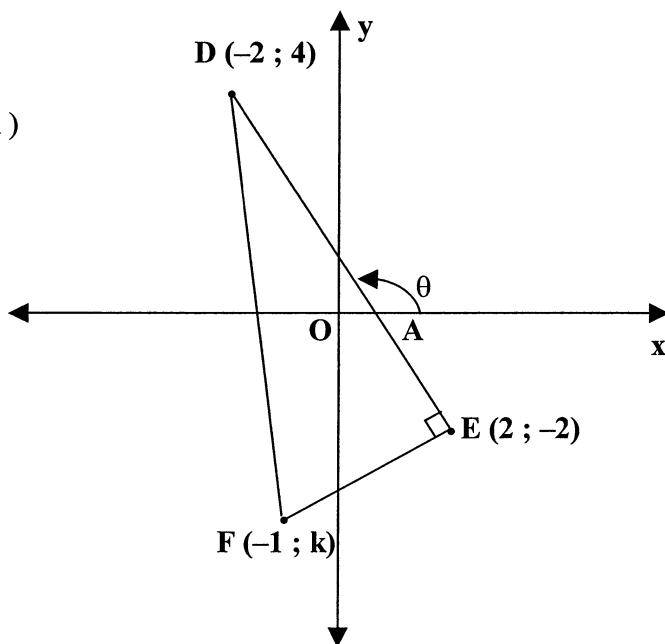
1.1   Calculate the gradient of DE.   (2)

1.2   Calculate the size of θ, rounded off to TWO decimal digits.   (3)

1.3   Show that $k = -4$.   (3)

1.4   Hence calculate the coordinates of M, the midpoint of FE.   (2)

1.5   Determine the equation of the straight line parallel to DE which passes through M.   (4)

1.6   Calculate the area of $\triangle$ DEF.   (7)

                                                                              [21]

VRAAG 2

2.1   Die sirkel met die middelpunt by die oorsprong,

        gaan deur A (–3 ; 4)

        en sny die x-as by C.

        BD is 'n raaklyn aan die sirkel by A.

        B is 'n punt op die x-as.

        CD is ewewydig aan die y-as.
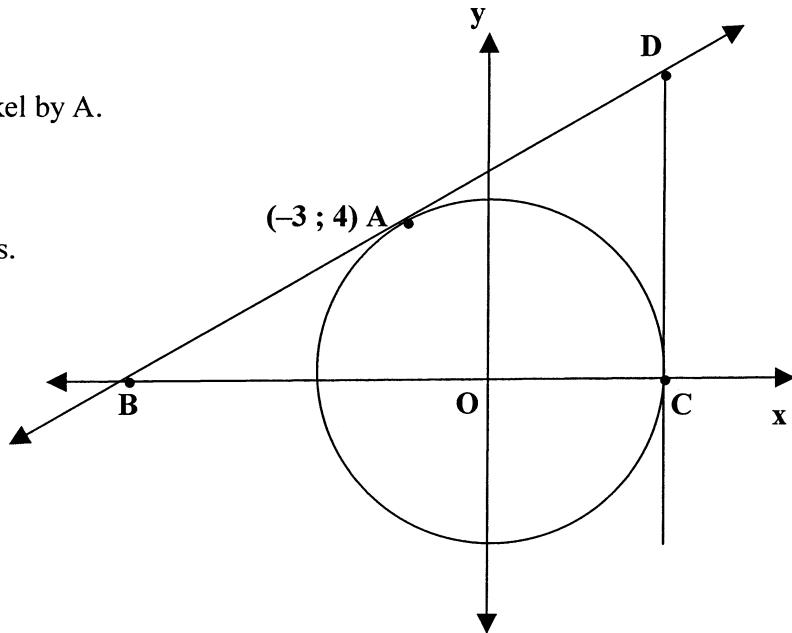
2.1.1   Bepaal die vergelyking van die sirkel.   (3)

2.1.2   Skryf die koördinate van C.   (2)

2.1.3   Bereken die gradiënt van BD.   (3)

2.1.4   Bepaal vervolgens die vergelyking van BD.   (3)

2.1.5   Bepaal die lengte van CD.   (2)

2.2   R (1 ; – 4), Q (– 1 ; 1) en S (1 ; 4) is drie punte in 'n Cartesiese vlak.

        Bepaal die vergelyking van die lokus van P (x ; y) sodat die gradiënt van PS gelyk
   is aan twee keer die gradiënt van RQ.   (5)

                                                                                [18]

QUESTION 2

2.1   The circle with the centre at the origin,

passes through A (–3 ; 4)

and cuts the *x*-axis at C.

BD is a tangent to the circle at A.

B is a point on the *x*-axis.

CD is parallel to the *y*-axis.
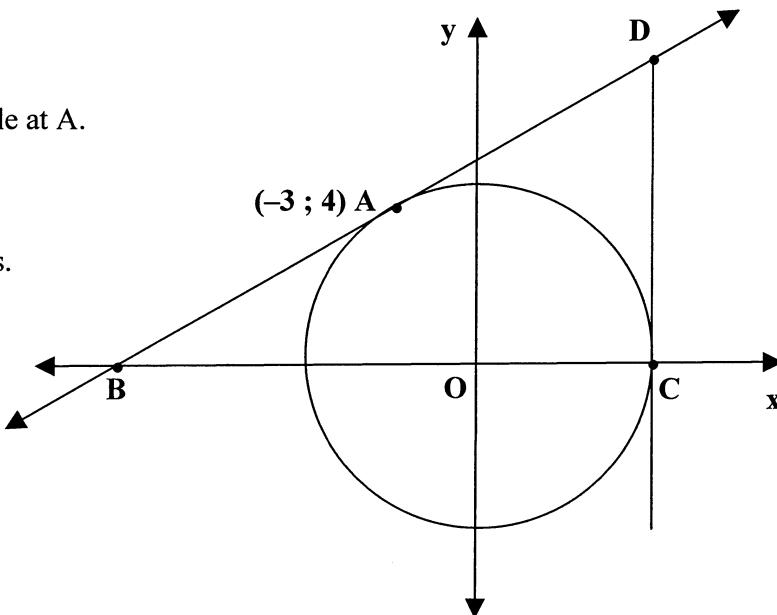
2.1.1   Determine the equation of the circle.   (3)

2.1.2   Write the coordinates of C.   (2)

2.1.3   Calculate the gradient of BD.   (3)

2.1.4   Hence, determine the equation of BD.   (3)

2.1.5   Determine the length of CD.   (2)

2.2   R (1 ; – 4), Q (– 1 ; 1) and S (1 ; 4) are three points in a Cartesian plane.

Determine the equation of the locus of P (x ; y) such that the gradient of PS
is equal to twice the gradient of RQ.   (5)

                                                                            [18]

TRIGONOMETRIE

VRAAG 3

Beantwoord hierdie vraag sonder die gebruik van 'n sakrekenaar.

3.1   As $4\cos\theta + 3 = 0$ en $0° < \theta < 180°$, **gebruik 'n diagram** om die waarde
te bepaal van $12\sin\theta . \tan\theta$ (6)

3.2   Vereenvoudig:

3.2.1 $\dfrac{\cot(90° - \alpha) . \sin(180° + \alpha)}{\tan(360° - \alpha)}$ (4)

3.2.2 $\sec 120° (\text{cosec } 315° + \cot 210°)$ (6)
                                                                                                     [16]

VRAAG 4

Gegee: $f(x) = \sin 2x$ en $g(x) = \cos x$

4.1   Gebruik die gegewe assestelsel op die diagramvel om die krommes van f en g te
skets as $x \in [0°;180°]$. Toon duidelik alle afsnitte met die asse en al die draaipunte.   (8)

4.2   Gebruik die grafieke in VRAAG 4.1 om die volgende vrae te beantwoord as
$x \in [0° ; 180°]$:

4.2.1   Skryf die waardeversameling(terrein) van g.   (2)

4.2.2   Bepaal die waarde van $f(180°) - g(180°)$.   (1)

4.2.3   Vir watter waardes van x sal beide $f(x)$ en $g(x)$ negatief wees?   (3)

4.3   Skryf die periode van die kromme van h as $h(x) = 3\tan x$.   (1)
                                                                                                     [15]

TRIGONOMETRY

QUESTION 3

Answer this question without the use of a calculator.

3.1   If $4 \cos \theta + 3 = 0$ and $0° < \theta < 180°$, **use a diagram** to determine the value of
        $12 \sin \theta \cdot \tan \theta$ (6)

3.2   Simplify:

        3.2.1 $\dfrac{\cot (90° - \alpha) \cdot \sin (180° + \alpha)}{\tan (360° - \alpha)}$ (4)

        3.2.2 $\sec 120° \, (\operatorname{cosec} 315° + \cot 210°)$ (6)
                                                                                                              [16]

QUESTION 4

Given: $f(x) = \sin 2x$ and $g(x) = \cos x$

4.1   Use the set of axes given on the diagram sheet to sketch the curves of f and g
        if $x \in [0°; 180°]$. Show clearly all intercepts with the axes and all the turning points.   (8)

4.2   Use the graphs in QUESTION 4.1 to answer the following questions if
        $x \in [0° ; 180°]$:

        4.2.1   Write the range of g.   (2)

        4.2.2   Determine the value of $f(180°) - g(180°)$.   (1)

        4.2.3   For which values of x will both $f(x)$ and $g(x)$ be negative?   (3)

4.3   Write the period of the curve of h if $h(x) = 3 \tan x$.   (1)
                                                                                                              [15]

VRAAG 5

5.1   Gebruik fundamentele trigonometriese identiteite en nie 'n diagram nie om die volgende identiteit te bewys:

$$\frac{\sin\theta \, . \, \tan\theta + \cos\theta}{\cos\theta} = \sec^2\theta$$ (5)

5.2   Gegee: $\sin\theta = \tan 323°$ en $\theta \in [\, 0° \, ; \, 270°]$

Bereken die waarde van die volgende, afgerond tot TWEE desimale syfers.

5.2.1 θ (3)

5.2.2 $\sec(\theta + 10°)$ (2)

[10]

VRAAG 6

6.1   Gebruik die diagram op die diagramvel of teken die diagram oor in jou antwoordeboek om te bewys dat

$$c^2 = a^2 + b^2 - 2ab.\cos C$$

(6)
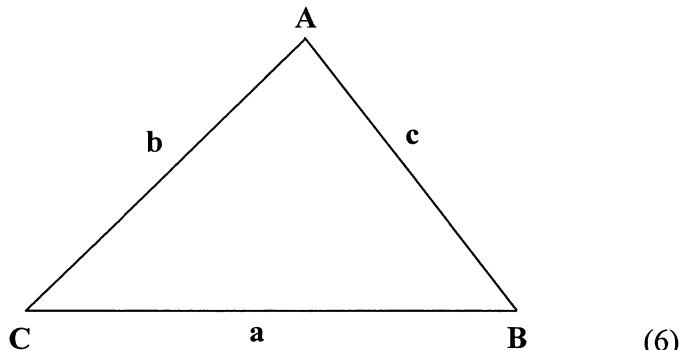
6.2   Mnr. Miti het deur die regering se grondherverdelingsprogram 'n stuk grond wat in die vorm van 'n driehoek PQR is, ontvang.

PQ = 0,7 km

QR = 1 km

RP = 0,5 km

Bereken, afgerond tot TWEE desimale syfers:

6.2.1   Die grootte van $\hat{P}$ (5)

6.2.2   Die area van $\triangle$ PQR (3)

QUESTION 5

5.1   Use fundamental trigonometric identities and not a diagram to prove the following
        identity:

$$\frac{\sin \theta \,.\, \tan \theta + \cos \theta}{\cos \theta} = \sec^2 \theta$$ (5)

5.2   Given: $\sin \theta = \tan 323°$ and $\theta \in [\, 0° \;;\; 270°]$

        Calculate the value of the following, rounded off to TWO decimal digits.

        5.2.1 θ (3)

        5.2.2 $\sec (\, \theta + 10°\,)$ (2)
                                                                                                          [10]

QUESTION 6

6.1   Use the diagram on the diagram

        sheet or redraw the diagram in

        your answer book to prove that

        $c^2 = a^2 + b^2 - 2\, ab.\cos C$

                                                                                                            (6)
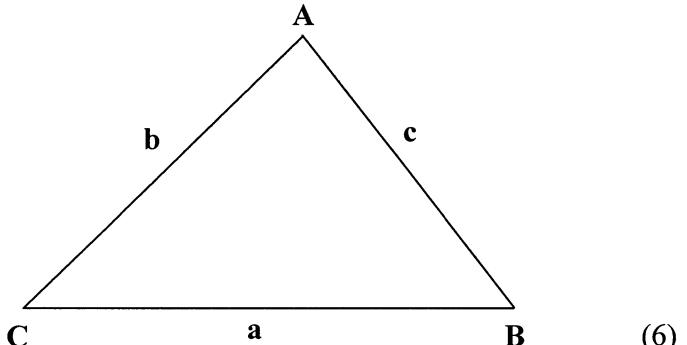
6.2   Mr Miti received, through the government's land distribution programme, a piece of
        land which is in the form of a triangle PQR.

        PQ = 0,7 km

        QR = 1 km

        RP = 0,5 km

        Calculate, rounded off to TWO decimal digits:

        6.2.1   The size of $\hat{P}$ (5)

        6.2.2   The area of $\triangle$ PQR   (3)

6.3   In die diagram langsaan, is BC die middellyn van sirkel BCD.

$\overset{\wedge}{BCD} = \theta$

$\overset{\wedge}{ACB} = \alpha$

AB = BC

BD = p eenhede
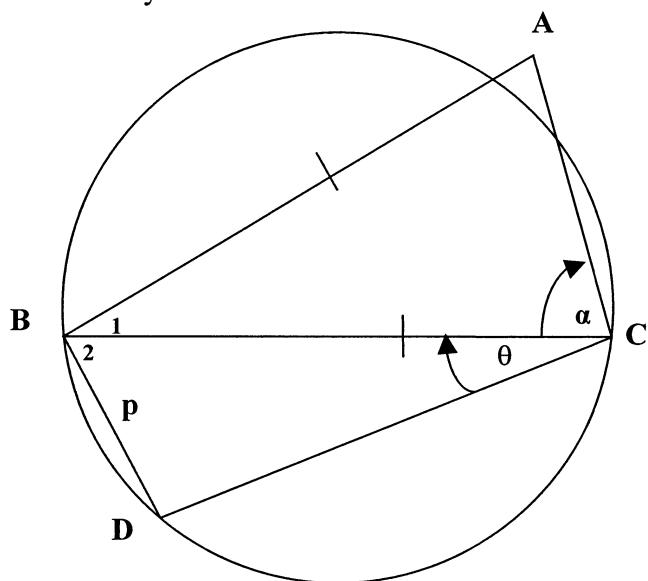
6.3.1   Skryf, sonder verstrek van redes, die grootte van $\hat{D}$.   (1)

6.3.2   Druk BC uit in terme van p en θ.   (2)

6.3.3   Bepaal, sonder verstrek van redes, die grootte van $\overset{\wedge}{B_1}$ in terme van α.   (1)

6.3.4   Bewys vervolgens dat $AC = \dfrac{p \cdot \sin 2\alpha}{\sin\theta \cdot \sin\alpha}$ (4)

[22]

6.3   In the diagram alongside, BC is the diameter of circle BCD.

$\overset{\wedge}{BCD} = \theta$

$\overset{\wedge}{ACB} = \alpha$

AB = BC

BD = p units
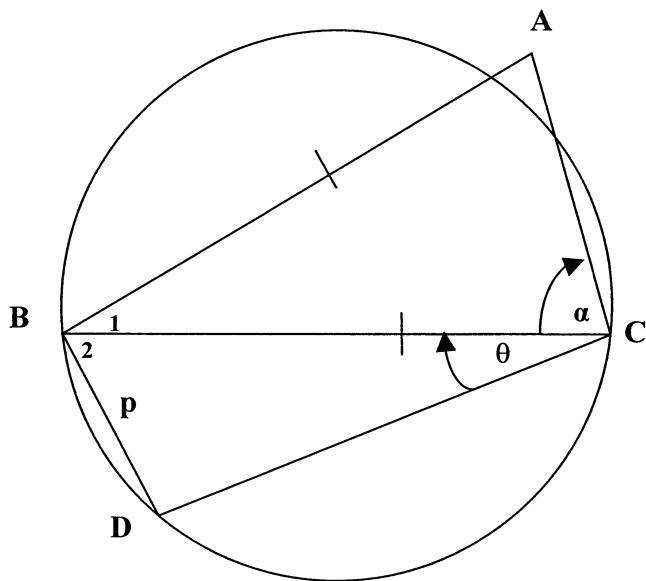
6.3.1   Write, without stating reasons, the size of $\hat{D}$.   (1)

6.3.2   Express BC in terms of p and θ.   (2)

6.3.3   Determine, without stating reasons, the size of $\overset{\wedge}{B_1}$ in terms of α.   (1)

6.3.4   Hence, prove that $AC = \dfrac{p \cdot \sin 2\alpha}{\sin\theta \cdot \sin\alpha}$ (4)

                                                                                                          [22]

EUKLIDIESE MEETKUNDE

LET WEL :   – **DIAGRAMME VIR DIE BEWYS VAN TEORIE MAG OP DIE DIAGRAM-VELLE GEBRUIK WORD OF IN JOU ANTWOORDEBOEK OORGETEKEN WORD.**
           – **MAAK DIE DIAGRAMVELLE VAN DIE VRAESTEL LOS EN PLAAS DIT IN JOU ANTWOORDEBOEK.**
           – **GEE 'n REDE VIR ELKE BEWERING.**

VRAAG 7

In die diagram langsaan is

MON die middellyn van die sirkel

SMEN met middelpunt O.

$\hat{M}_2 = 55°$

Bepaal met verstrek van redes, die grootte van die volgende:
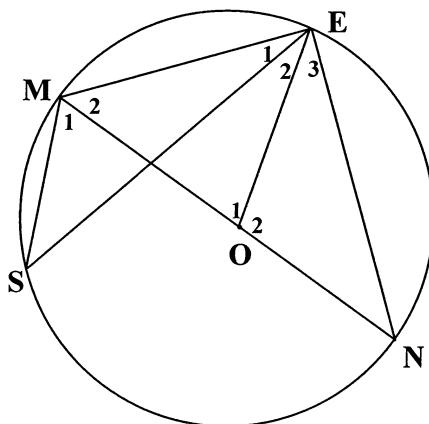
7.1 $\hat{O}_2$ (2)

7.2 $\hat{S}$ (5)

                                                                     [7]

VRAAG 8

8.1   In die diagram langsaan is C, B en A punte

          op die sirkel met middelpunt O. DA is 'n

          raaklyn aan die sirkel by A. Gebruik die

          diagram op diagramvel of teken die diagram

          oor in jou antwoordeboek en bewys die stelling

          wat beweer dat
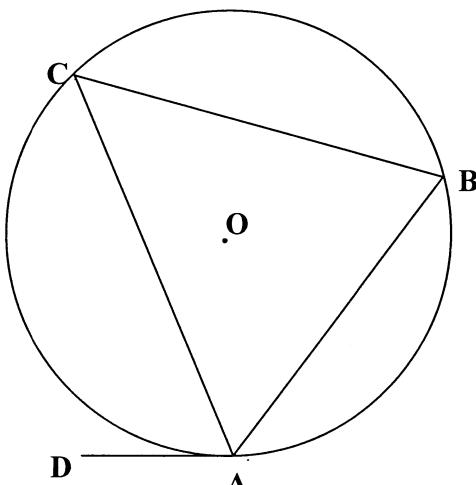
          $D\hat{A}C = \hat{B}$ (6)

EUCLIDEAN GEOMETRY

NOTE :	– DIAGRAMS FOR PROVING THEORY MAY BE USED ON THE DIAGRAM SHEETS, OR REDRAWN IN YOUR ANSWER BOOK.
	– DETACH THE DIAGRAM SHEETS FROM THE QUESTION PAPER AND PLACE THEM IN YOUR ANSWER BOOK.
	– GIVE A REASON FOR EACH STATEMENT.

QUESTION 7

In the diagram alongside,

MON is a diameter of the circle

SMEN with centre O.

$\hat{M}_2 = 55°$

Determine, stating reasons, the size of the following:
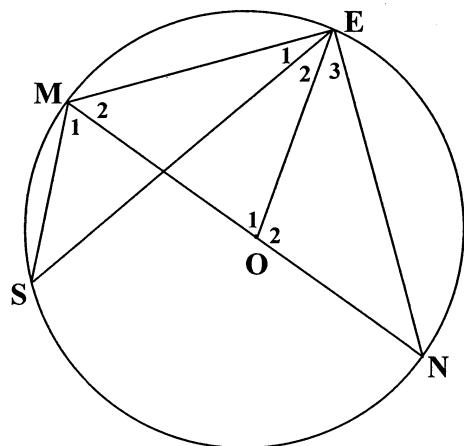
7.1 $\hat{O}_2$ (2)

7.2 $\hat{S}$ (5)

[7]

QUESTION 8

8.1   In the diagram alongside C, B and A are points on the circle with centre O. DA is a tangent to the circle at A. Use the diagram on the diagram sheet or redraw the diagram in your answer book to prove the theorem which states that

$D\hat{A}C = \hat{B}$ (6)
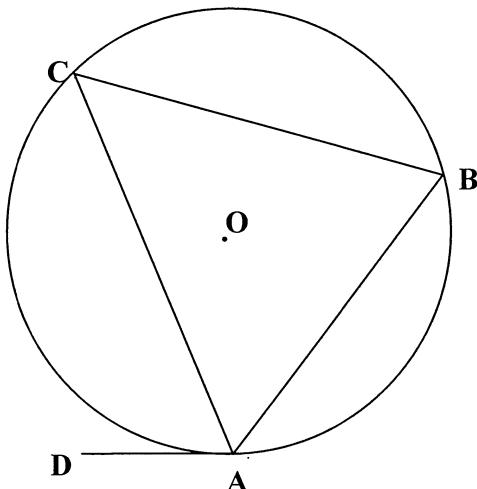
8.2   In die onderstaande diagram raak twee sirkels mekaar uitwendig by A.

        BAC is 'n gemene raaklyn aan albei sirkels by A.

        PAQ is 'n dubbele koord.

        QR is 'n middellyn van die groter sirkel.

        PT is 'n raaklyn aan die kleiner sirkel.

        QRT en NAR is reguitlyne.
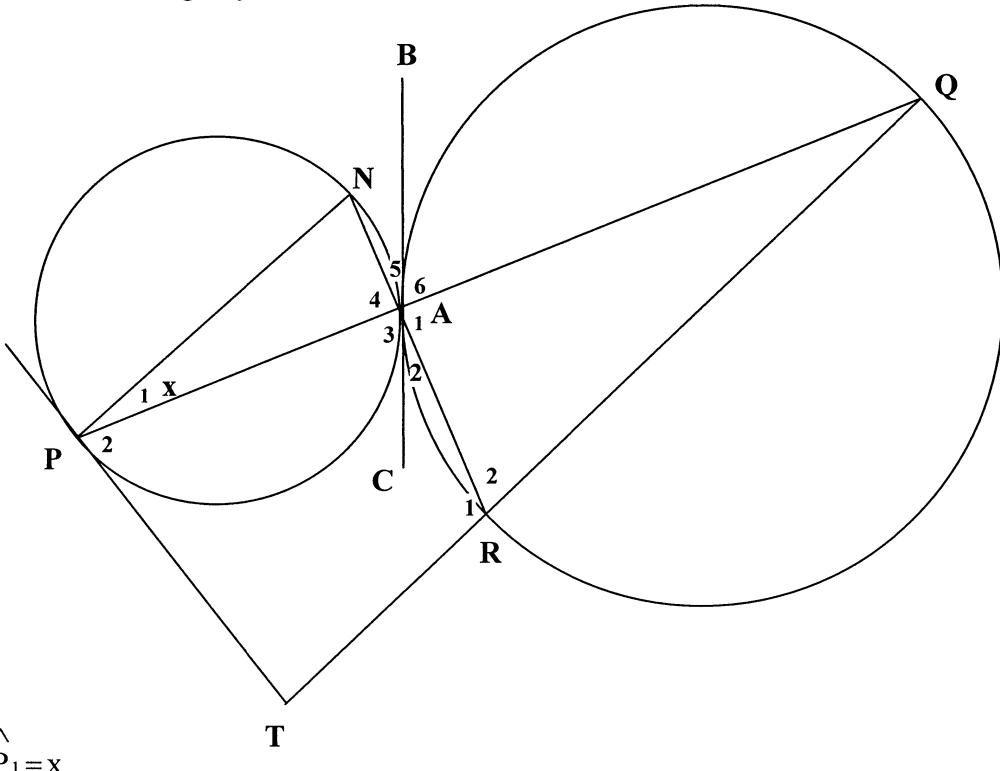
Laat $\hat{P}_1 = x$

8.2.1   Noem, met opgaaf van redes, DRIE ander hoeke elk gelyk aan x.   (5)

8.2.2   Waarom is PN ‖ RQ?   (1)

8.2.3   Bewys dat:

            (a)   PN is 'n middellyn van die kleiner sirkel   (4)

            (b)   APTR is 'n koordevierhoek   (4)

                                                                                              [20]

8.2   In the diagram below two circles touch each other externally at A.

BAC is a common tangent to both circles at A.

PAQ is a double chord.

QR is a diameter of the larger circle.

PT is a tangent to the smaller circle.

QRT and NAR are straight lines.
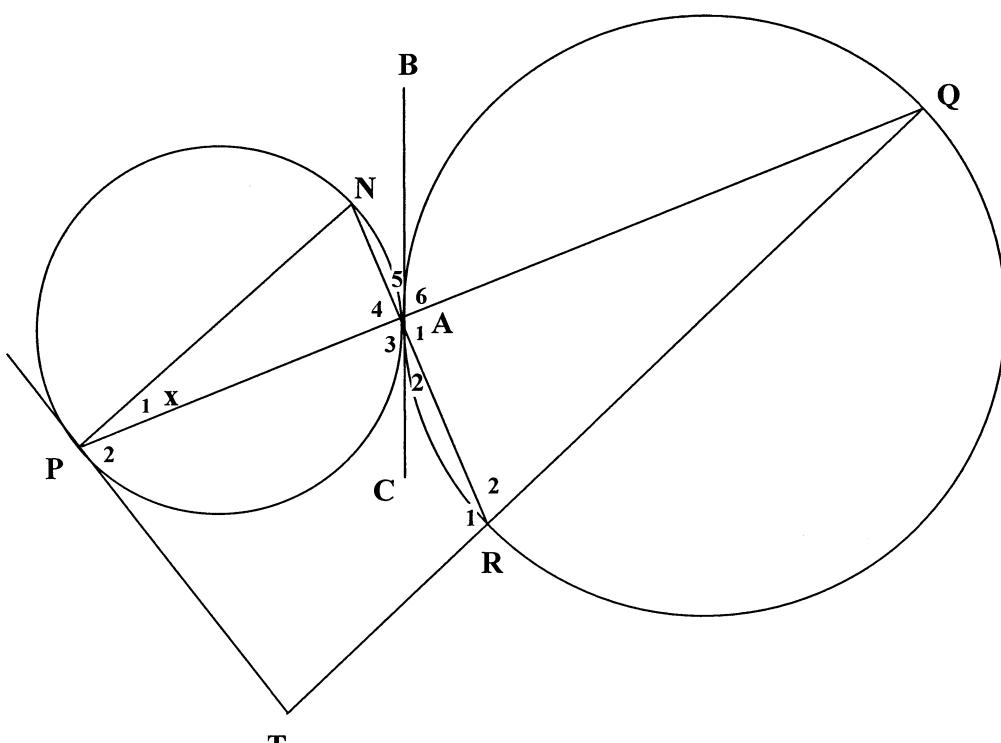
Let $\hat{P}_1 = x$

8.2.1   Name, stating reasons, THREE other angles each equal to x.   (5)

8.2.2   Why is PN || RQ?   (1)

8.2.3   Prove that:

(a) PN is a diameter of the smaller circle   (4)

(b) APTR is a cyclic quadrilateral   (4)

[20]

VRAAG 9

9.1   In die onderstaande diagram is $\triangle$ ABC en $\triangle$ DEF gelykhoekig,

met $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ en $\hat{C} = \hat{F}$.

M is 'n punt op AB sodat   AM = DE en N is 'n punt op AC sodat AN = DF.

Gebruik die diagram op die diagramvel of teken die diagram oor in jou
antwoordeboek om die stelling te bewys wat beweer dat :

$$\frac{AB}{DE} = \frac{AC}{DF}$$

(6)
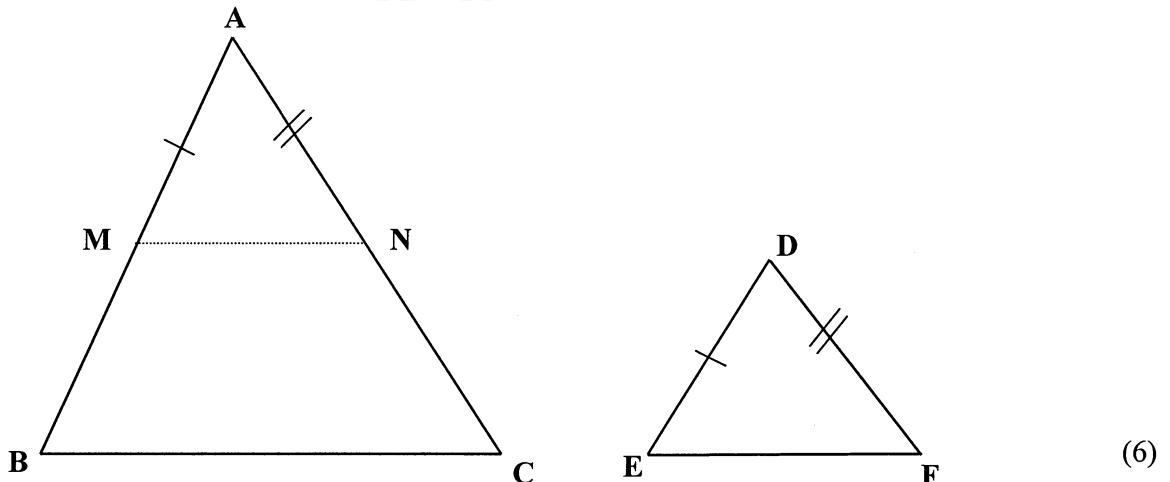
QUESTION 9

9.1   In the diagram below $\triangle$ ABC and $\triangle$ DEF are equiangular

with $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.

M is a point on AB such that AM = DE and N is a point on AC such that AN = DF.

Use the diagram on the diagram sheet or redraw the diagram in your answer book, to prove the theorem which states that:

$$\frac{AB}{DE} = \frac{AC}{DF}$$

(6)
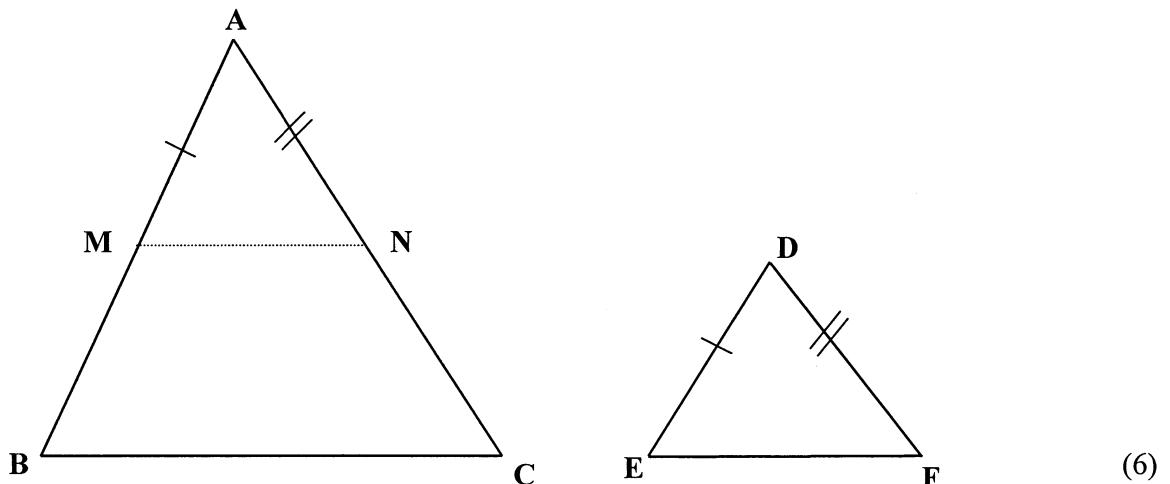
9.2 In die onderstaande diagram is PQCB 'n koordevierhoek.

Koorde BP en CQ is verleng om

by punt A te ontmoet sodat AQ = BC.
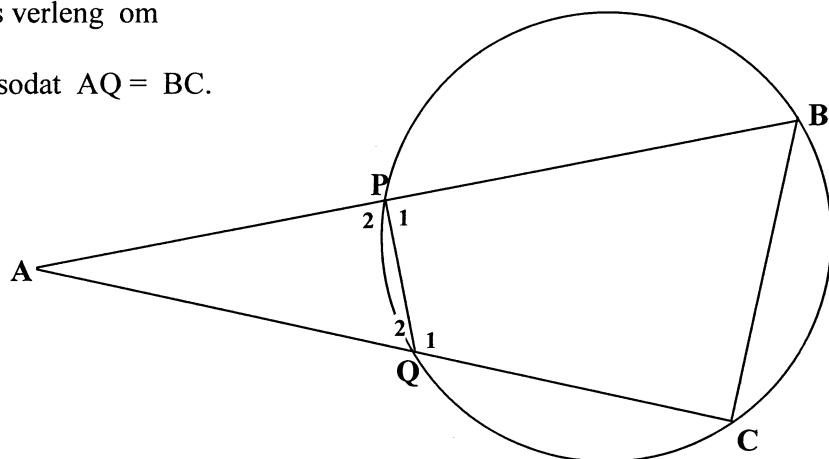
9.2.1 Bewys dat $\triangle$ APQ ||| $\triangle$ ACB (4)

9.2.2 Bewys vervolgens dat AQ^2 = AB.PQ (3)

9.3 In die diagram langsaan is

RF || KG

ED || KH

RH = 3 eenhede

RK = 9 eenhede

HF = 2 eenhede.

GE : EK = 1 : 3
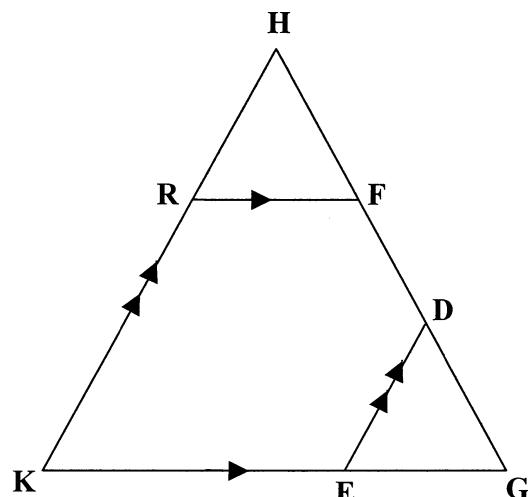
Bereken (met verstrek van redes) die lengtes van:

9.3.1 FG (3)

9.3.2 FD (5)

[21]

TOTAAL: 150

9.2   In the diagram below, PQCB is a cyclic quadrilateral.

Chords BP and CQ are produced to meet

at A such that AQ = BC.
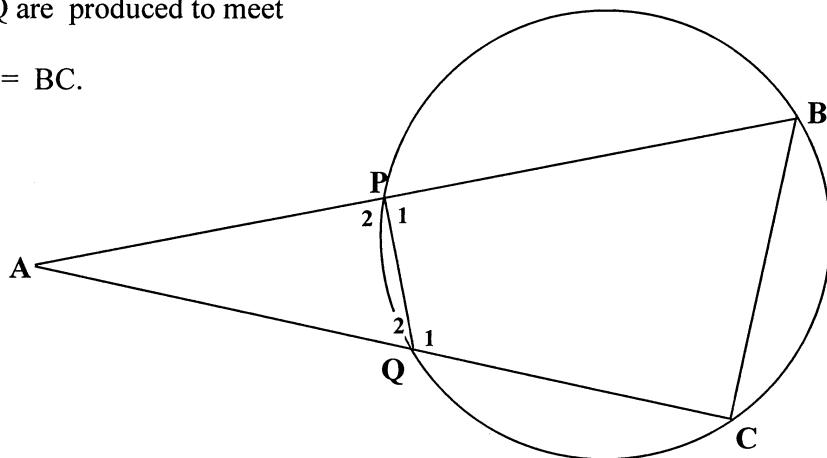
9.2.1   Prove that $\triangle$ APQ ||| $\triangle$ ACB   (4)

9.2.2   Hence, prove that $AQ^2 = AB.PQ$ (3)

9.3   In the diagram alongside,

RF || KG

ED || KH

RH = 3 units

RK = 9 units

HF = 2 units.

GE : EK = 1 : 3
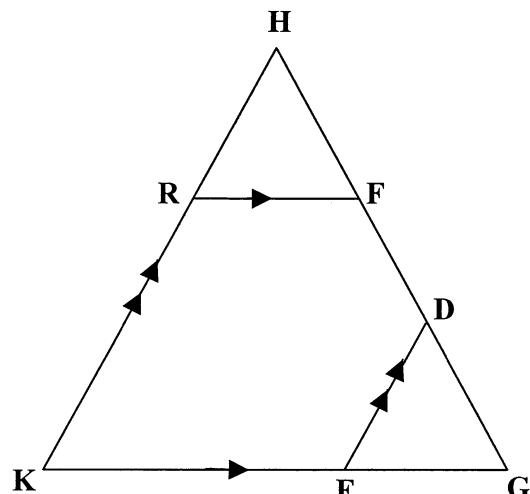
Calculate (stating reasons) the lengths of:

9.3.1   FG   (3)

9.3.2   FD   (5)

                                                                                      [21]

                                                              TOTAL: 150

Mathematics Formula Sheet (HG and SG)
Wiskunde Formuleblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}(a+1) \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = a.r^{n-1} \quad S_n = \frac{a\left(1 - r^n\right)}{1 - r} \qquad S_n = \frac{a\left(r^n - 1\right)}{r - 1} \qquad S_\infty = \frac{a}{1 - r} \quad ; \quad r \neq 1$$

$$A = P\left(1 + \frac{r}{100}\right)^n \qquad A = P\left(1 - \frac{r}{100}\right)^n$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

$$x^2 + y^2 = r^2 \qquad (x - p)^2 + (y - q)^2 = r^2$$

In $\triangle ABC$:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \triangle ABC = \tfrac{1}{2}ab.\sin C$$

SENIOR CERTIFICATE EXAMINATION - 2004 - *(DIAGRAM SHEET)*
SENIORSERTIFIKAAT-EKSAMEN - 2004 - (DIAGRAMVEL)

DEPARTMENT OF EDUCATION

DEPARTEMENT VAN ONDERWYS

REPUBLIC OF SOUTH AFRICA
REPUBLIEK VAN SUID-AFRIKA

SENIOR CERTIFICATE EXAMINATION/SENIORSERTIFIKAAT-EKSAMEN
MATHEMATICS SG/WISKUNDE SG
PAPER II/VRAESTEL II

DIAGRAM SHEET/DIAGRAMVEL

INSTRUCTION

This diagram sheet must be handed in with your answer book. Please ensure that your details are complete.

INSTRUKSIE

Hierdie diagramvel moet saam met jou antwoordeboek ingelewer word. Maak asseblief seker dat jou besonderhede volledig is.

EXAMINATION NUMBER
EKSAMENNOMMER

CENTRE NUMBER
SENTRUMNOMMER

EXAMINATION NUMBER
EKSAMENNOMMER

QUESTION 1/VRAAG 1

D (–2 ; 4)

θ

O   A

x

E (2 ; –2)

F (–1 ; k)

QUESTION 2/VRAAG 2

y   D

(–3 ; 4) A

B   O   C   x

QUESTION 4.1/VRAAG 4.1
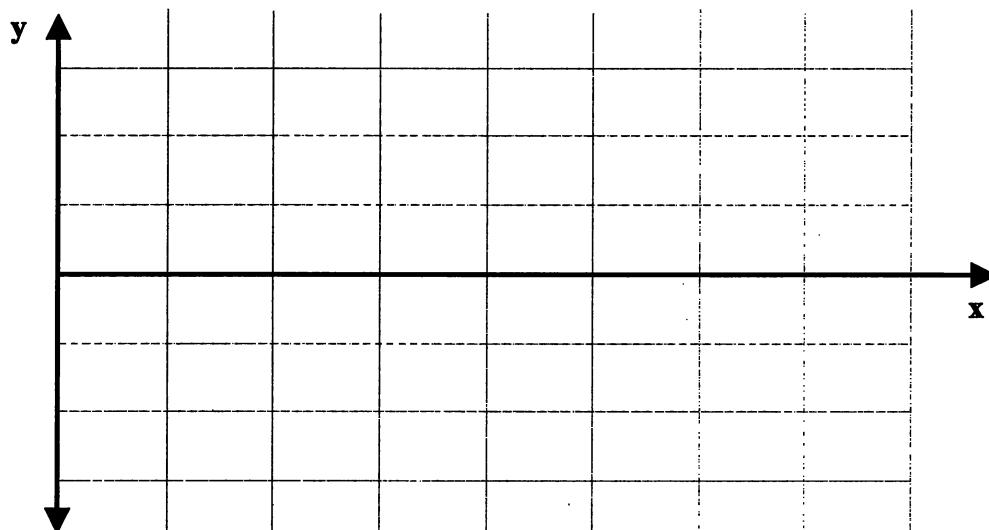
y

x

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QUESTION 6.1/VRAAG 6.1
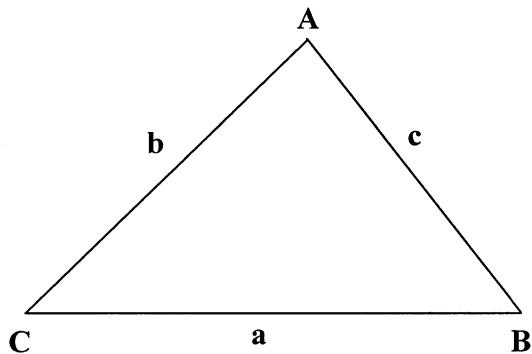
QUESTION 6.3/VRAAG 6.3
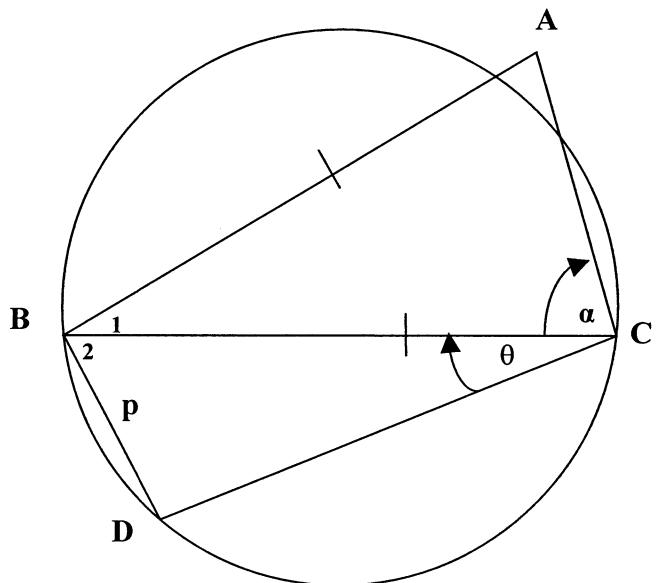
QUESTION 7/VRAAG 7
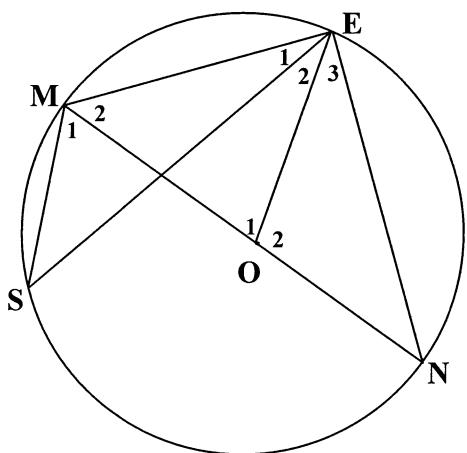
QUESTION 8.1/VRAAG 8.1
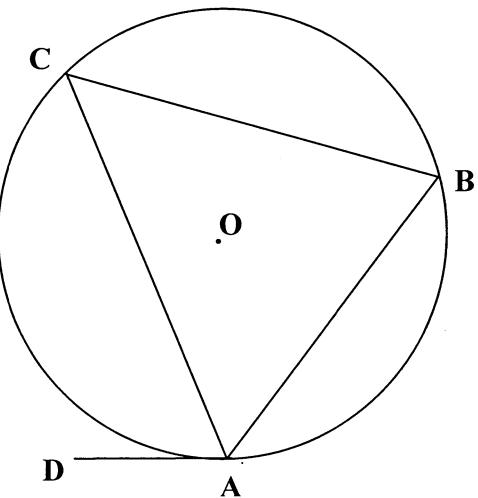
EXAMINATION NUMBER
EKSAMENNOMMER

QUESTION 8.2/VRAAG 8.2

QUESTION 9.1/VRAAG 9.1

SENIOR CERTIFICATE EXAMINATION - 2004 - (DIAGRAM SHEET)
SENIORSERTIFIKAAT-EKSAMEN - 2004 - (DIAGRAMVEL)
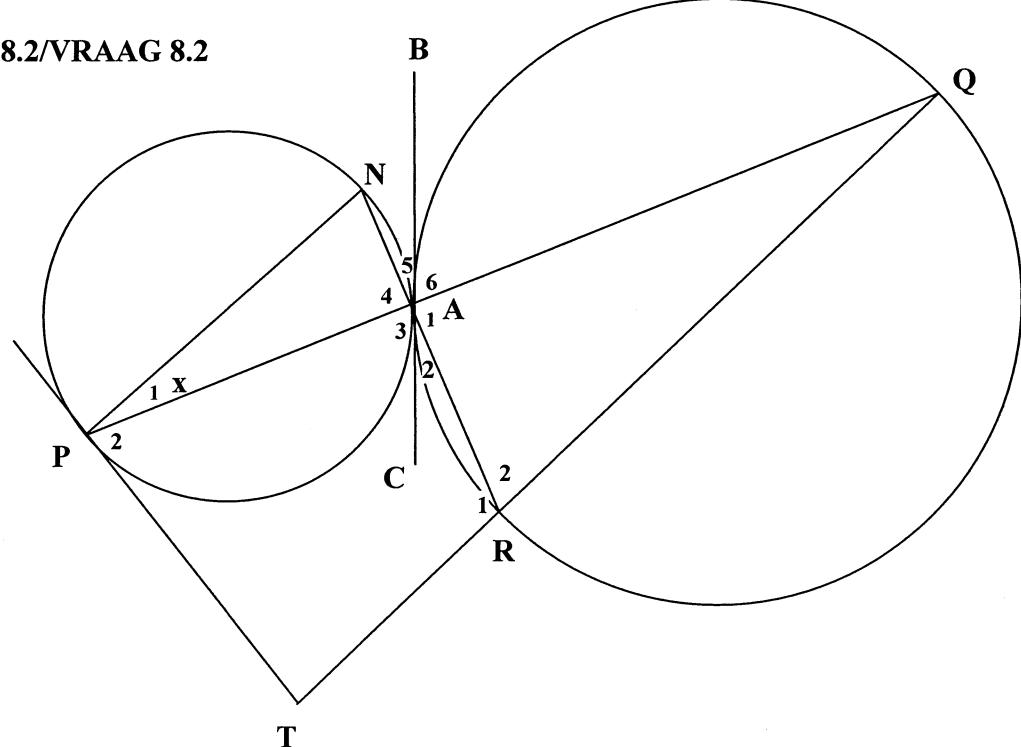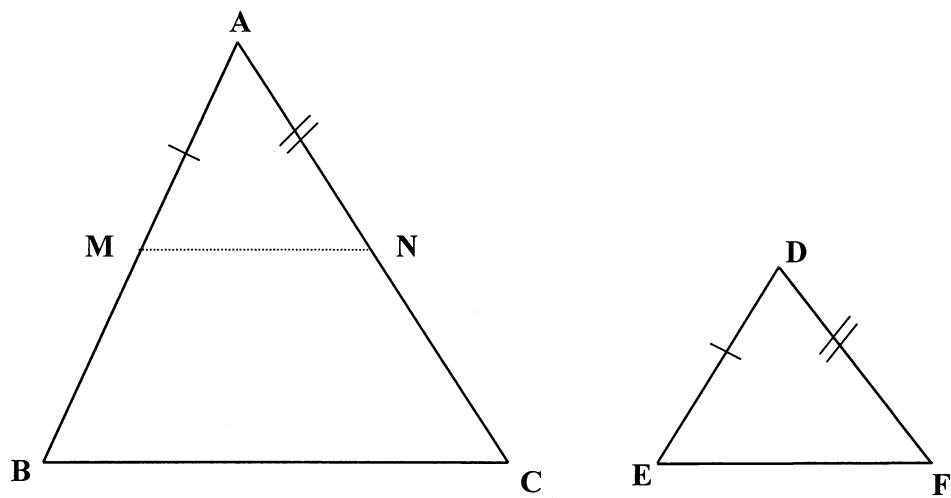
EXAMINATION NUMBER
EKSAMENNOMMER

QUESTION 9.2/VRAAG 9.2

QUESTION 9.3/VRAAG 9.3