QUESTION 1

1.1.1

$(x-1)^2 = 16$

$x^2 - 2x + 1 = 16$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x = 5 \quad \text{or} \quad x = -3$

- ✓ product
- ✓ standard form
- ✓ factorization/Formula
- ✓ answers

If standard form not indicated but answer correct, full marks.

Alt.solution :

$(x-1)^2 = 16$

$x - 1 = \pm 4$

$x - 1 = 4 \quad \text{or} \quad x - 1 = -4$

$x = 5 \quad \text{or} \quad x = -3$

- ✓ taking square roots
- ✓ for ±
- ✓✓ each solution

If $x - 1 = 4$
$\therefore \quad x = 5$

max 2 marks

(4)

$(x-1)^2 - 16 = 0$

$(x-1-4)(x-1+4) = 0$

$(x-5)(x+3) = 0$

$x = 5 \text{ or } x = -3$

(3)

1.1.2

$\sqrt{4x-3} = 5$

$4x - 3 = 25$

$4x = 28$

$x = 7$

- ✓ squaring both sides
- ✓ transposing - 3
- ✓ answer

1.2

$x^2 - 2xy - 4y = 0$ .....(1)

$x + 2y - 3 = 0$

$x = 3 - 2y$ .....(2)

sub. (2) in (1)

$(3-2y)^2 - 2y(3-2y) - 4y = 0$

$9 - 12y + 4y^2 - 6y + 4y^2 - 4y = 0$

$8y^2 - 22y + 9 = 0$

$(2y-1)(4y-9) = 0$

$y = \frac{1}{2} \quad \text{or} \quad y = \frac{9}{4}$

$x = 2 \quad \text{or} \quad x = -\frac{3}{2}$

- ✓ making x the subject of the formula in the linear equation
- ✓ correct substitution
- ✓ simplification (brackets removed)
- ✓ standard form
- ✓ factorisation
- ✓✓ y-values
- ✓✓ x-values

Alt solution:

$$y = \frac{3-x}{2}$$

$$x^2 - 2x\left(\frac{3-x}{2}\right) - 4\left(\frac{3-x}{2}\right) = 0$$

$$x^2 - 3x + x^2 - 6 + 2x = 0$$

$$2x^2 - x - 6 = 0$$

$$(2x+3)(x-2) = 0$$

$$x = -\frac{3}{2} \quad \textit{or} \quad x = 2$$

$$y = \frac{9}{4} \quad \textit{or} \quad y = \frac{1}{2}$$

✓ making y the subject of the formula in the linear equation

✓ correct substitution

✓ simplification (brackets removed)

✓ standard form/further simplification

✓ factorisation

✓✓ x – values

✓✓ y – values

(9)
[16]

If $x = 3 - 2y$

$$(2y-3)^2 - 2y(2y-3) - 4y = 0$$

$$4y^2 - 12y + 9 - 4y^2 + 6y - 4y = 0$$

$$-10y = -9$$

$$y = \frac{9}{10}$$

$$x = 1\tfrac{1}{5} \quad \textit{or} \quad 1{,}2 \quad \textit{or} \quad -1{,}2$$

✓ x subject

✓ simplification

✓ further simplification

✓ finding y

✓ substitution **max :** $\frac{5}{9}$

QUESTION 2

2.1

$$x^2 - 5x + 4 = 5$$

$$x^2 - 5x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-1)}}{2}$$

$$= \frac{5 \pm \sqrt{29}}{2}$$

$$x = 5{,}19 \quad \textit{or} \quad x = -0.19$$

✓ standard form

✓ correct formula

✓ substitution.

✓ simplification

✓✓ answers

If error results in factorisation or wrong formula: Max 3 marks i.e. for standard form & calculator

– 1 for incorrect rounding off

(6)

Alt. Solution:
Completing the square.

$$x^2 - 5x = 1$$

$$x^2 - 5x + \left(-\tfrac{5}{2}\right)^2 = 1 + \left(-\tfrac{5}{2}\right)^2$$

$$(x - \frac{5}{2})^2 = \frac{29}{4}$$

$$x = \frac{5}{2} \pm \sqrt{\frac{29}{4}}$$

$$x = \frac{5 \pm \sqrt{29}}{2}$$

$$x = 5{,}19 \quad \textit{or} \quad x = -0{,}19$$

✓ for adding $\left(-\tfrac{5}{2}\right)^2$ on both sides

✓ $(x - \frac{5}{2})^2$ ✓ $\frac{29}{4}$

✓ simplification

✓✓ answers

2.2	$2kx^2 - 3x - k = 0$ $x = -1: \quad 2k(-1)^2 - 3(-1) - k = 0$ $2k + 3 - k = 0$ $k = -3$	✓ substitution ✓ simplification ✓ answer If $x = 1$ $\therefore k = 3$ 2 marks	(3)
2.3	$3x^2 + (k+5)x + 3 = 0$ $\Delta = b^2 - 4ac$ $\Delta = (k+5)^2 - 4(3)(3)$ $= k^2 + 10k + 25 - 36$ $\Delta = k^2 + 10k - 11$ for equal roots : $\Delta = 0$: $k^2 + 10k - 11 = 0$ $(k-1)(k+11) = 0$ $k = 1$ or $k = -11$ - **Alt . solution:** $\Delta = b^2 - 4ac$ $\Delta = (k+5)^2 - 4(3)(3)$ for equal roots : $\Delta = 0$: $(k+5)^2 = 36$ $k + 5 = \pm 6$ $k = 1$ or $k = -11$	✓ formula ✓ sub. in Δ ✓ simplification ✓ equating delta to zero ✓ factorisation ✓ k-values $\Delta \geq 0$ breakdown **max:** $\frac{3}{6}$ ✓ formula ✓ sub. in Δ ✓ equating delta to zero ✓ simplification ✓ finding square roots ✓ k-values	(6) **[15]**

QUESTION 3			
3.1	$f(x) = x^3 - 3x^2 + 2ax - 1$ $f(1) = 7$ $1 - 3 + 2a - 1 = 7$ $2a = 10$ $a = 5$	✓ $f(1)=7$ ✓ substitution ✓ answer If $f(1) = 0$ Max. 1 mark	(3)
3.2	$f(x) = x^3 - 3x^2 + 10x - 1$ $x = -3$ $f(-3) = (-3)^3 - 3(-3)^2 + 10(-3) - 1$ $= -27 - 27 - 30 - 1$ $= -85$	✓ substitution for a ✓ $f(-3)$ / method ✓ answer CA for value of a from 3.1	(3) **[6]**
	$x + 3 \overline{)\, x^3 - 3x^2 + 10x - 1}$ $\quad x^2 - 6x + 28$ $x^3 + 3x^2$ $-6x^2 + 10x$ $-6x^2 - 18x$ $28x - 1$ $28x + 84$ -85 Remainder $= -85$	✓✓ answer	

QUESTION 4

4.1.1

$y_C = OC = f(0) = 3$ ✓ *OC* answer.

$y = -x^2 - 2x + 3$

$0 = x^2 + 2x - 3$ ✓ equating to zero

$= (x+3)(x-1)$ ✓ factorisation

$x = -3$ ***or*** $x = 1$ ✓ *x*-values

If not equating to 0 but correct *x* values, full 3 marks

$AB = AO + OB$

$= 3 + 1$

$= 4$ ✓ *AB* :answer

(5)

4.1.2.

$x = -\frac{b}{2a}$ **OR** $f'(x) = 0$ **OR** $x = \frac{x_A + x_B}{2}$

$= -\frac{-2}{2(-1)}$ | $-2x - 2 = 0$ | $= \frac{-3+1}{2}$

$= -1$ | $x = -1$ | $= -1$

✓ formula / $f'(x)$

If -1 without x Penalise 1 mark

✓ answer

(2)

4.1.3

$ST = f(-1) = -(-1)^2 - 2(-1) + 3$ ✓ $f(-1)$ ✓ substitution

$= -1 + 2 + 3$ ✓ simplification

$= 4$

NB.: 4 is given

Alt. Solution:

$ST = y_T = \frac{-\Delta}{4a}$ ✓✓ method/ formula

$ST = \frac{-(4+12)}{-4}$ ✓ substitution

$= 4$

(3)

4.1.4(a)

$m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$ **OR** $m_{AC} = \frac{OC}{OA}$

$= \frac{3-0}{0-(-3)}$ | $= \frac{3}{3}$

$= \frac{3}{3}$ | $= 1$

$= 1$

✓ formula

✓ substitution

✓ answer

Answer only: full marks.

(3)

4.1.4(b)

$m = m_{AC}$ (parallel lines)

$m = 1$ ✓ answer

$y = x + c$

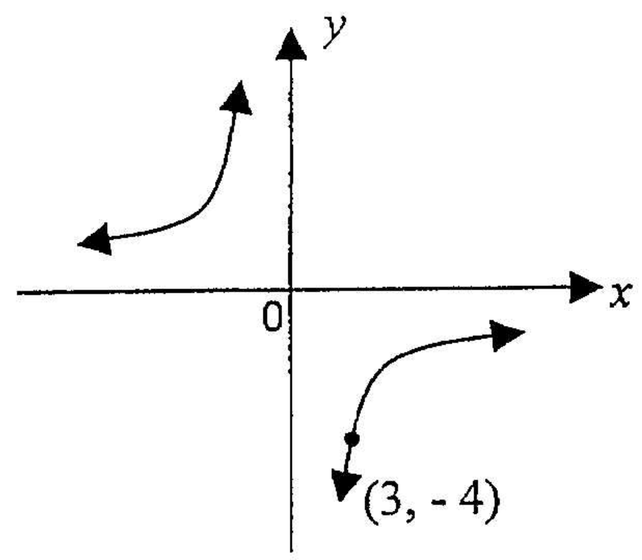
$B(1;0)$: $0 = 1 + c$ ✓ subst. of $m = 1$ in str. line equation.

$c = -1$ ✓ subst. B(1 ; 0) in str. line equation.

✓ answer

Answer only: full marks.

(4)

4.1.5	$-x^2-2x+k=0$ equal roots: the graph touches $x-$axis $\therefore$ move the graph 4 units downwards $\therefore k=3-4=-1$	✓✓ showing understanding of roots in relation to x-axis ✓ value of k If formula (Δ) is used, 1 mark **KEY: touch, tangent, turns at x-axis** **-Accept if shown by means of sketch**	(3)
4.2.1		✓ shape ✓ showing any valid point (check: product of co-ordinates = – 12) If only one arm shown, 1 mark	(2)
4.2.2	$y\neq 0$ **OR** $y<0$ or $y>0$ **OR** $(-\infty;0)\cup(0;\infty)$ **OR** $\{y/y\neq 0\}$	✓✓ answer (CA in terms of 4.2.1) (package) [If $y\in$ R only: 1 mark]	(2) **[24]**

QUESTION 5

5.1.1	$\sqrt{5}\left(\sqrt{45}+2\sqrt{80}\right)$ $=\sqrt{5}\left(\sqrt{9.5}+2\sqrt{16.5}\right)$ $=\sqrt{5}\left(3\sqrt{5}+2.4\sqrt{5}\right)$ $=\sqrt{5}\left(3\sqrt{5}+8\sqrt{5}\right)$ $=\sqrt{5}\left(11\sqrt{5}\right)$ $=11(5)$ $=55$ - **Alt. Solution:** $\sqrt{5}\left(\sqrt{45}+2\sqrt{80}\right)$ $=\sqrt{225}+2\sqrt{400}$ $=15+40$ $=55$	✓ splitting factors under root sign ✓ simplification (at least one of the 3 steps) Answer only: 1 mark ✓ answer ✓✓ multiplying by $\sqrt{5}$ (each term) ✓ answer	(3)

5.1.2	$\frac{2^{3n+2}\cdot 8^{n-3}}{4^{3n-2}}$ $=\frac{2^{3n+2}\cdot(2^3)^{n-3}}{(2^2)^{3n-2}}$ $=\frac{2^{3n+2}\cdot 2^{3n-9}}{2^{6n-4}}$ $=2^{6n-7-6n+4}$ $=2^{-3}$ $=\frac{1}{8}$	✓ writing as base 2 ✓ exponential laws ✓ simplification/exp. law ✓ 2^{-3} ✓ answer **Accept 0,125 if it follows from $\frac{1}{2^3}$**	(5)
5.1.3	$3\log 2+\log 125$ $=\log 2^3+\log 125$ $=\log(2^3\cdot 125)$ $=\log 1000$ $=\log 10^3$ $=3\log 10$ $=3$ **Alt. Solution** $3\log 2+\log 5^3$ $=3\log 2+3\log 5$ $=3(\log 2+\log 5)$ $=3\log 10$ $=3$	✓ log law ✓ log law (single log) ✓ simplification ($\log 1000$) ✓ answer ✓ $125=5^3$ ✓ log law ✓ simplification (log 10) ✓ answer	(4)
5.1.4	$8^{\frac{2}{3}}+\log_2 32$ $=(2^3)^{\frac{2}{3}}+\log_2 2^5$ $=2^2+5\log_2 2$ $=4+5$ $=9$	✓ writing as base 2 ✓ exp. law ✓ log law ✓ $4+5$ ✓✓ 4 ✓✓ 5	(4)
5.2.1	$3^x+3^{x-1}=4$ $3^x+3^x\cdot 3^{-1}=4$ $3^x(1+\frac{1}{3})=4$ $3^x\cdot\frac{4}{3}=4$ $3^x=3$ $\therefore x=1$	✓ split factors ✓ common factor ✓ simplification ✓ answer $x+x-1=4$ Breakdown : zero	(4)

Alt. Solution:

$3^{x-1}(3+1)=4$

$3^{x-1}\cdot 4=4$

$3^{x-1}=1$

$3^{x-1}=3^0$

$x-1=0$

$x=1$

✓✓ common factor ; other factor

✓ $1=3^0$

✓ answer

Alt. Solution:

$3^x+3^{x-1}=3^1+3^0$

$\therefore 3^x=3^1$ **and** $3^{x-1}=3^0$

$\therefore\ x=1$ **and** $x-1=0$

$x=1$

Mathematical inconsistency
Max. 2 marks

Answer only:
2 marks

5.2.2

$\log(2x+1)-\log(x-1)=1$

$\log\frac{2x+1}{x-1}=1$

$\log\frac{2x+1}{x-1}=\log 10$

$\frac{2x+1}{x-1}=10$

$2x+1=10x-10$

$-8x=-11$

$x=\frac{11}{8}$

✓ log law

$\frac{2x+1}{x-1}=1$
Breakdown : $\frac{1}{4}$

✓ log definition
✓ simplification

$(2x+1)(x-1)=10$

Breakdown: max $\frac{2}{4}$

✓ answer

(4)

[24]

QUESTION 6

6.1.1

$T_n=a+(n-1)d$

$T_5=0 \Rightarrow a+4d=0$.....(1)

$T_{13}=16 \Rightarrow a+12d=16$.....(2)

(2) − (1) $\therefore\ 8d=16$

$d=2$

✓ formula
✓ substitution
✓ substitution
✓ answer
answer only: full marks

$d=\frac{T_{13}-T_5}{13-5}=\frac{16}{8}=2$ ✓ ✓ ✓

(4)

6.1.2. from (1): $a=-4d=-4(2)=-8$

✓ substitution
✓ answer

(2)

6.1.3.

$S_n=\frac{n}{2}[2a+(n-1)d]$

$S_{21}=\frac{21}{2}[2(-8)+(21-1)(2)]$

$=\frac{21}{2}(-16+40)$

$=\frac{21}{2}(24)$

$=252$

✓ formula

✓✓ subst. n, d, a

✓ answer

(4)

Expansion of terms (i.e. writing out of terms) and then correct answers, **full marks in each case**

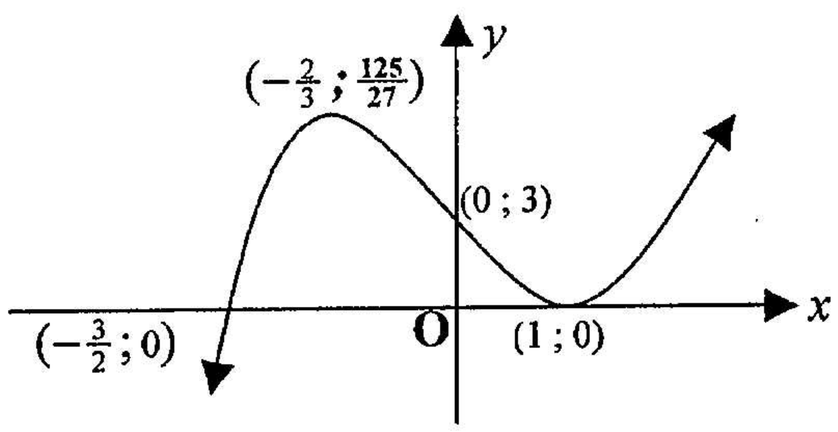
6.2	$r = \frac{1}{3}$ $T_n = ar^{n-1}$ $T_{12} = ar^{11} = 3 \cdot \left(\frac{1}{3}\right)^{11} = 1{,}69 \times 10^{-6}$ *or* $\frac{1}{59049}$ (or 0,000016935) (or 3^{-10})	✓ value of r ✓ formula ✓ substitution ✓ answer	(4)
6.3	$\frac{T_n}{T_{n-1}} = r = 2$ $S_n = \frac{a(r^n - 1)}{r - 1}$ $S_{20} = \frac{1(2^{20} - 1)}{2 - 1}$ $S_{20} = 1048575$	✓ $r = 2$ ✓ formula ✓ substitution ✓ answer Interpreting the question in terms of the context yields T_n as correct: ✓ $T_n = ar^{n-1}$ ✓ $T_{20} = 1(2)^{19}$ ✓ ✓ $= 524288$ ✓	(4)
6.4	$A = P\left(1 - \frac{r}{100}\right)^n$ $25\,000 = 50\,000\left(1 - \frac{4}{100}\right)^n$ $\frac{1}{2} = (0{,}96)^n$ $\log 0{,}5 = n \log 0{,}96$ $\frac{\log 0{,}5}{\log 0{,}96} = n$ $16{,}979 = n$ *after nearly 17 years*	✓ formula ✓✓ substitution ✓ (0,96) ✓ log laws ✓ simplification (n = ...) ✓ answer (either answer) If wrong formula i.e. $A = P\left(1 + \frac{r}{100}\right)^n$ Max. marks 4: ✓ ✓subst. ✓ (1,04) ✓ apply logs both sides	(7) **[25]**

QUESTION 7

7.1	$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \to 0} \frac{-2(x+h)^2 - (-2x^2)}{h}$ $= \lim_{h \to 0} \frac{-2(x^2 + 2xh + h^2) + 2x^2}{h}$ $= \lim_{h \to 0} \frac{-2x^2 - 4xh - 2h^2 + 2x^2}{h}$ $= \lim_{h \to 0} \frac{-4xh - 2h^2}{h}$ $= \lim_{h \to 0} \quad (-4x - 2h)$ $= \quad -4x$	✓ formula ✓ substitution ✓ simplification ✓ dividing by h ✓ answer Max. penalty of 1 for notational errors No marks for answer only	

	Alt. Solution: $f(x) = -2x^2$ $f(x+h) = -2(x+h)^2 = -2x^2 - 4xh - 2h^2$ $\frac{f(x+h)-f(x)}{h} = \frac{-2x^2-4xh-2h^2+2x^2}{h}$ $= \frac{-4xh-2h^2}{h}$ $= -4x - 2h$ $f'(x) = \lim_{h\to 0} (-4x-2h)$ $= -4x$	✓ $f(x+h)$ ✓ substitution ✓ simplification ✓ $\lim_{h\to 0} (-4x-2h)$ ✓ answer	(5)
7.2	$m = f'(x)$ $6 = -4a$ $a = -\frac{3}{2}$	✓✓ method and substitution ✓ answer If $x = -\frac{3}{2}$?	(3)
7.3.1	$y = (2x-1)^2$ $= 4x^2 - 4x + 1$ $\frac{dy}{dx} = 8x - 4$	✓ product ✓✓ for $8x$ and -4 Max. penalty of 1 for abuse of equal sign	(3)
7.3.2.	$y = 4\sqrt{x} + x^3$ $= 4x^{\frac{1}{2}} + x^3$ $\frac{dy}{dx} = 2x^{-\frac{1}{2}} + 3x^2$	✓ $x^{\frac{1}{2}}$ ✓✓ each derivative	(3) **[14]**

QUESTION 8			
8.1	$f(x) = 2x^3 - x^2 - 4x + 3$ $f(1) = 2(1)^3 - (1)^2 - 4(1) + 3$ $= 2 - 1 - 4 + 3$ $= 0$	✓ $f(1)$ ✓ subst. ✓ answer (= 0) Long division: - Correctly: $\frac{3}{3}$ - with error: $\frac{1}{3}$	(3)
8.2	$f(x) = (x-1)(2x^2 + x - 3)$ $= (x-1)(x-1)(2x+3)$	✓ $2x^2$ & -3 ✓ x ✓ factors	(3)
8.3	$(1\ ;\ 0)$ and $(-\frac{3}{2}\ ;\ 0)$	✓✓ for each pair Full Marks if: $x = -\frac{3}{2}\ ;\ 1$	(2)

8.4	Turning points where $f'(x) = 0$ $\therefore f'(x) = 6x^2 - 2x - 4 = 0$ $\therefore \quad 3x^2 - x - 2 = 0$ $(3x + 2)(x - 1) = 0$ $x = -\frac{2}{3} \quad \text{or} \quad x = 1$ $f\left(-\frac{2}{3}\right) = \frac{125}{27} \quad \text{or} \quad 4\frac{17}{27} \quad \text{or} \quad 4{,}63$ $f(1) \quad = 0$ $\therefore \quad (1;0) \quad ; \quad D\left(-\frac{2}{3};\frac{125}{27}\right)$	✓ derivative ✓ $= 0$ ✓ factors ✓ x-values ✓ value of $f(-\frac{2}{3})$ ✓ value of $f(1)$	(6)
8.5		✓ x-intercepts ✓✓ t.ps ✓ shape	(4) **[18**

QUESTION 9			
9.1	$A = xy = 200$ $y = \frac{200}{x}$	✓ y in terms of x	(1)
9.2.	$Length\ of\ fencing = x + x + \frac{200}{x}$ $= 2x + \frac{200}{x}$ $Cost = C = 100\left(2x + \frac{200}{x}\right)$ $= 200x + \frac{20000}{x}$	✓ for perimeter or $(2x + y)$ ✓ multiplying by 100	(2)
9.3	$C = 200x + 20000x^{-1}$ $\frac{dC}{dx} = 200 - 20000x^{-2}$ $200 - 20000x^{-2} = 0$ $\frac{20000}{x^2} = 200$ $x^2 = 100$ $x = 10$ $PQ = 10\ m$	✓ writing in exponential form ✓ derivative ✓ derivative = 0 ✓ simplification ✓ either answer	(5) **[8]**