DEPARTMENT OF EDUCATION
REPUBLIC OF SOUTH AFRICA

DEPARTEMENT VAN ONDERWYS
REPUBLIEK VAN SUID-AFRIKA

SENIOR CERTIFICATE EXAMINATION - 2004
SENIORSERTIFIKAAT-EKSAMEN - 2004

MATHEMATICS P1 : ALGEBRA
WISKUNDE V1 : ALGEBRA

STANDARD GRADE
STANDAARDGRAAD

OCTOBER/NOVEMBER 2004
OKTOBER/NOVEMBER 2004

301-2/1

Marks: 150
Punte : 150

3 Hours
3 Ure

This question paper consists of 7 pages and 1 information sheet.
Hierdie vraestel bestaan uit 7 bladsye en 1 inligtingsblad.

MATHEMATICS SG: Paper 1
Algebra

301 2 1   SG

INSTRUKSIES AAN KANDIDATE

Lees die volgende instruksies sorgvuldig deur voordat die vrae beantwoord word:

1.   Hierdie vraestel bestaan uit **9** vrae. Beantwoord **AL** die vrae.

2.   Toon duidelik **AL** die berekeninge, diagramme, grafieke, ensovoorts wat jy gebruik het om die antwoorde te bepaal.

3.   'n Goedgekeurde sakrekenaar (nie-programmeerbaar en nie-grafies) mag gebruik word, tensy anders vermeld.

4.   Indien nodig, moet antwoorde tot **TWEE** desimale plekke afgerond word, tensy anders vermeld.

5.   Grafiekpapier word **NIE** in hierdie vraestel benodig nie.

6.   Nommer die antwoorde **PRESIES** soos die vrae genommer is.

7.   Diagramme is nie noodwendig volgens skaal geteken nie.

8.   Dit is tot jou eie voordeel om leesbaar te skryf en om jou werk netjies aan te bied.

9.   **'n Inligtingsblad met formules is ingesluit aan die einde van die vraestel.**

INSTRUCTIONS TO CANDIDATES

Read the following instructions carefully before answering the questions:

1.   This paper consists of **9** questions. Answer **ALL** the questions.

2.   Clearly show **ALL** calculations, diagrams, graphs, et cetera you have used in determining the answers.

3.   An approved calculator (non-programmable and non-graphical) may be used unless stated otherwise.

4.   If necessary, answers should be rounded off to **TWO** decimal places, unless stated otherwise.

5.   Graph paper is **NOT** required in this question paper.

6.   Number the answers **EXACTLY** as the questions are numbered.

7.   Diagrams are not necessarily drawn to scale.

8.   It is in your own interest to write legibly and to present the work neatly.

9.   **An information sheet with formulae is included at the end of the question paper.**

VRAAG 1

1.1 Los op vir x:

    1.1.1 $(x-1)^2 = 16$ (4)

    1.1.2 $\sqrt{4x-3} = 5$ (3)

1.2 Los op vir x en y in die volgende gelyktydige vergelykings:

$$x + 2y - 3 = 0$$
$$x^2 - 2xy - 4y = 0$$ (9)
[16]

VRAAG 2

2.1 Bepaal die wortels van $x^2 - 5x + 4 = 5$ afgerond tot **TWEE** desimale syfers.   (6)

2.2 As $x = -1$ 'n wortel is van $2kx^2 - 3x - k = 0$, bepaal k.   (3)

2.3 Bepaal die waardes van k waarvoor die wortels van $3x^2 + (k+5)x + 3 = 0$ gelyk is.   (6)
[15]

VRAAG 3

Gegee: $f(x) = x^3 - 3x^2 + 2ax - 1$

3.1 As $f(x)$ deur $(x-1)$ gedeel word, is die res 7. Bereken die waarde van a.   (3)

3.2 Bereken vervolgens die res as $f(x)$ deur $(x+3)$ gedeel word.   (3)
[6]

QUESTION 1

1.1   Solve for x:

    1.1.1 $(x-1)^2 = 16$ (4)

    1.1.2 $\sqrt{4x-3} = 5$ (3)

1.2   Solve for x and y in the following simultaneous equations:

$$x + 2y - 3 = 0$$
$$x^2 - 2xy - 4y = 0$$

    (9)
    [16]

QUESTION 2

2.1   Determine the roots of $x^2 - 5x + 4 = 5$ rounded off to **TWO** decimal places.   (6)

2.2   If $x = -1$ is a root of $2kx^2 - 3x - k = 0$, determine k.   (3)

2.3   Determine the values of k for which $3x^2 + (k+5)x + 3 = 0$ has equal roots.   (6)
    [15]

QUESTION 3

Given: $f(x) = x^3 - 3x^2 + 2ax - 1$

3.1   Calculate the value of a if there is a remainder of 7 when $f(x)$ is divided by $(x-1)$.   (3)
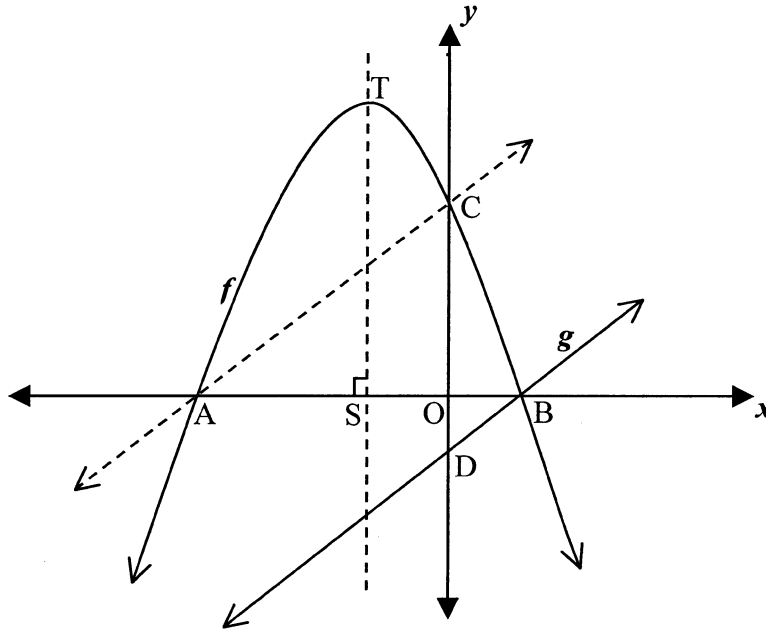
3.2   Hence, calculate the remainder when $f(x)$ is divided by $(x+3)$.   (3)
    [6]

VRAAG 4

4.1 Die skets toon die grafieke van $f(x) = -x^2 - 2x + 3$ en $g(x) = mx + c$. A en B is die afsnitte op die x-as. C en D is die afsnitte op die y-as. T is die draaipunt van die grafiek van f.

4.1.1 Bepaal die lengtes van OC en AB. (5)

4.1.2 Bepaal die vergelyking van die as van simmetrie van f. (2)

4.1.3 Toon dat die lengte van ST = 4 eenhede. (3)

4.1.4 Die grafiek van g is ewewydig aan AC.

Bereken:

(a) Die gradiënt van AC (3)
(b) Die waardes van m en c (4)

4.1.5 Verduidelik hoe jy die grafiek van f sou gebruik om die waarde van k te vind waarvoor die vergelyking $-x^2 - 2x + k = 0$ gelyke wortels het. Skryf die waarde van k in hierdie geval. (3)
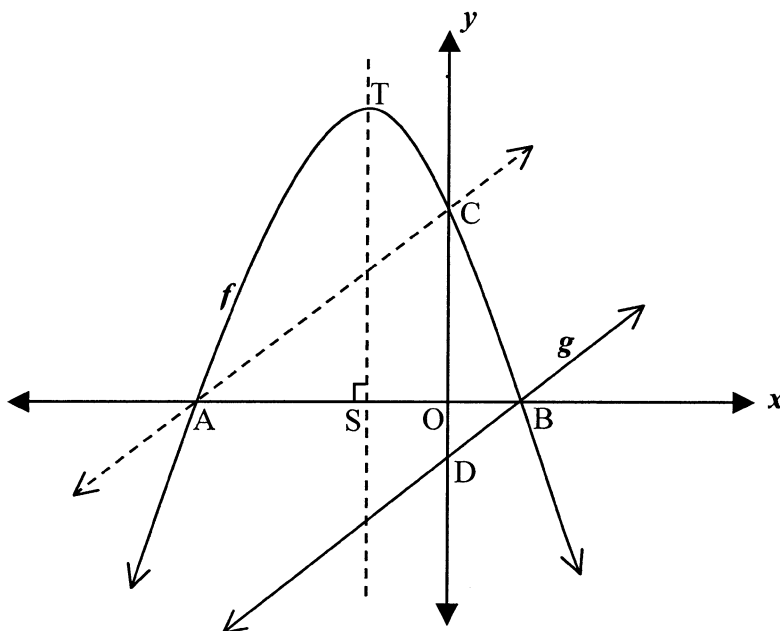
4.2 Gegee: $h(x) = -\dfrac{12}{x}$

4.2.1 Teken die grafiek van h. Toon die koördinate van een punt op die grafiek. (2)

4.2.2 Gee die waardeversameling van h. (2)

[24]

QUESTION 4

4.1   The sketch shows the graphs of $f(x) = -x^2 - 2x + 3$ and $g(x) = mx + c$. A and B are the intercepts on the x-axis. C and D are the intercepts on the y-axis. T is the turning point of the graph of f.

4.1.1   Determine the lengths of OC and AB.   (5)

4.1.2   Determine the equation of the axis of symmetry of the graph of f.   (2)

4.1.3   Show that the length of ST = 4 units.   (3)

4.1.4   The graph of g is parallel to AC.

Calculate:

(a)   The gradient of AC   (3)
(b)   The values of m and c (4)

4.1.5   Explain how you would use the graph of f to find the value of k for which the equation $-x^2 - 2x + k = 0$ has equal roots. Write the value of k in this case.   (3)

4.2   Given: $h(x) = -\dfrac{12}{x}$

4.2.1   Draw the graph of h. Show the co-ordinates of one point on the graph.   (2)

4.2.2   Give the range of h.   (2)
[24]

VRAAG 5

5.1 **Sonder om 'n sakrekenaar te gebruik,** bereken die waarde van elk van die volgende in sy eenvoudigste vorm:

5.1.1 $\sqrt{5}\left(\sqrt{45} + 2\sqrt{80}\right)$ (3)

5.1.2 $\dfrac{2^{3n+2}.8^{n-3}}{4^{3n-2}}$ (5)

5.1.3 $3\log 2 + \log 125$ (4)

5.1.4 $8^{\frac{2}{3}} + \log_2 32$ (4)

5.2 Los op vir x, **sonder om 'n sakrekenaar te gebruik:**

5.2.1 $3^x + 3^{x-1} = 4$ (4)

5.2.2 $\log(2x+1) - \log(x-1) = 1$ (4)

[24]

VRAAG 6

6.1 Die vyfde term van 'n rekenkundige ry is nul en die dertiende term is gelyk aan 16.

Bepaal:

6.1.1 Die gemene verskil (4)

6.1.2 Die eerste term (2)

6.1.3 Die som van die eerste 21 terme (4)

6.2 Bepaal die twaalfde term van die meetkundige ry

$3 ; 1 ; \dfrac{1}{3} ; \ldots$ (4)

QUESTION 5

5.1   **Without using a calculator,** calculate the value of each of the following in its simplest form:

5.1.1 $\sqrt{5}\left(\sqrt{45}+2\sqrt{80}\right)$ (3)

5.1.2 $\dfrac{2^{3n+2}.8^{n-3}}{4^{3n-2}}$ (5)

5.1.3 $3\log 2+\log 125$ (4)

5.1.4 $8^{\frac{2}{3}}+\log_2 32$ (4)

5.2   Solve for x, **without using a calculator**:

5.2.1 $3^x+3^{x-1}=4$ (4)

5.2.2 $\log(2x+1)-\log(x-1)=1$ (4)

[24]

QUESTION 6

6.1   The fifth term of an arithmetic sequence is zero and the thirteenth term is equal to 16.

    Determine:

6.1.1   The common difference   (4)
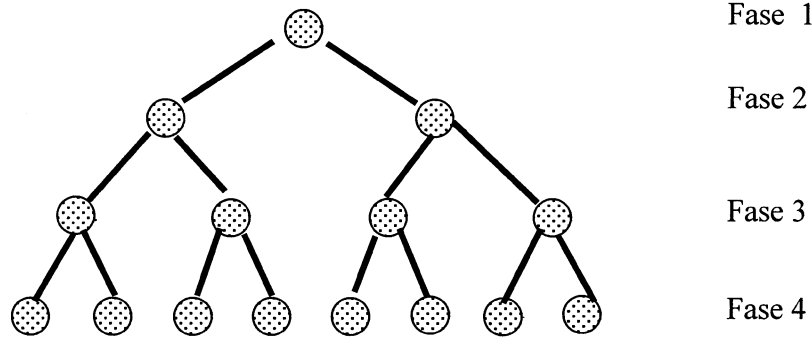
6.1.2   The first term   (2)

6.1.3   The sum of the first 21 terms   (4)

6.2   Determine the twelfth term of the geometric sequence
$3;1;\dfrac{1}{3};\ldots$ (4)

6.3   Selle verdeel deurentyd en vermeerder dus in getal. 'n Sel verdeel en word twee nuwe selle. Die proses word herhaal en vorm so 'n meetkundige ry. Die volgende skets verteenwoordig hierdie selverdeling.

Fase 1

Fase 2

Fase 3

Fase 4

        Hoeveel selle sal daar in totaal wees na twintig fases?   (4)

6.4   'n Fabriek is langs 'n park in 'n stad gebou. Die hoof van die park sê dat die gasse wat deur die skoorstene van die fabriek vrygelaat word, die voëlbevolking van sy park doodmaak. Die voëls kan ook nie eiers lê nie. Hy skat dat die voëlgetalle jaarliks met 4% afneem. As hy nou 50 000 voëls in die park het, na hoeveel jaar sal hierdie bevolking tot 25 000 verminder het?   (7)

    [25]

VRAAG 7

7.1   Bepaal $f'(x)$ vanaf **eerste beginsels** as $f(x) = -2x^2$.   (5)

7.2   Die gradiënt van die raaklyn aan die kromme van $f(x) = -2x^2$ by 'n punt $(a;b)$ op die kromme is 6. Bepaal die waarde van a.   (3)
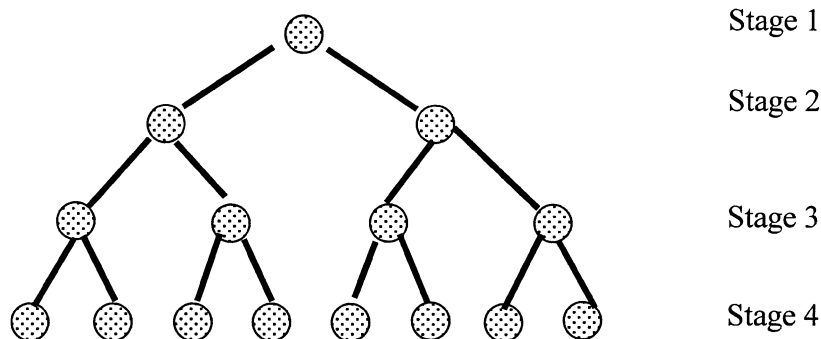
7.3   Bepaal $\dfrac{dy}{dx}$ in elk van die volgende:

       7.3.1 $y = (2x - 1)^2$ (3)

       7.3.2 $y = 4\sqrt{x} + x^3$ (3)

    [14]

6.3   Cells are continually dividing and thus increasing in number. A cell divides and becomes two new cells. The process repeats itself forming a geometric sequence. The following sketch represents this cell division.

How many cells will there be altogether after twenty stages?   (4)

6.4   A factory has been built next to a park in a city. The head of the park says that the gases given off by the chimneys of the factory kill the bird population of his park. Also, the birds are unable to lay eggs. He estimates that the bird numbers decrease by 4% per year. If he now has 50 000 birds in the park, after how many years will this population have been decreased to 25 000?   (7)
                                                                                         [25]

QUESTION 7

7.1   Determine $f'(x)$ from **first principles** if $f(x) = -2x^2$.   (5)

7.2   The gradient of the tangent to the curve of $f(x) = -2x^2$ at a point $(a;b)$ on the curve is 6. Determine the value of a.   (3)

7.3   Determine $\dfrac{dy}{dx}$ in each of the following:

        7.3.1 $y = (2x - 1)^2$ (3)

        7.3.2 $y = 4\sqrt{x} + x^3$ (3)
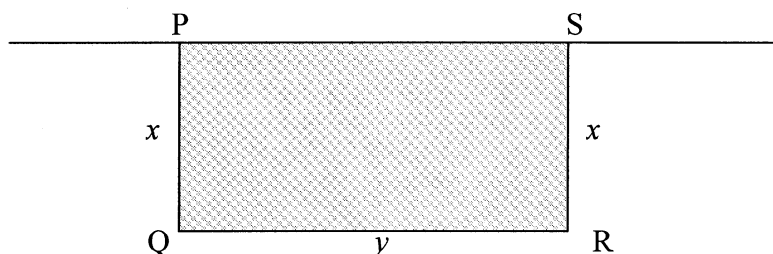   **[14]**

VRAAG 8

Gegee: $f(x) = 2x^3 - x^2 - 4x + 3$

8.1	Toon dat ($x-1$) 'n faktor van $f(x)$ is.	(3)
8.2	Faktoriseer vervolgens $f(x)$ volledig.	(3)
8.3	Skryf die koördinate van die x-afsnitte van f.	(2)
8.4	Bepaal die koördinate van die draaipunte van f.	(6)
8.5	Teken 'n netjiese sketsgrafiek van f en toon die koördinate van die draaipunte sowel as die x–afsnitte.	(4)
		[18]

VRAAG 9

'n Hoenderboer wil 'n reghoekige hok PQRS bou om sy kuikens te huisves. Hy wil die oppervlakte 200 vierkante meter maak. Een van die sye, naamlik PS, is langs die muur van 'n bestaande gebou. Die oorblywende drie sye moet omhein word. Omheiningsmateriaal kos R100 per meter. Hy wil die afmetings van die reghoek bereken sodat hy so min geld as moontlik aan die heining spandeer. Laat PQ = x en QR= y.

9.1	Bepaal y in terme van x.	(1)
9.2	Toon dat die totale koste C (in Rande) gegee is deur:	
	$$C = 200x + \frac{20\,000}{x}$$	(2)
9.3	Bepaal PQ sodat C so klein as moontlik is.	(5)
		[8]

TOTAAL: **150**

QUESTION 8

Given: $f(x) = 2x^3 - x^2 - 4x + 3$

8.1	Show that $(x-1)$ is a factor of $f(x)$.	(3)
8.2	Hence factorise $f(x)$ completely.	(3)
8.3	Write the co-ordinates of the x-intercepts of f.	(2)
8.4	Determine the co-ordinates of the turning points of f.	(6)
8.5	Draw a neat sketch graph of f indicating the co-ordinates of the turning points as well as the x-intercepts.	(4)

[18]

QUESTION 9

A chicken farmer wishes to build a rectangular enclosure PQRS to house his chickens. He wants the area to be 200 square metres. One of the sides, namely PS, is along the wall of an existing building. The remaining three sides must be fenced. Fencing material costs R100 per metre. He wants to calculate the dimensions of the rectangle so that he spends as little money as possible on the fencing. Let PQ = x and QR= y.

9.1	Determine y in terms of x.	(1)
9.2	Show that the total cost C (in Rands) is given by: $$C = 200x + \frac{20\,000}{x}$$	(2)
9.3	Determine PQ so that C is as small as possible.	(5)

[8]

TOTAL: **150**

Information Sheet (HG and SG)
Inligtingsblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}(a + l) \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = a.r^{n-1} \qquad S_n = \frac{a(1 - r^n)}{1-r} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \qquad S_\infty = \frac{a}{1-r}$$

$$A = P\left(1 + \frac{r}{100}\right)^n \qquad A = P\left(1 - \frac{r}{100}\right)^n$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$x^2 + y^2 = r^2$$

$$(x - p)^2 + (y - q)^2 = r^2$$

In $\triangle ABC$:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area \triangle ABC = \frac{1}{2}ab.\sin C$$