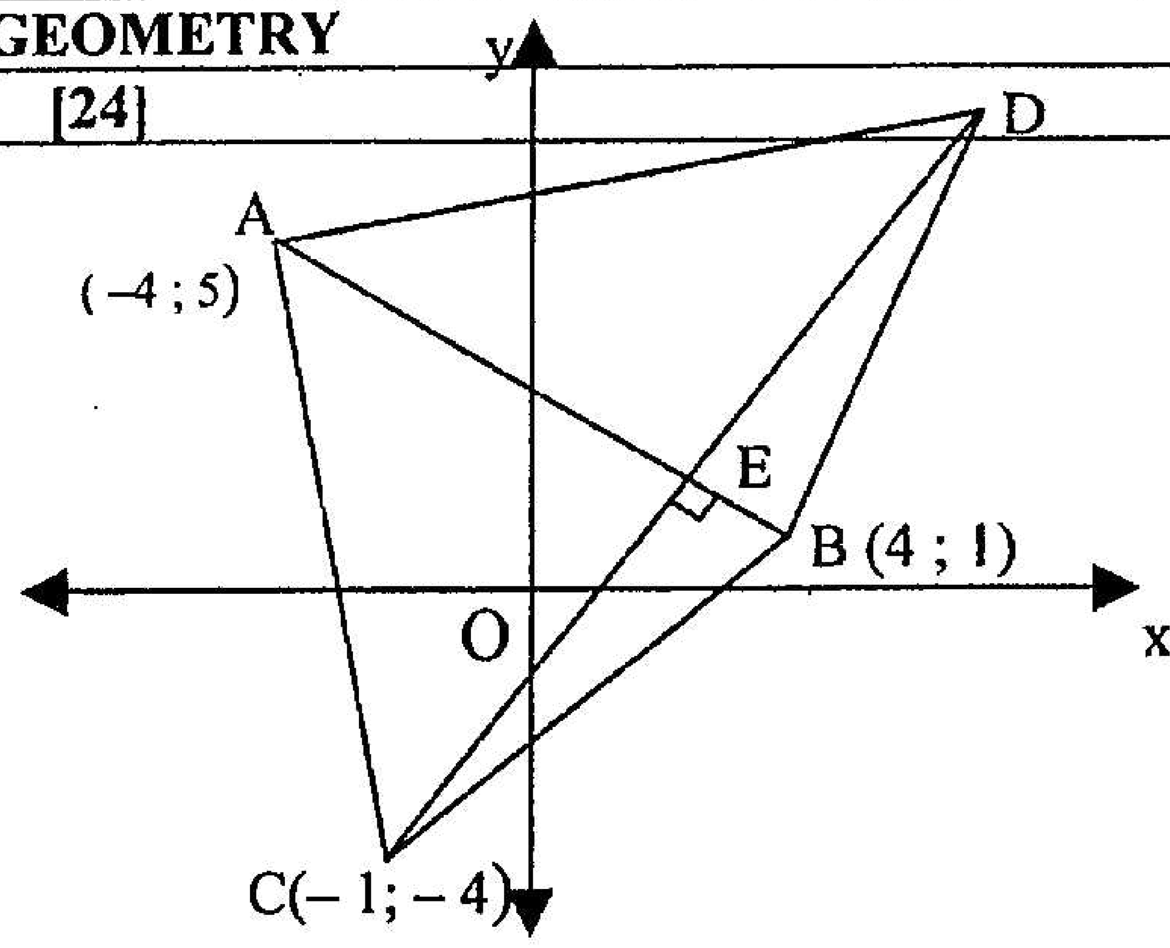
MATHEMATICS HG

PAPER II

ANALYTICAL GEOMETRY

QUESTION 1 [24]

1.1.1		Substitution in incorrect formula no marks. Maximum marks if // gradients $(\frac{4}{6})$ If D assumed maximum $(\frac{3}{6})$
	$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$ ✓M	correct formula
	$= \frac{1-5}{4+4}$	
	$= -\frac{1}{2}$ ✓A	Calculate gradient correctly
	Equation of CD: $m_{CD} = 2$ ✓CA	Deducing the perpendicular gradient
	$y - y_1 = m(x - x_1)$ ✓M **or** $y = mx + c$ ✓M	an appropriate formula of straight line
	$y - (-4) = 2(x - (-1))$ ✓CA **or** $y = 2x + c$; $-4 = 2(-1) + c$ ✓CA	Substitution of point C and gradient of CD
	$y + 4 = 2x + 2$; $y = 2x - 2$ ✓CA **or** $c = -4 + 2$; $y = 2x - 2$ ✓CA	Simplifying equation correctly
	(6)	
1.1.2	Equation of AB:	
	$y - y_1 = m_{AB}(x - x_1)$	
	$y - 5 = -\frac{1}{2}(x + 4)$ ✓CA	Sustitituting into formula of straight line
	$y = -\frac{1}{2}x + 3$ ✓CA .. ...Eq (2)	Simplifying equation correctly
	$y = 2x - 2$ ....(1)	
	Subt. (2) from (1)	
	✓M (1) – (2): $0 = \frac{5}{2}x - 5$ ✓CA	subtracting/ equating the equations
	$\frac{5}{2}x = 5$	Simplifying
	$x = 2$ ✓CA	Calculating x correctly
	Substitute in (1): $y = 2(2) - 2 = 2$ ✓CA	Calculating y correctly
	∴ E (2;2)	point E need not be in co-ordinate form.
	OR	

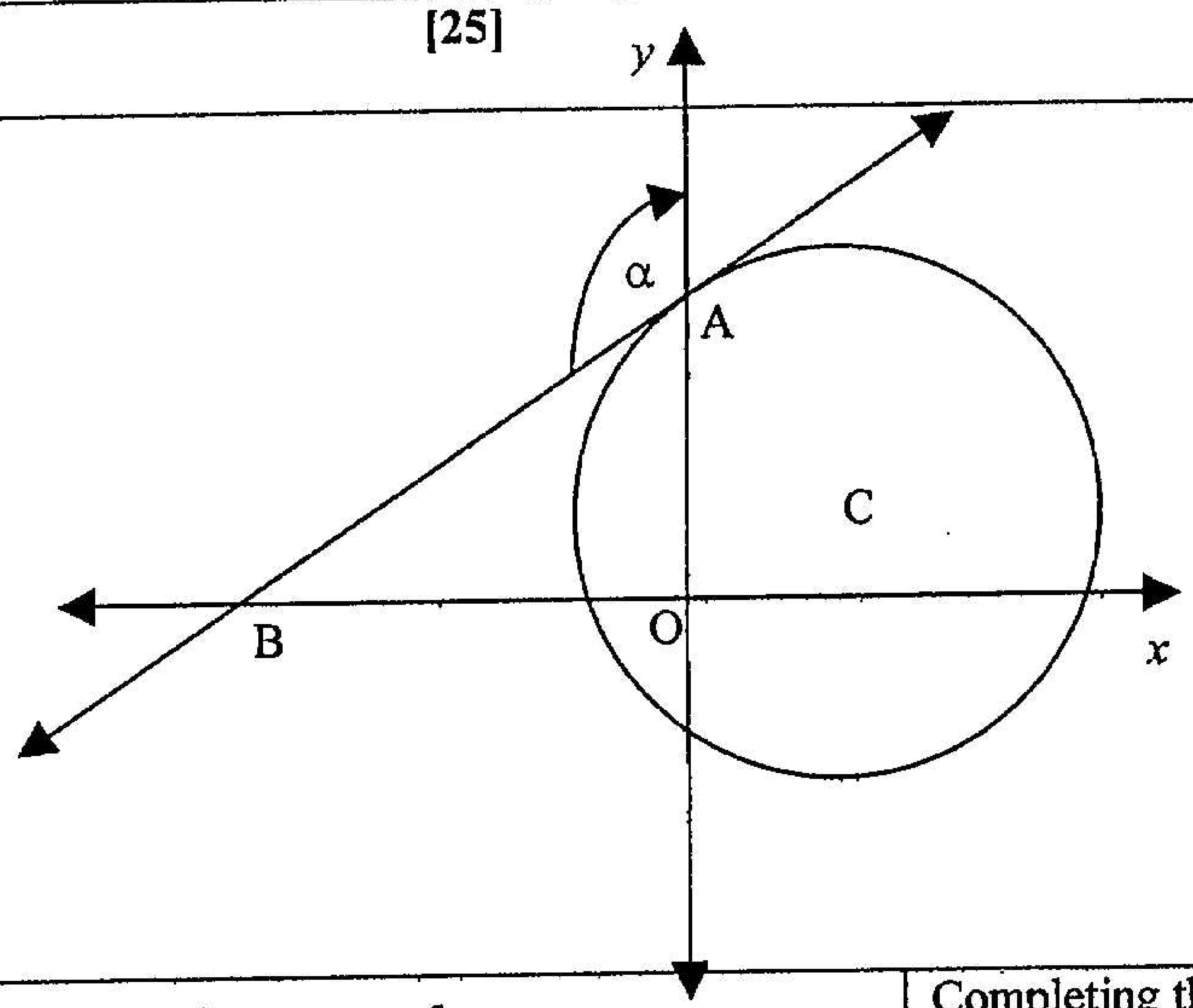
	Equation of AB:	
	$y - y_1 = m_{AB}(x - x_1)$ ✓M	Sustitituting into formula of straight line
	$y - 5 = -\frac{1}{2}(x + 4)$ ✓CA	Simplifying equation correctly
	$y = -\frac{1}{2}x + 3$	
	$2y = -x + 6$ .. ...Eq (2)	
	$y = 2x - 2$ ....(1)	
	Subst. (1) into (2) ✓M	Equating the equations
	$2(2x - 2) = -x + 6$ ✓CA	Simplifying
	$4x - 4 = -x + 6$ $5x = 10$ ✓CA $x = 2$	Calculating x correctly
	$y = 2(2) - 2$ ✓CA $= 2$	Calculating y correctly
	OR	
	Using gradients: Let E be (p ; q) $m_{CE} = 2$	
	$\frac{q + 4}{p + 1} = \frac{2}{1}$ ✓M	Sustitituting into gradient formula
	$q = 2p - 2$ ✓CA ....(1)	Simplifying equation correctly
	$m_{EB} = -\frac{1}{2}$	
	$\frac{q - 1}{p - 4} = -\frac{1}{2}$ ✓CA	Simplifying equation correctly
	$q = -\frac{1}{2}p + 3$ ...... (2)	
	$2p - 2 = -\frac{1}{2}p + 3$ ✓CA	Equating the equations
	$2\frac{1}{2}p = 5$ ✓CA $p = 2$	Calculating x correctly
	Substitute in (1): $q = 2(2) - 2$ $= 2$ ✓CA E (2 ; 2)	Calculating y correctly
	OR	point E need not be in co-ordinate form.

	Let E be (x ; y)	
	$m_{CE} \cdot m_{BE} = -1$	
	$(\frac{y+4}{x+1})(\frac{y\quad 1}{x\quad 4}) = \quad 1$ ✓M	Sustitituting into gradient formula
	$\frac{y^2+3y\quad 4}{x^2\quad 3x\quad 4} = \quad 1$	
	$y^2 + 3y - 4 = -x^2 + 3x + 4$ ✓CA	Simplifying equation correctly
	But $y = 2x - 2$	
	$(2x-2)^2 + 3(2x-2) - 4 = -x^2 + 3x + 4$ ✓CA	Substituting Eq. 1 into Eq. 2
	$4x^2 - 8x + 4 + 6x - 6 - 4 = -x^2 + 3x + 4$	
	$5x^2 - 5x - 10 = 0$	Simplifying
	$x^2 - x - 2 = 0$ ✓CA	
	$(x-2)(x+1) = 0$	
	$x = 2$ ✓CA or $x = -1$	Calculating x correctly
	$y = 2$ ✓CA or $y = -4$	Calculating y correctly
	$\therefore$ E $(2;2)$	point E need not be in co-ordinate form.
	OR	
	$CE^2 + EB^2 = CB^2$ ✓M	Sustitituting into formula of straight line
	$(x+1)^2 + (y+4)^2 + (x-4)^2 + (y-1)^2 = (-1-4)^2 + (-4-1)^2$	
	$x^2 + 2x + 1 + y^2 + 8y + 16 + x^2 - 8x + 16 + y^2 - 2y + 1 = 50$	Simplifying equation correctly
	$2x^2 - 6x + 2y^2 + 6y - 16 = 0$ ✓CA	
	$x^2 - 3x + y^2 + 3y - 8 = 0$	
	Substitute $y = 2x - 2$: ✓CA	
	$x^2 - 3x + (2x-2)^2 + 3(2x-2) - 8 = 0$	Substituting Eq. 2 into Eq. 1
	$x^2 - 3x + 4x^2 - 8x + 4 + 6x - 6 - 8 = 0$	
	$5x^2 - 5x - 10 = 0$ ✓CA	Simplifying
	$x^2 - x - 2 = 0$	
	$(x-2)(x+1) = 0$	
	$x = 2$ ✓CA or $x = -1$	Calculating x correctly
	$y = 2$ ✓CA	Calculating y correctly
		point E need not be in co-ordinate form.
	OR	

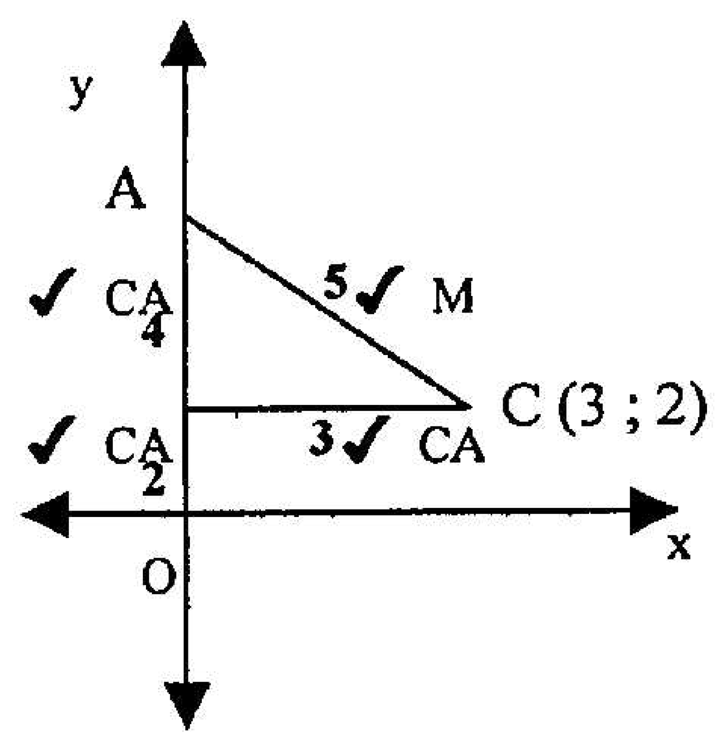
	OR $\Delta BED \equiv \Delta BEC$ $DB^2 = CB^2$ ✓M $(x-4)^2 + (y-1)^2 = (-1-4)^2 + (-4-1)^2$ ✓CA $x^2 - 8x + 16 + y^2 - 2y + 1 = 50$ $x^2 - 8x + y^2 - 2y - 33 = 0$ Substitute $y = 2x - 2$: ✓CA $x^2 - 8x + (2x-2)^2 - 2(2x-2) - 33 = 0$ $x^2 - 8x + 4x^2 - 8x + 4 - 4x + 4 - 33 = 0$ $5x^2 - 20x - 25 = 0$ $x^2 - 4x - 5 = 0$ $(x-5)(x+1) = 0$ $x = 5$ or $x \neq -1$ ✓CA $y = 8$ Hence E $\left(\frac{-1+5}{2} ; \frac{-4+8}{2}\right)$ E $(2 ; 2)$ ✓CA ✓CA (6)	Recognising DB = CB Simplifying equation correctly Substituting Eq. 2 into Eq. 1 Calculating co–ordinates of D Calculating x correctly Calculating y correctly point E need not be in co-ordinate form.
1.1.3	$x_E = \frac{x_D + x_C}{2}$ $y_E = \frac{y_D + y_C}{2}$ $x_D = 2x_E - x_C = 2(2) - (-1) = 5$ ✓CA $y_D = 2y_E - y_C = 2(2) - (-4) = 8$ ✓CA **or** by inspection D(5 ; 8) $m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-4-5}{-1-(-4)}$ $= \frac{-9}{3} = -3$ ✓A $\therefore y - y_1 = m(x - x_1)$ $y - 8 = -3(x-5)$ ✓CA ✓CA $y = -3x + 15 + 8$ $y = -3x + 23$ ✓CA **or** $y = mx + c$ $y = -3x + c$ ✓CA ✓CA $8 = -3(5) + c$ $y = -3x + 23$ ✓CA (6)	Calculating x_D correctly Calculating y_D correctly Finding the gradient correctly at any of the steps. Substituting co-ordinates of D and gradient Simplifying correctly to an equivalent form.

1.2	$m_{CB} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-4-1}{-1-4}$ $= \frac{-5}{-5} = 1$ ✓A $m_{altitude} = -1$ ✓CA Equat. alt. from A to BC $\therefore\ y - y_1 = m(x - x_1)$ **or** $y = mx + c$ $y - 5 = -1(x+4)$ ✓A $y = -x + c$ ✓A $y = -x - 4 + 5$ $5 = -(-4) + c$ $y = -x + 1$ $y = -x + 1$ $\therefore$ x- intercept = 1 ✓CA and x- intercept of CD = 1 (from 1.1.1) ✓CA $\therefore$ x-intercept lies on the altitude ✓CA	Finding the gradient correctly at any of the steps. Deducing the gradient of the altitude Substituting co-ordinates of A Determining x-intercept of the altitude Determining x-intercept of CD A justified conclusion
	OR x-intercept CD = 1 ✓CA Let F be the point (1;0) ✓M $m_{AF} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{CB} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{5-0}{-4-1}$ $= \frac{-4-1}{-1-4}$ $= \frac{5}{-5} = -1$ ✓CA $= \frac{-5}{-5} = 1$ ✓A $m_{AF} \cdot m_{BC} = -1$ ✓CA $\therefore$ x-intercept lies on the altitude. ✓CA (6)	Determining x-intercept of CD Writing down co-ordinates of F Finding the gradient of AF correctly at any of the steps. Finding the gradient of CB correctly at any of the steps. Mutiplying the gradients A justified conclusion Yes / no only – no marks

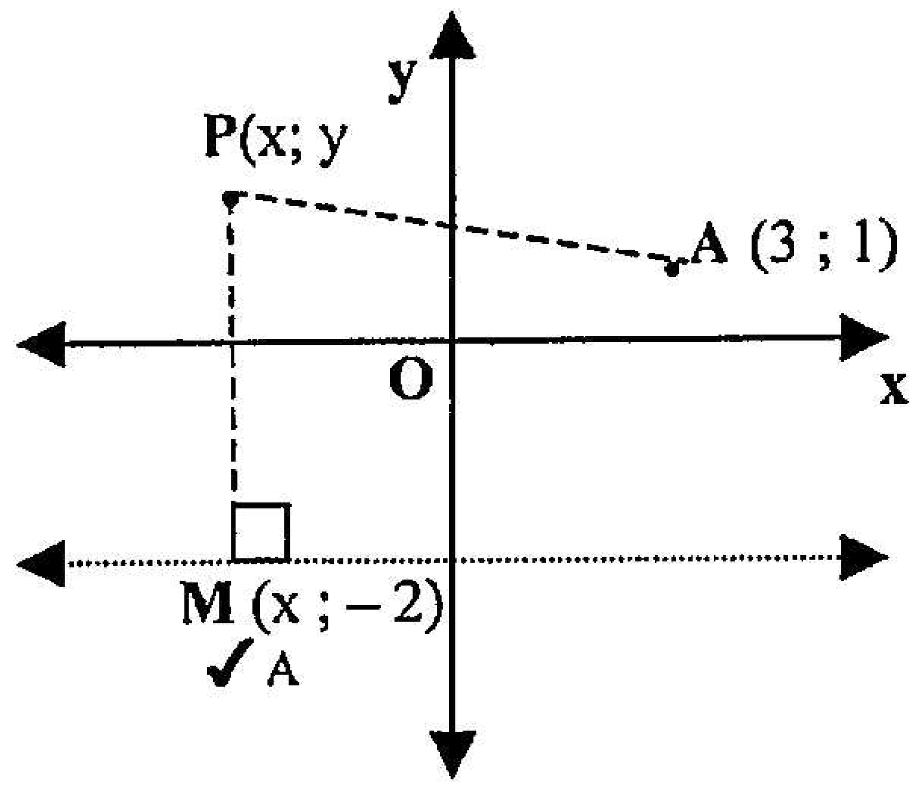
Question 2 [25]



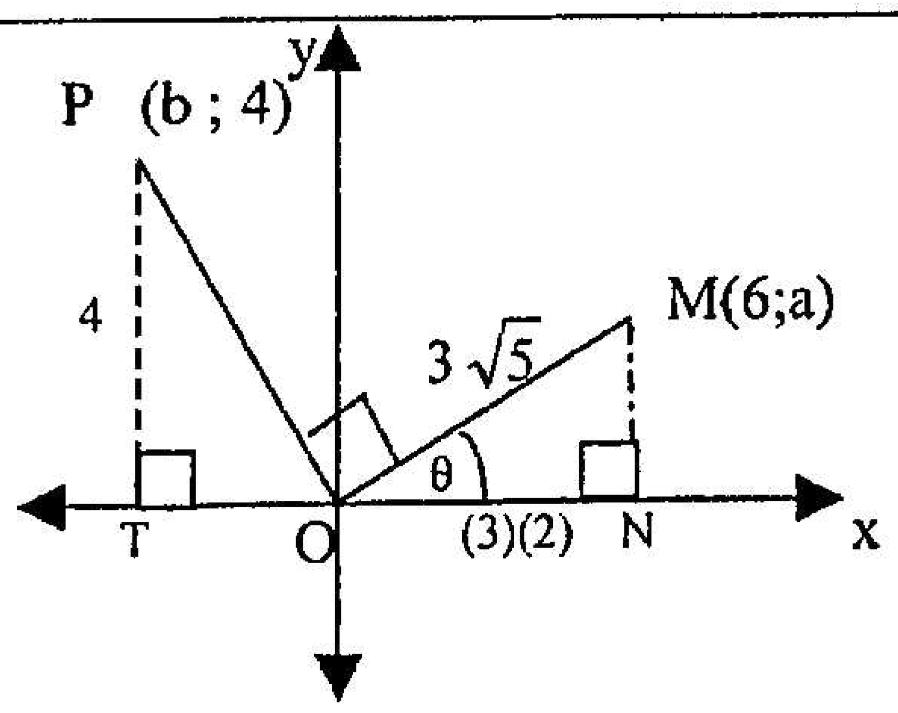
2.1.1	✓A ✓A ✓A $x^2 - 6x + 9 + y^2 - 4y + 4 = 12 + 13$ $(x-3)^2 + (y-2)^2 = 25$ ✓CA ✓CA C (3 ; 2) (5)	Completing the square of x correctly Completing the square of y correctly Adding correctly to RHS of equation x-value of C y-value of C Correct answer only – full marks C need not be in co–ordinate form
2.1.2	For A, $x = 0$ ✓M $y^2 - 4y - 12 = 0$ ✓A $(y-6)(y+2) = 0$ $y = 6$ or $y = -2$ ✓CA A (0 ; 6) ✓CA $m_{AC} = \frac{6-2}{0-3}$ $= -\frac{4}{3}$ ✓A $m_{AB} = \frac{3}{4}$ ✓CA $y = \frac{3}{4}x + 6$ ✓M ✓CA **OR**	Substituting x = 0 in given equation Simplifying equation Solving for y Substitution of appropriate value of A Finding the gradient of AC correctly at any of the steps. Determining the gradient of the tangent Substitution into appropriate formula Correct substitution

	OR $(9) + (y-2)^2 = 25$ ✓M $(y-2)^2 = 16$ ✓A $y - 2 = \pm 4$ ✓CA $y = -2$ (N/A) or $y = 6$ A (0 ; 6) $m_{AC} = \frac{6-2}{0-3}$ ✓CA $= -\frac{4}{3}$ ✓A $m_{AB} = \frac{3}{4}$ ✓ CA $y = \frac{3}{4}x + 6$ (3✓M, 6✓ CA) **OR**	Substituting x = 0 into equation from 2.1.1 Simplifying equation Solving for y Substitution of appropriate value of A Finding the gradient of AC correctly at any of the steps. Determining the gradient of the tangent Substitution into appropriate formula Correct substitution
	 ∴ A (0 ; 6) $m_{AC} = -\frac{4}{3}$ ✓ CA $m_{AB} = \frac{3}{4}$ ✓ CA $y = \frac{3}{4}x + 6$ (3✓ CA) ✓ CA **OR**	Method mark, use of Pythagoras See sketch for marks radius 5; $x_C = 3$; $y_C = 2$ A 4 vertical units higher than C Finding the gradient of AC correctly Determining the gradient of the tangent Substitution into appropriate formula Correct substitution

	OR For A, $x = 0$ ✓M $y^2 - 4y - 12 = 0$ ✓A $(y-6)(y+2) = 0$ $y = 6$ or $y = -2$ ✓CA $A(0;6)$ $x^2 - 6x + y^2 - 4y = 12$ Gradient of circle = $\frac{dy}{dx}$ $2x - 6 + 2y\frac{dy}{dx} - 4\frac{dy}{dx} = 0$ ✓M $\frac{dy}{dx}(2y-4) = 6 - 2x$ $\frac{dy}{dx} = \frac{6-2x}{2y-4}$ ✓CA at A (0;6) $m = \frac{6}{8} = \frac{3}{4}$ ✓CA $y = \frac{3}{4}x + 6$ ✓CA ✓CA (8)	Substituting $x = 0$ in given equation Simplifying equation Solving for y Finding $\frac{dy}{dx}$ (Using implicit differentiation) Writing $\frac{dy}{dx}$ as the subject of the formula Finding the gradient of AB correctly at any of the steps. Substitution into appropriate formula Correct substitution
2.1.3	$\tan A\hat{B}O = \frac{3}{4}$ ✓M $A\hat{B}O = 36{,}9°$ ✓CA $\therefore \alpha = 90° + 36{,}9°$ $= 126{,}9°$ ✓CA (3)	Substituting correct gradient into correct inclination formula. Calculating ∠ of inclination Calculating α correctly Penalty of 1 mark for incorrect rounding off

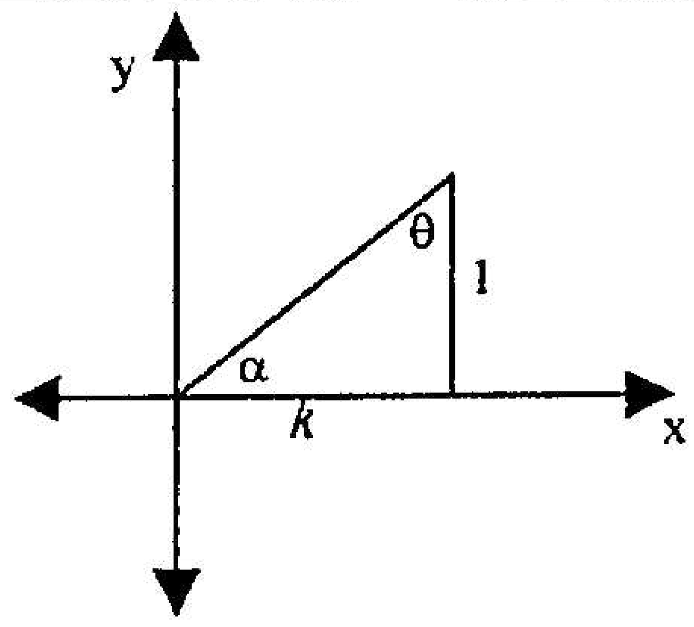
2.2.1	 $AP = PM$ ✓M $AP^2 = PM^2$ ✓A ✓CA $(x - 3)^2 + (y - 1)^2 = (x - x)^2 + (y - (-2))^2$ $x^2 - 6x + 9 + y^2 - 2y + 1 = 0 + y^2 + 4y + 4$ ✓CA $-6y = -x^2 + 6x - 6$ ✓CA **OR** $y = \frac{1}{6}x^2 - x + 1$ **OR** $(x - 3)^2 - 3 = 6y$ (6)	Determining co-ordinates of M correctly. Equating the two lengths Correct use of the distance formula LHS , RHS Simplification Final form of the equation. Any accepted form of the equation co-ordinates of M incorrect - max $(\frac{5}{6})$
2.2.2	a parabola ✓CA or parabolic ✓CA (1)	Shape depends on 2.2.1
2.2.3	$y = \frac{1}{6}(x^2 - 6x + 9 - 9 + 6)$ ✓M $= \frac{1}{6}(x - 3)^2 - \frac{1}{2}$ minimum value is $-\frac{1}{2}$ ✓CA **or** $\frac{dy}{dx} = \frac{1}{3}x - 1$ ✓M $x = 3$ $y = \frac{1}{6}(3)^2 - 3 + 1$ minimum value is $-\frac{1}{2}$ ✓CA or min. $= \frac{4(\frac{1}{6})(1) - (-1)^2}{4(\frac{1}{6})}$ ✓M $= \frac{-1}{2}$ ✓CA (2)	Completing the square Answer from correct manipulation Differentiating Answer from correct manipulation Substitution into minimumvalue of y – formula. Answer from correct manipulation ANSWER ONLY : FULL MARKS Answer depends on 2.2.1

Question 3 [23]

3.1.1	 	
	$\cos\theta = \frac{2}{\sqrt{5}}$ ✓ A	Writing cos θ as subject or in sketch
	$r = \sqrt{5}$; $x = 2$ $y = \sqrt{(\sqrt{5})^2 - 2^2} = 1$ ✓ M	Calculating value of y from cos θ
	$x_M = 3(2) = 6 \quad y_M = a = 3(1) = 3$ ✓ CA	Calculating value of a
	OR $\cos\theta = \frac{2}{\sqrt{5}}$ ✓ A	Writing cos θ as subject or in sketch
	$\tan\theta = \frac{a}{6} = \frac{1}{2}$ ✓ M $\therefore$ $a = 3$ ✓ CA (3)	Correct use of tan θ Calculating value of a
3.1.2	$-\tan(90° - \theta) = \frac{4}{b}$ ✓ M	Use of tan (90° – θ)
	$-\cot\theta = \frac{4}{b}$ ✓ A	Writing in terms of co-function
	$-\frac{2}{1} = \frac{4}{b}$ ✓ CA	Substitution
	$b = -\frac{4}{2} = -2$ ✓ CA	Simplification
	OR In Δ POM	
	$PM^2 = PO^2 + OM^2$ ✓ M	Use of pythagoras
	$(b-6)^2 + (4-3)^2 = b^2 + 4^2 + 6^2 + 3^2$ ✓ A	Substitution into the distance formula
	$b^2 - 12b + 36 + 1 = b^2 + 16 + 36 + 9$ ✓ CA	Multiplying out
	$-12b = 24$ $b = -2$ ✓ CA	Simplification
	OR	
	Δ PTO ||| Δ ONM ✓ M	Use of ||| Δ's
	$\frac{4}{\|b\|} = \frac{2}{1}$ ✓ A	Substitution of ratios
	$-2b = 4$ ✓ CA	Cross multiplying
	$b = -2$ ✓ CA	Simplification
	OR	

	OR $\cos(\theta + 90°) = -\sin\theta$ ✓ M	Use of co-function
	$\frac{1}{\sqrt{5}} = \frac{b}{\sqrt{b^2 + 16}}$ ✓ A	Substitution
	$\frac{1}{5} = \frac{b^2}{b^2 + 16}$ ✓ CA	Squaring both sides
	$5b^2 = b^2 + 16$	
	$4b^2 = 16$ $b = -2$ ✓ CA	Simplification
	OR	
	$\sin(90° - \theta) = \frac{4}{OP}$ ✓ M	Use of $\sin(90° - \theta)$
	$\cos\theta = \frac{4}{OP}$ ✓ A	Use of co-function
	$\frac{2}{\sqrt{5}} = \frac{4}{OP}$ ✓ CA	
	$OP = 2\sqrt{5}$	Algebraic manipulation
	$b^2 = 20 - 16$	
	$b = -2$ ✓ CA	Simplification
	OR	
	$m_{OM} = \frac{1}{2}$ ✓ M	Use of gradient of OM
	$m_{OP} = -2$ ✓ A	Writing down the gradient of the perpendicular
	$\frac{4}{b} = -2$ ✓ CA	Substitution
	$b = -2$ ✓ CA	Simplification
	OR	
	$m_{OM} = \frac{1}{2}$ ✓ M	
	$m_{OP} = -2$ ✓ A	
	$y = -2x$ ✓ CA	
	$4 = -2(b)$	
	$b = -2$ ✓ CA	
	(4)	

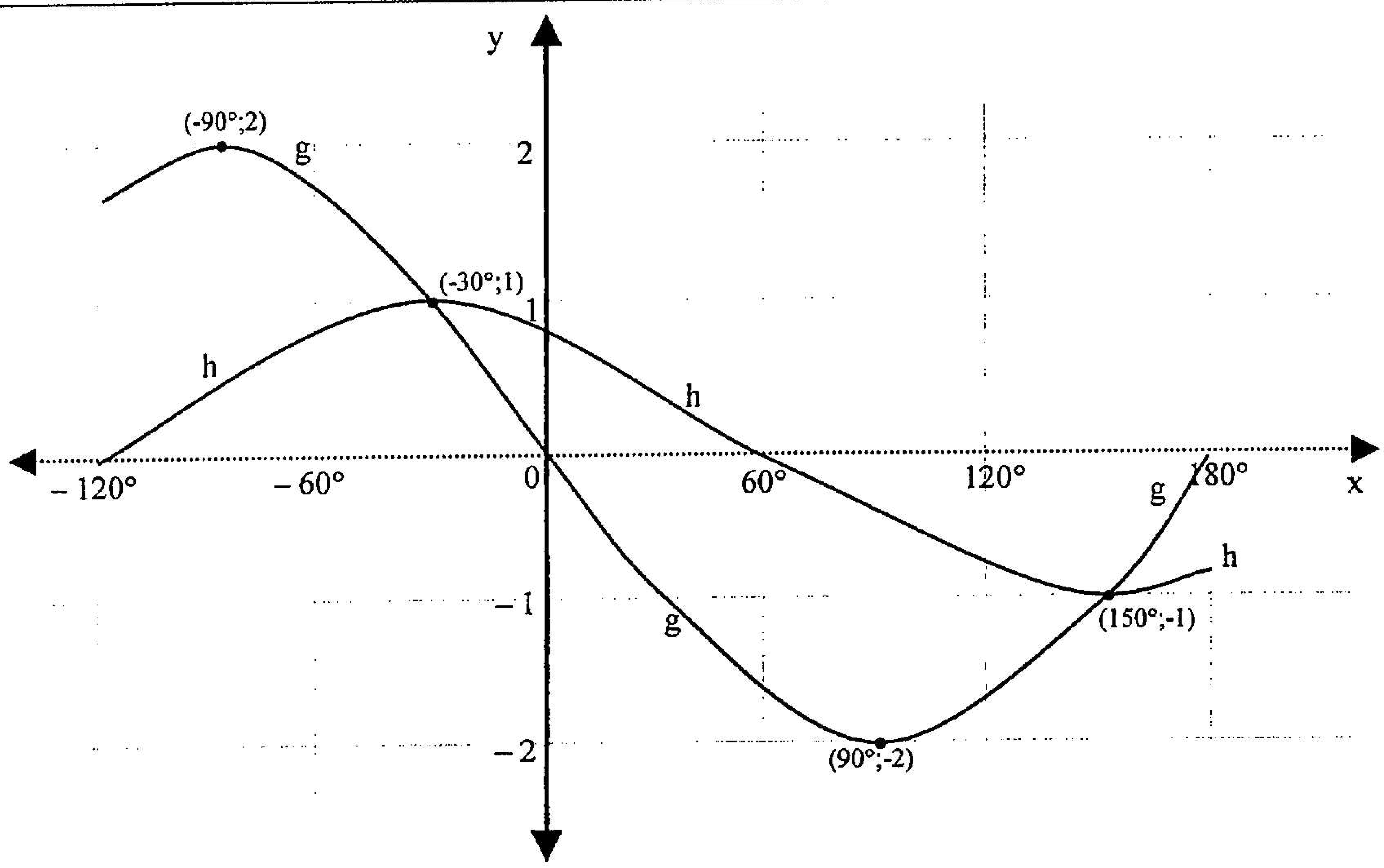
3.2	$\dfrac{(\tan(-420°))(\cos 156°)}{(\sin 492°)(\sec 294°)}$	
	$= \dfrac{(-\tan 60°)(-\cos 24°)}{(\sin 48°)(\sec 66°)}$ ✓A ✓A ✓A ✓A	Reducing correctly, with correct signs
	$= \dfrac{(-\sqrt{3})(-\cos 24°)}{(2\sin 24°.\cos 24°)(\operatorname{cosec} 24°)}$ ✓A ✓A ✓A	Substitution of tan 60° Expansion of sin2A Use of co-function ratio
	$= \dfrac{\sqrt{3}}{2\sin 24°.\dfrac{1}{\sin 24°}}$ ✓CA	Application of identities
	$= \dfrac{\sqrt{3}}{2}$ ✓CA	Simplification
	OR	
	$\dfrac{(\tan(-420°))(\cos 156°)}{(\sin 492°)(\sec 294°)}$	
	$= \dfrac{(-\tan 60°)(-\cos 24°)}{(\sin 132°)(\sec 66°)}$ ✓A ✓A ✓A ✓A	Reducing correctly, with correct signs
	$= \dfrac{(-\sqrt{3})(-\sin 66°)}{(2\sin 66°.\cos 66°)(\sec 66°)}$ ✓A ✓A ✓A	Substitution of tan 60° Expansion of sin2A Use of co-function ratio
	$= \dfrac{\sqrt{3}}{2\cos 66°.\dfrac{1}{\cos 66°}}$ ✓A	Application of identities
	$= \dfrac{\sqrt{3}}{2}$ ✓A	Simplification
	(9)	

3.3	$\cot\alpha = k$	
		
	$r^2 = x^2 + y^2$ $= k^2 + 1^2$ $r = \sqrt{k^2+1}$ ✓ A	Application of pythagoras
	LHS: $\dfrac{\text{cosec}^2(180^\circ+\alpha).\cos(\theta-720^\circ)}{\cos(90^\circ-\theta)}$	
	$= \dfrac{\text{cosec}^2\alpha\ .\ \cos\theta}{\sin\theta}$ ✓ A ✓ A ✓ A	Correct reduction
	$= (\text{cosec}^2\alpha)(\cot\theta)$ ✓ A	Use of identities
	$= (k^2+1).(\frac{1}{k})$ ✓ CA ✓ CA	Substitution
	$= \dfrac{k^2+1}{k} = k + \dfrac{1}{k}$	
	or	
	$r^2 = x^2 + y^2$ $= k^2 + 1^2$ $r = \sqrt{k^2+1}$ ✓ A	Application of pythagoras
	$\alpha + \theta = 90^\circ$ $\alpha = 90^\circ - \theta$	
	$\dfrac{\text{cosec}^2(180^\circ+\alpha).\cos(\theta-720^\circ)}{\cos(90^\circ-\theta)}$	
	$= \dfrac{\text{cosec}^2\alpha\ .\ \sin\alpha}{\cos\alpha}$ ✓ A ✓ A ✓ A	Correct reduction
	$= \text{cosec}\,\alpha\ .\ \sec\alpha$ ✓ A	Use of identities
	$= (\sqrt{k^2+1})(\dfrac{\sqrt{k^2+1}}{k})$ ✓ CA ✓ A	Substitution
	$= \dfrac{k^2+1}{k}$ (7)	

	OR	
	$\dfrac{\operatorname{cosec}^2\alpha \, . \, \cos\theta}{\sin\theta}$ ✓A ✓A ✓A	Correct reduction
	$= \sec^2\theta.\cot\theta$ ✓A	
	$= (1 + \tan^2\theta).\cot\theta$ ✓A	Use of identities
	$= (1 + \dfrac{k^2}{1}).\dfrac{1}{k}$ ✓CA ✓CA	Substitution
	$= \dfrac{1}{k} + k$	
	OR	
	$r^2 = x^2 + y^2$ $= k^2 + 1^2$ $r = \sqrt{k^2 + 1}$ ✓A	Application of pythagoras
	$\alpha + \theta = 90°$ $\alpha = 90° - \theta$	
	$\dfrac{\operatorname{cosec}^2\alpha \, . \, \cos\theta}{\sin\theta}$ ✓A ✓A ✓A	Correct reduction
	$= \sec^2\theta.\cot\theta$	
	$= \dfrac{1}{\cos^2\theta}.\dfrac{\cos\theta}{\sin\theta}$ ✓A	
	$= \dfrac{1}{\sin\theta.\cos\theta} = \operatorname{cosec}\theta.\sec\theta$	Use of identities
	$= \dfrac{\sqrt{k^2+1}.\sqrt{k^2+1}}{k} = \dfrac{k^2+1}{k}$ ✓CA ✓CA	Substitution
	$= \dfrac{1}{k} + k$	

Question 4 [24]		
4.1	$\cos(x + 30^\circ) = -2\sin x$	
	✓ M $\cos x \cdot \cos 30^0 - \sin x \cdot \sin 30^\circ + 2\sin x = 0$	Expanding correctly
	$\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x + 2\sin x = 0$ ✓ A	Substitution of special angle ratios
	$\frac{\sqrt{3}}{2}\cos x + \frac{3}{2}\sin x = 0$ ✓ CA	Simplifying
	$\sin x = -\frac{\sqrt{3}}{3}\cos x$	
	✓ CA $\tan x = -\frac{\sqrt{3}}{3} = -\frac{1}{\sqrt{3}}$	Writing in terms of tan x
	ref. angle $= 30^\circ$ ✓ CA	Finding the reference angle
	✓ CA ✓ A $x = 150^\circ + n.180^\circ, n \in Z$	Solution ; General form of solution
	OR	
	✓ CA ✓ A $x = -30^\circ + n.180^\circ, n \in Z$	
	OR	
	✓ CA ✓ A $x = 150^\circ + n.360^\circ$ / $330^\circ + n.360^\circ, n \in Z$ (7)	Both forms must be given

4.2



for each graph:

x-intercepts ✓ A ✓ A

y – intercept ✓ A ✓ A

shape ✓ A ✓ A (g must past through (0 ; 0))

turning points ✓ A ✓ A (coordinates need not be written in)

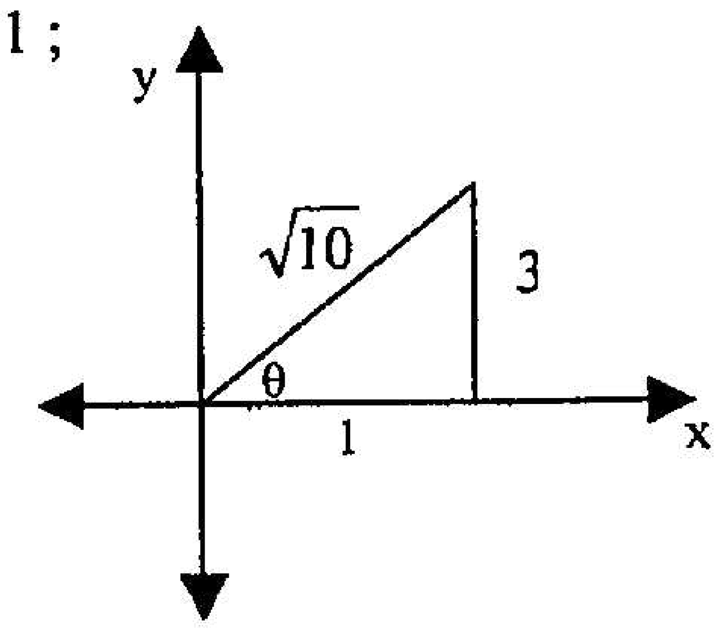
(8)

Penalty of 1 if graphs drawn out of the domain

No penalty if y–axis not to scale, but graph understood

No penalty if y-intercept not indicated on the graph

4.3.1	$2\sin x + \cos x . \cos 30° \geq \sin x . \sin 30°$ $\cos x \cos 30° - \sin x \sin 30° \geq -2\sin x$ ✓ M $\cos(x + 30°) \geq -2\sin x$ ✓ A ✓ CA ✓ CA ✓ A $x \in [-30° ; 150°]$ (Notation) **OR** ✓ CA ✓ CA ✓ A $-30° \leq x \leq 150°$ (Notation) (5)	Rewriting inequality to resemble graphs Writing LHS as cos (x + 30°) Correct notation; correct end-points of interval Answer only full marks
4.3.2	✓ CA ✓ CA ✓ A $x \in [-120° ; -90°)$ (Notation) ✓ CA ✓ CA ✓ A **OR** $-120° \leq x < -90°$ (Notation) (3)	Correct notation; correct end-points of interval If answer is NO : 1 mark only
4.4.	0 ✓ CA (1)	Answer from graph Do not penalise if written as (0 ; 0)

Question 5 [19]		
5.1.1	$\cos(A - B) = \cos A . \cos B + \sin A . \sin B$ ✓ A (1)	Correct expansion
5.1.2	$\sin(A - B) = \cos[90° - (A - B)]$ $= \cos[(90° - A) - (-B)]$ ✓ M $= \cos(90° - A).\cos(-B) + \sin(90° - A) . \sin(-B)$ ✓ A $= \sin A . \cos B - \cos A . \sin B$ ✓ A (3)	Correct use of co-function formula Correct expansion Accurate answer only – 1 mark + B used penalty of 1 mark
5.2.1	$\sec\theta = \sqrt{10}$, $\therefore r = \sqrt{10}$; $x = 1$;  ✓ A ✓ A $y^2 = 10 - 1$, $y = 3$ $\sqrt{10}\sin(A - \theta) = \sqrt{10}(\sin A . \cos\theta - \cos A . \sin\theta)$ ✓ A $= \sqrt{10}(\sin A \frac{1}{\sqrt{10}}$ ✓ A $- \cos A \frac{3}{\sqrt{10}})$ ✓ CA $= \sin A - 3\cos A$ **OR**	Use of Pythagoras or indicated on the sketch Correct calculation of y Correct expansion of sin (A – θ) Substitution of cos θ and sin θ

	OR $\tan\theta = \sqrt{\sec^2\theta - 1}$ ✓ A $= \sqrt{10-1} = 3$ ✓ A $\sqrt{10}\sin(A-\theta)$ $= \sec\theta(\sin A.\cos\theta - \cos A.\sin\theta)$ ✓ A ✓ A $= \sin A - \cos A.\frac{\sin\theta}{\cos\theta}$ $= \sin A - \cos A.\tan\theta$ ✓ CA $= \sin A - 3\cos A$ (5)	Correct ue of identity Correct substitution Correct substitution of $\sec\theta$; correct expansion of $\sin(A-\theta)$ Multplying and simplifying
5.2.2	**DO NOT MARK THIS QUESTION** **GIVE 9 MARKS FOR 5.2.2 ONLY IF CANDIDATES STARTED WITH 5.2.1** $6\cos A + 3 = 2\sin A$ $2(\sin A - 3\cos A) = 3$ ✓ M $(\sin A - 3\cos A) = 1,5$ $\therefore \sqrt{10}\sin(A-\theta) = 1,5$ ✓ M $\sin(A-\theta) = 1,5 \div \sqrt{10} = 0,4743..$ ✓ CA ref angle $= 28,316°$ ✓ CA $A - \theta = 28,316°$ ✓ CA or $A - \theta = 180° - 28,316° = 151,684°$ ✓ CA but $\theta = 71,565°$ ✓ A $\therefore A = 28,316° + 71,565°$. or $151,684° + 71,565°$ $A = 99,9°$ ✓ CA or $-136,8°$ ✓ CA (9)	Rearranging equation and taking out common factor Substituting from 5.2.1 Dividing by $\sqrt{10}$ Calculating the reference angle Writing solution in correct quadrants Calculating θ from $\sec\theta$ Determining values of A If rounding off error occurs ref. angle = 30°. Penalty of 1.

Question 6 [21]

6.1.1

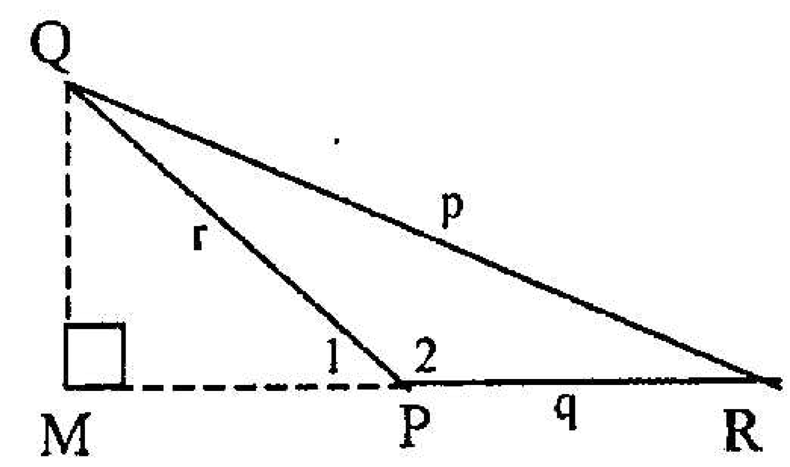
Draw QM ⊥ RP produced. ✓ M

$p^2 = QM^2 + MR^2$ ✓ A
$= QM^2 + (q + MP)^2$ ✓ A
$= QM^2 + q^2 + 2q.MP + MP^2$
$= q^2 + r^2 + 2q.MP \quad (QM^2 + MP^2 = r^2)$ ✓ A

but $\frac{MP}{r} = \cos P_1 = \cos(180° - P_2)$
$= -\cos P_2$ ✓ A

$\therefore MP = -r\cos P$ ✓ A
$\therefore p^2 = q^2 + r^2 - 2qr\cos P$

Or shown on diagram



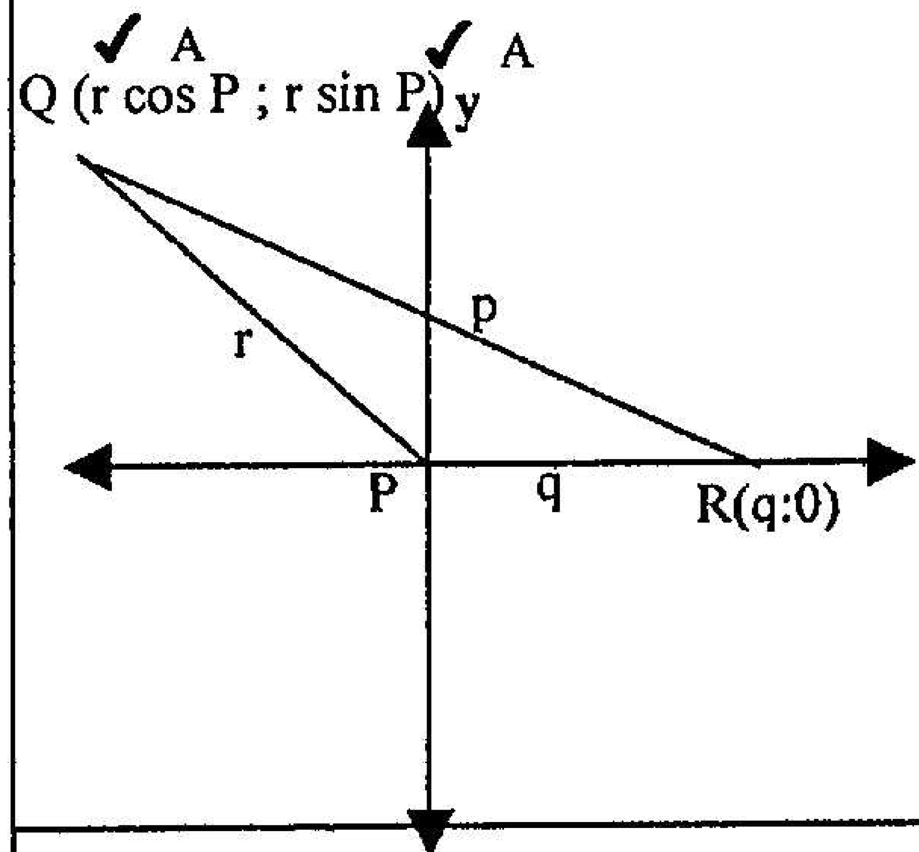
acute angle drawn, penalty 2 marks

OR

Draw Δ PQR with P at the origin and PR on the X-axis.

$p^2 = (r\cos P - q)^2 + (r\sin P)^2$ ✓ M ✓ A (distance formula)
$= r^2\cos^2 P - 2qr\cos P + q^2 + r^2\sin^2 P$ ✓ A
$= r^2(\cos^2 P + \sin^2 P) - 2qr\cos P + q^2$ ✓ A
$= q^2 + r^2 - 2qr\cos P$

(6)



no axis drawn, penalty 1 mark

6.1.2

RHS $\frac{4.\text{area}\,\Delta PQR}{q^2 + r^2 - p^2}$

$= \frac{4.[\frac{1}{2}qr\sin P]}{2qr\cos P}$ ✓ A ✓ A (from 6.1.1)

$= \frac{\sin P}{\cos P}$ ✓ A

$= \tan P = \text{LHS}$

(3)

OR area $\Delta PQR = \frac{1}{2}qr.\sin P$ ✓ A

$\therefore \sin P = \frac{2(\text{area}\,\Delta PQR)}{qr}$

and $\cos P = \frac{q^2 + r^2 - p^2}{2qr}$ ✓ A

$\tan P = \frac{\sin P}{\cos P}$

$= \frac{\frac{2(\text{area}\,\Delta PQR)}{qr}}{\frac{q^2 + r^2 - p^2}{2qr}}$ ✓ A

$= \frac{4(\text{area}\,\Delta PQR)}{q^2 + r^2 - p^2}$

OR

area $\Delta PQR = \frac{1}{2}qr.\sin P$ ✓ A

$\frac{\text{area}\,\Delta PQR}{\cos P} = \frac{\frac{1}{2}qr.\sin P}{\cos P}$

$\frac{2(\text{area}\,\Delta PQR)}{qr.\cos P} = \tan P$ ✓ A

$\frac{2(\text{area}\,\Delta PQR)}{qr(\frac{q^2 + r^2 - p^2}{2qr})} = \tan P$ ✓ A

$\frac{4(\text{area}\,\Delta PQR)}{q^2 + r^2 - p^2} = \tan P$

6.2.1

$\frac{BD}{p} = \cos\alpha$ ✓ A

$BD = p\cos\alpha$ ✓ A (2)

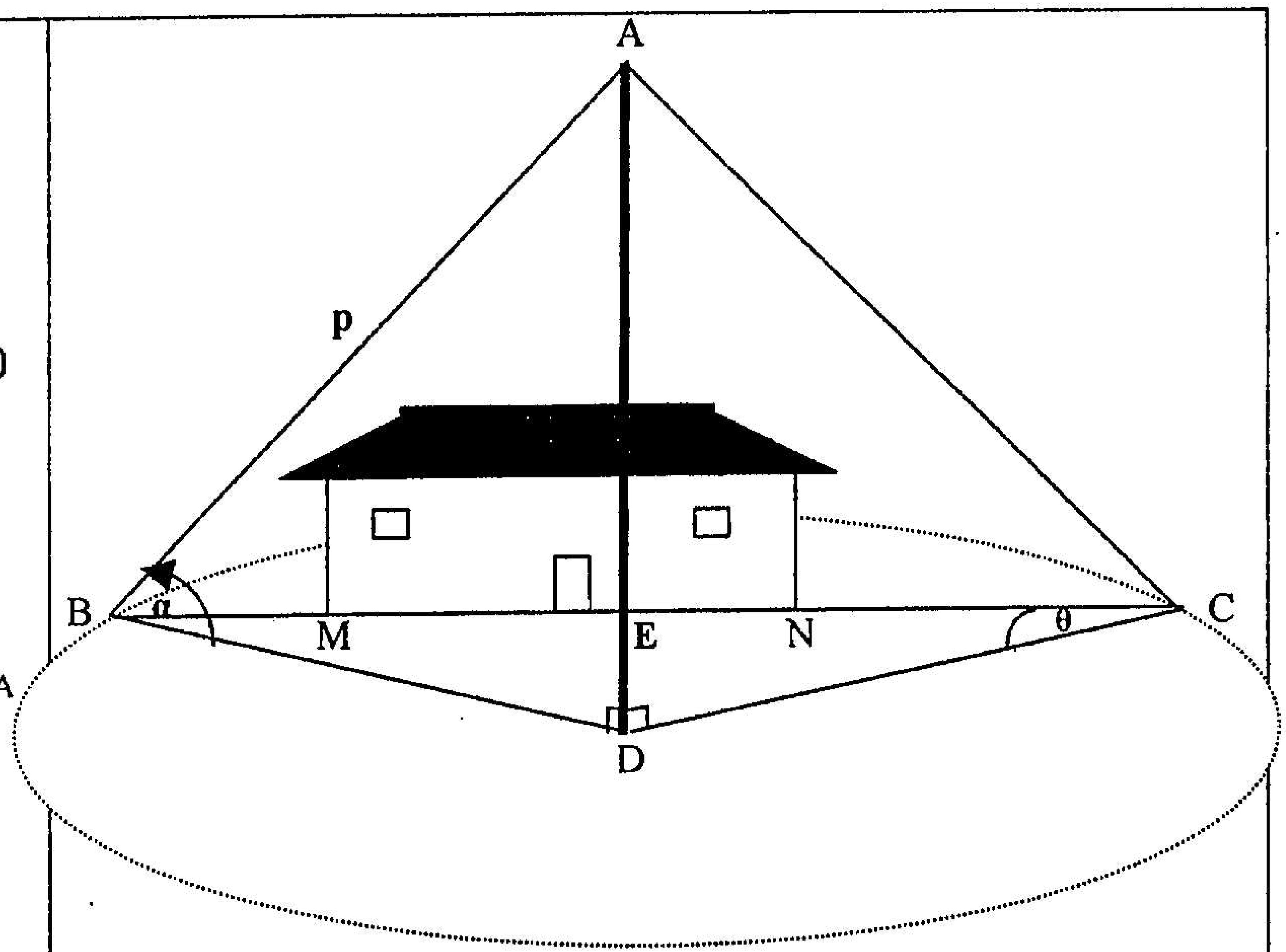
OR

$\frac{BD.}{\sin(90° - \alpha)} = \frac{p}{\sin 90°}$ ✓ A

$BD = \frac{p.\sin(90° - \alpha)}{\sin 90°}$ ✓ A

OR

$BD = p\sin(90° - \alpha)$



answer only – full marks

6.2.2

$B\hat{C}D = \theta$

$B\hat{D}C = 180^0 - 2\theta$

In ΔBCD,

$\frac{BC}{\sin(180° - 2\theta)} = \frac{BD}{\sin\theta}$ ✓ M

✓ A

$BC = \frac{BD.\sin 2\theta}{\sin\theta}$

✓ CA ✓ CA

$= \frac{p\cos\alpha.2\sin\theta\cos\theta}{\sin\theta}$

$= 2p.\cos\alpha.\cos\theta$

OR

$BC^2 = BD^2 + DC^2 - 2BD.DC.\cos(180° - 2\theta)$ ✓ M

$= 2BD^2 + 2BD^2 \cdot \cos 2\theta$ ✓ A

$= 2BD^2(1 + \cos 2\theta)$

$= 2BD^2(2\cos^2\theta)$ ✓ CA

$= 4p^2.\cos^2\alpha.\cos^2\theta$ ✓ CA

$BC = 2p.\cos\alpha.\cos\theta$

OR

$BC = 2BE$ ✓ M

$= 2BD.\cos\theta$ ✓ A ✓ A

$= 2p\cos\alpha.\cos\theta$ ✓ CA (4)

6.2.3

$BC = 2p.\cos\alpha.\cos\theta$

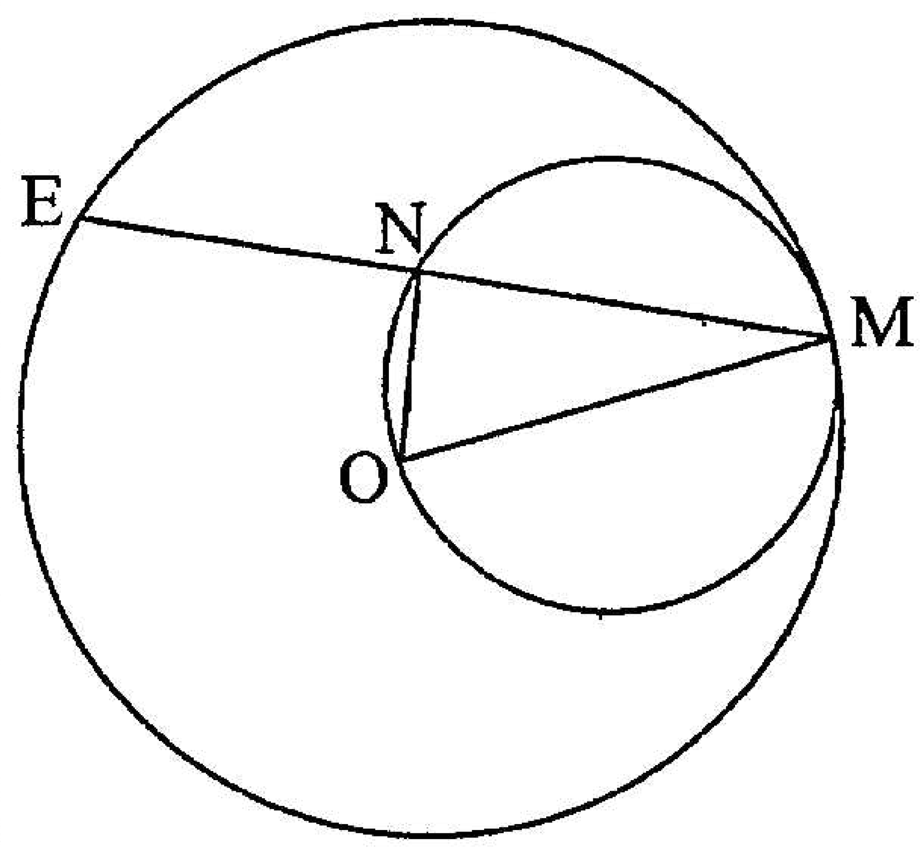
$29{,}5 = 2(21{,}2)(\cos 45°)(\cos\theta)$ ✓ A — Correct substitution

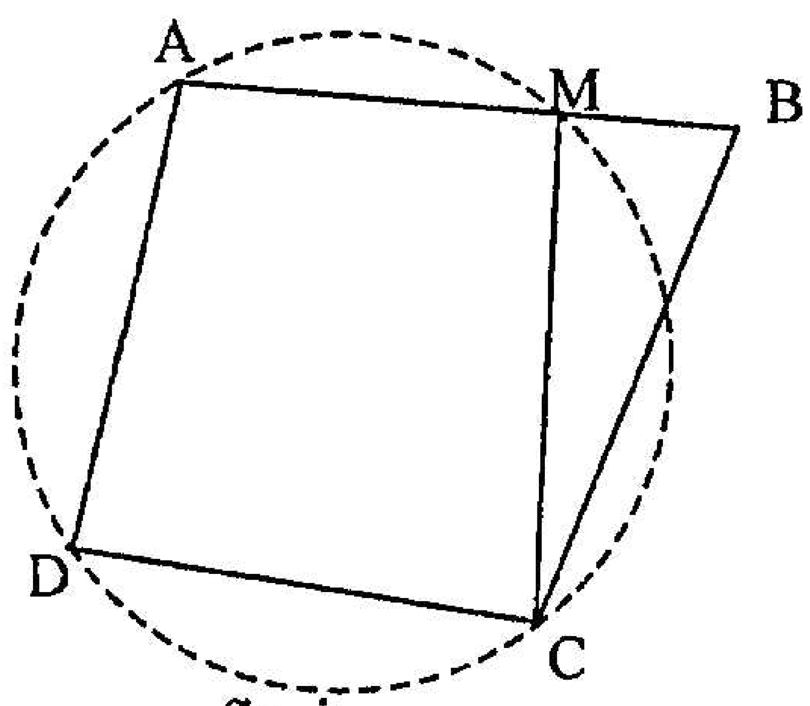
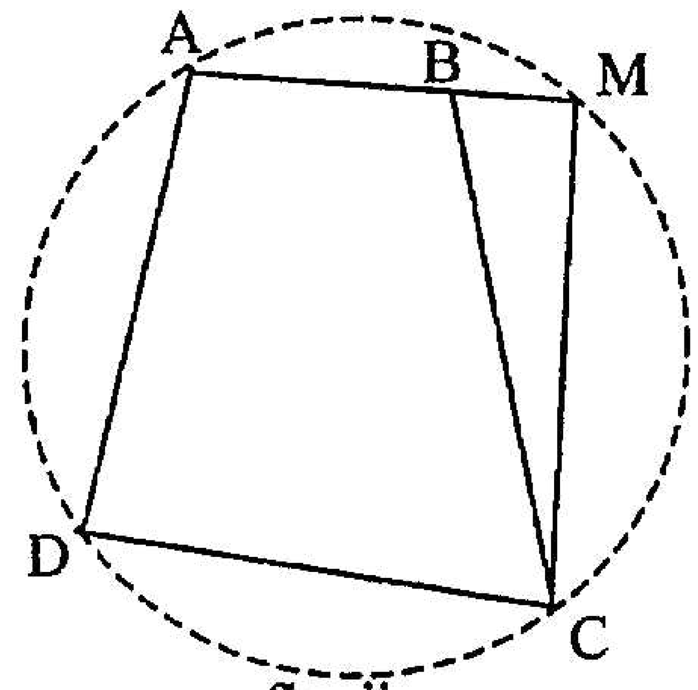
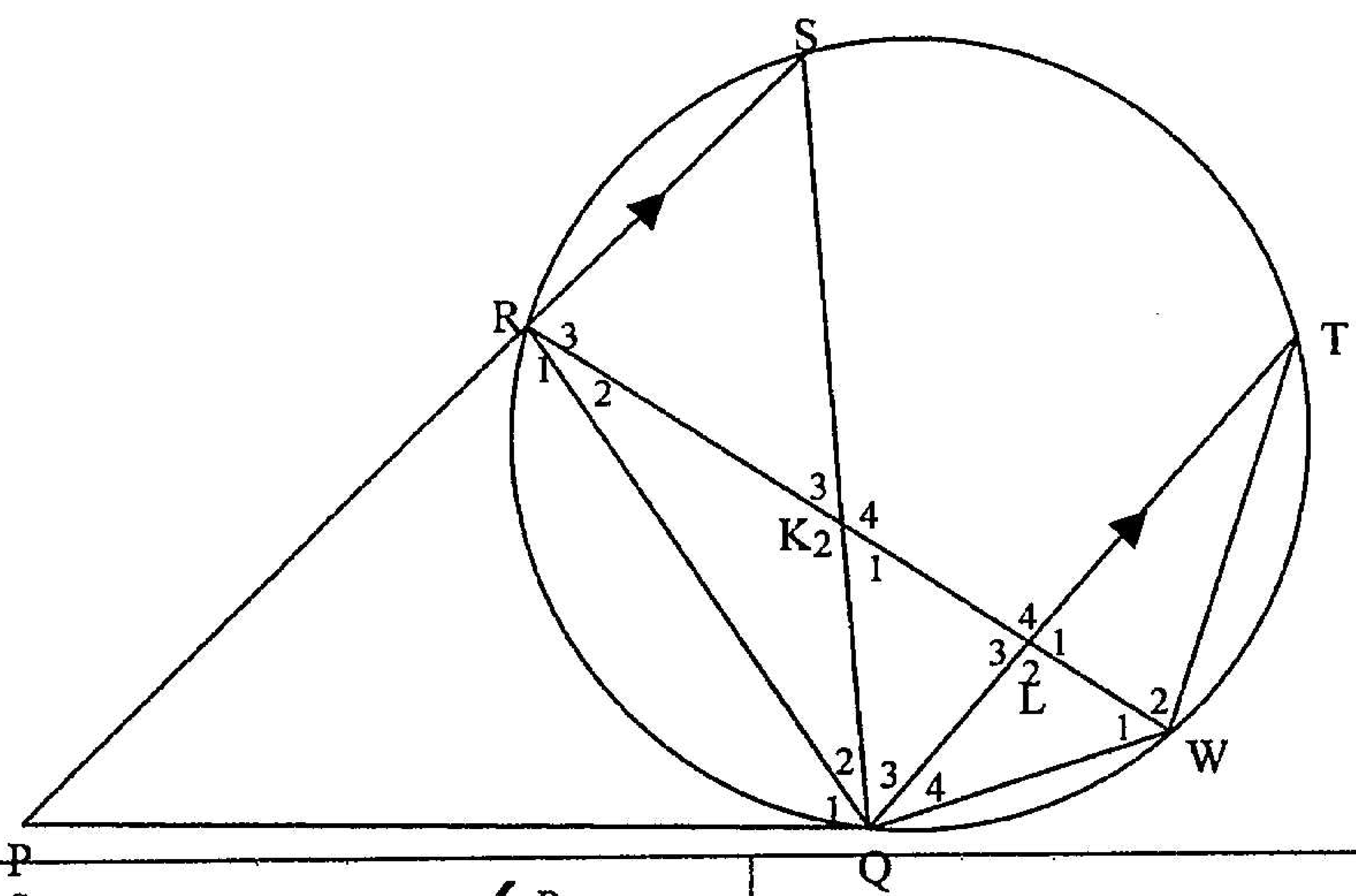
$\cos\theta = \frac{29{,}5}{2(21{,}2).\cos 45}$ ✓ A — Manipulation

$\theta = 10{,}3°$ ✓ CA

incorrect rounding off – penalty 1 mark

OR

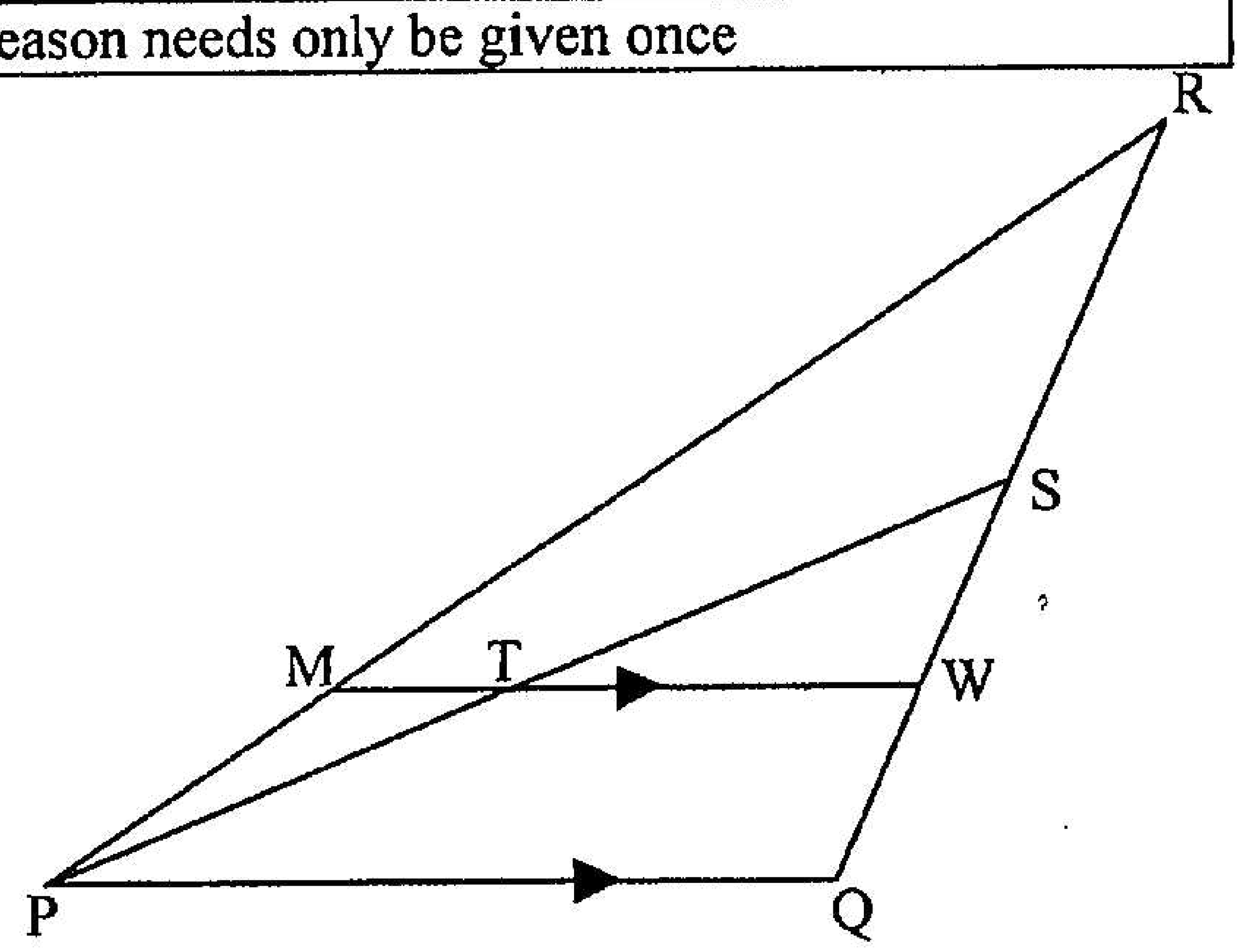
	OR BD = p cos α $\cos\theta = \dfrac{\frac{1}{2}BC}{BD}$ ✓ A $= \dfrac{\frac{1}{2}(29,5)}{(21,2)(\cos 45°)}$ ✓ A θ = 10,3° ✓ CA (3)	Manipulation Correct substitution Correct ∠
6.2.4	Let ED be the shortest distance from D to BC. θ = 10,3° $\frac{1}{2}BC = 14,7$ $\dfrac{ED}{\frac{1}{2}BC} = \tan\theta$ ✓ A ED = 14,75 tan 10,3 ✓ CA = 3 m ✓ CA (3)	**OR** $\sin\theta = \dfrac{ED}{BD}$ ✓ A ED = BD.sin θ = p .cos α. sin $10,3^0$ = 21,2 x cos 45^0 sin $10,3^0$ ✓ CA = 3 m ✓ CA no penalty for incorrect rounding off
Question 7		**[27]**
7.1	✓ S/R $O\hat{N}M = 90°$ (∠ in a semicircle) ✓ S/R EN = NM (line from centre ⊥ chord) $NM = \frac{1}{2}(2x^2 - 2) = x^2 - 1$ units ✓ S $OM^2 = NM^2 + NO^2$ (Pythagoras) $= (x^2 - 1)^2 + (2x)^2$ ✓ S $= x^4 - 2x^2 + 1 + 4x^2$ ✓ CA $= x^4 + 2x^2 + 1$ **OR** $(x^2 + 1)^2$ $OM = \sqrt{(x^2 + 1)^2} = (x^2 \div 1)$ units ✓ CA (6)	

7.2	 fig. i  fig. ii Const: Draw a circle through ACD and not passing though B Suppose the circle cuts AB at M. Join M to C. ✓ S Proof: $A\hat{M}C + \hat{D} = 180°$ (opp ∠ cycl quad) ✓ S /R but $\hat{B} + \hat{D} = 180°$ (given) ✓ S ∴ $A\hat{M}C = \hat{B}$ ✓ S Contradiction: ext. ∠ = int. opp ∠ ✓ S ∴ M must fall on B / B must be on the circumf. of circle ✓ S ∴ ABCD is a cyclic quadrilateral. (6)	only one case of figure needs to be drawn Construction may also be shown on sketch. also use of construction of ∠ at centre or any other suitable wording
7.3	 	
7.3.1	$\hat{Q}_3 = \hat{S}$ ✓ S (alt. angles. //) ✓ R $\hat{W}_1 = \hat{S}$ ✓ S (∠'s in same segm.) ✓ R ∴ $\hat{Q}_3 = \hat{W}_1$ ⇒ KQ is a tang. to circle LQW ✓ R (∠ betw.line & chord = ∠ subt. by chord) (5)	**OR** converse tan-chord theorem

7.3.2	$\hat{R}_1 = \hat{Q}_3 + \hat{Q}_2$ (alt. ∠'s //) ✓ S/R $\hat{L}_3 = \hat{W}_1 + \hat{Q}_4$ (ext. ∠ = sum int. opp. ∠'s) ✓ S/R but $\hat{Q}_3 = \hat{W}_1$ (proven 7.3.1) and $\hat{Q}_2 = \hat{Q}_4$ ✓ S (= chords subt. = ∠'s) ✓ R $\therefore \hat{R}_1 = \hat{L}_3$ (4)	**OR** $\hat{R}_1 = \hat{S} + \hat{Q}_2$ (ext. ∠ of Δ) ✓ S/R $\hat{L}_3 = \hat{W}_1 + \hat{Q}_4$ (ext. ∠ of Δ)✓ S/R but $\hat{W}_1 = \hat{S}$ (∠s same segment) ✓ S/R and $\hat{Q}_2 = \hat{Q}_4$ (= chords subt. = ∠'s) ✓ S/R $\therefore \hat{R}_1 = \hat{L}_3$
7.3.3	$\hat{R}_3 = \hat{L}_3$ (alt. ∠'s, //) $= \hat{W}_1 + \hat{Q}_4$ (ext. ∠ of Δ) ✓ S/R but $\hat{W}_1 = \hat{Q}_1$ (tan-chord theorem) ✓ S/R and $\hat{Q}_4 = \hat{Q}_2$ (= chords subt. = ∠'s) ✓ S/R $\therefore \hat{R}_3 = \hat{Q}_1 + \hat{Q}_2$ ∴ PRKQ is a cyclic quad. (ext. ∠ = int. opp ∠) ✓ R	**OR** $\hat{R}_1 + \hat{R}_2 + \hat{L}_3 = 180°$ (co-int. ∠'s, //) ✓ S/R but $\hat{L}_3 = \hat{Q}_4 + \hat{W}_1$ (ext ∠ of Δ) ✓ S/R and $\hat{Q}_4 = \hat{Q}_2$ (= chords subt. = ∠'s) and $\hat{W}_1 = \hat{Q}_1$ (tan-chord) ✓ S/R $\therefore \hat{R}_1 + \hat{R}_2 + \hat{Q}_2 + \hat{Q}_1 = 180°$ PRKQ is a cyclic quad. (opp ∠'s suppl.) ✓ R (4)
7.3.4	$\hat{L}_3 = \hat{W}_1 + \hat{Q}_4$ (ext ∠ of Δ) $= \hat{S} + \hat{Q}_4$ (∠s same segment) ✓ S $\therefore \hat{L}_3 \neq \hat{S}$ ✓ R ∴ RSLQ is not a cyclic quad (angles in same segm. not equal) **OR** $\hat{R}_1 = \hat{L}_3$ ✓ S (from 7.3.2) $\neq S\hat{L}Q$ ✓ R ∴ RSLQ is not a cyclic quad (ext. ∠ ≠ int. opp. ∠s) (2)	**OR** $\hat{R}_3 = \hat{Q}_3 + \hat{Q}_4$ (∠s same segment) $\therefore \hat{R}_3 \neq \hat{Q}_3$ ✓ S RSLQ is not a cyclic quad. (angles in same segm .not equal) ✓ R

Question 8 **[10]**

Reason needs only be given once



8.1

Let SW = x

$\frac{SW}{WQ} = \frac{ST}{TP}$ (line || one side Δ) / (intercept-midpt) ✓ S/R

$\therefore$ SQ = $2x$ ✓ S

RS = SQ = $2x$ ✓ S

$\therefore$ RW = $3x$ ✓ S

$\frac{RM}{RP} = \frac{RW}{RQ}$ ✓ S (line || one side Δ)

$= \frac{3}{4}$ ✓ S

OR

OR

$\frac{SW}{WQ} = \frac{ST}{TP} = \frac{1}{1}$ (line || one side Δ) ✓ S/R

$\therefore$ SQ = 2 SW ✓ S

RS = SQ = 2 SW ✓ S

$\therefore$ RW = 3 SW ✓ S

$\frac{RM}{RP} = \frac{RW}{RQ} = \frac{3SW}{4SW}$ ✓ S (line || one side Δ)

$= \frac{3}{4}$ ✓ S (6)

8.2

$\frac{\text{area } \Delta RPS}{\text{area } \Delta RMW} = \frac{\frac{1}{2}RP.RS\sin R}{\frac{1}{2}RW.RM\sin R}$ ✓ S

$= \frac{4}{3}.\frac{2}{3} = \frac{8}{9}$ ✓ S ✓ S ✓ CA

OR

$\Delta RPS = \frac{1}{2}\Delta PQR$ ✓ S

$\Delta RMW = \frac{9}{16}\Delta PQR$ ✓ S

$\frac{\Delta RPS}{\Delta RMW} = \frac{\frac{1}{2}}{\frac{9}{16}}$ ✓ S

$= \frac{8}{9}$ ✓ CA (4)

OR

Area ΔRPS = $\frac{1}{2}$ area ΔPQR ✓ S

$= \frac{1}{2}(\frac{1}{2}PQ)H$

$\frac{\text{Area}\Delta PRS}{\text{Area}\Delta RMW} = \frac{\frac{1}{4}PQ.H}{\frac{1}{2}.MW.\frac{3}{4}.H}$ ✓ S (for $\frac{3}{4}$H)

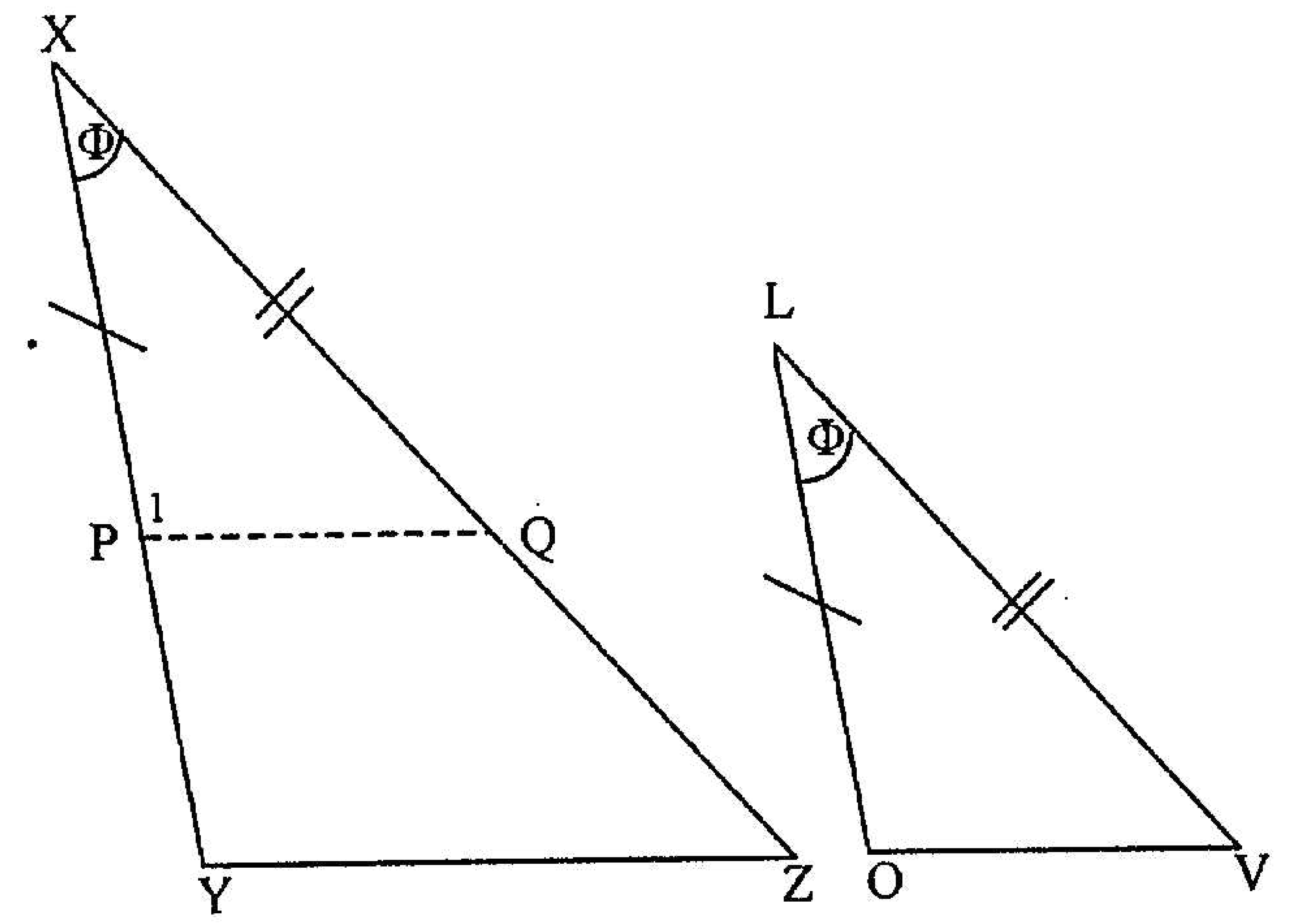
$= \frac{\frac{1}{4}PQ.H}{\frac{1}{2}.\frac{3}{4}PQ.\frac{3}{4}H}$ ✓ S (for $\frac{3}{4}$PQ)

$= \frac{8}{9}$ ✓ CA

ANSWER ONLY FULL MARKS

QUESTION 9 [28]

9.1



Constr: On XY and XZ cut off XP = LO and XQ = LV.
Join P to Q. ✓M

Proof: $\Delta XPQ \equiv \Delta LOV$ ✓S (s,∠, s) ✓R

$\therefore \hat{P}_1 = \hat{O}$ ✓S

$= \hat{Y}$ (given)

$\therefore$ PQ // YZ (corresp. ∠'s equal) ✓S/R

$\therefore \dfrac{XY}{XP} = \dfrac{XZ}{XQ}$ ✓S (line // one side Δ) ✓R

$\therefore \dfrac{XY}{LO} = \dfrac{XZ}{LV}$ (construction)

(7)

OR Construction may be shown on sketch.

OR
Constr: On XY cut off XP = LO and draw XP // LO.
Join P to Q. ✓M

Proof: $\hat{P}_1 = \hat{Y}$ (corresp. ∠'s, //) ✓S/R

and $\hat{Y} = \hat{O}$ (const.)

$\therefore \hat{P}_1 = \hat{O}$ ✓S

$\Delta XPQ \equiv \Delta LOV$ ✓S (∠,∠, s) ✓R

$\therefore \dfrac{XY}{XP} = \dfrac{XZ}{XQ}$ ✓S (line // one side Δ) ✓R

$\therefore \dfrac{XY}{LO} = \dfrac{XZ}{LV}$ (constr./congr.)

(7)

OR
Constr: On XY cut off XP = LO and draw $\hat{P}_1 = \hat{O}$ Join P to Q. ✓M

Proof: $\Delta XPQ \equiv \Delta LOV$ ✓S (∠,∠, s) ✓R

$\therefore \hat{P}_1 = \hat{O}$ ✓S

$= \hat{Y}$ (given)

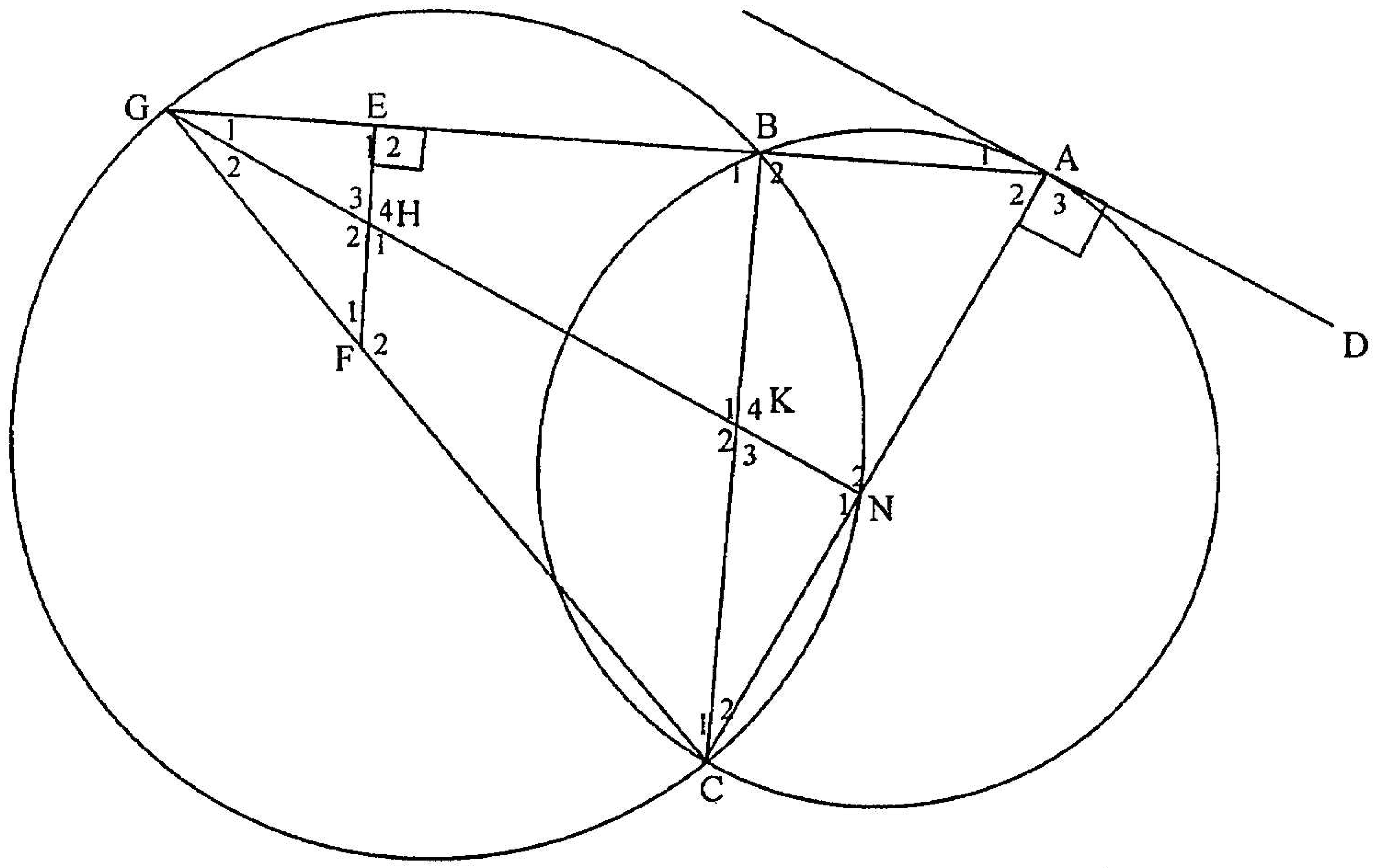
$\therefore$ PQ // YZ (corresp. ∠'s equal) ✓S/R

$\therefore \dfrac{XY}{XP} = \dfrac{XZ}{XQ}$ ✓S (line // one side Δ) ✓R

$\therefore \dfrac{XY}{LO} = \dfrac{XZ}{LV}$ (construction)

(7)

9.2



9.2.1

✓ S

$\therefore\ \hat{B}_2 = 90°$ (tan-chord) ✓ R

$\therefore\ \hat{N}_1 = \hat{B}_1 = 90°$ ✓ S (tan-chord) ✓ R

$\therefore$ GN and BC are altitudes of Δ AGC ✓ S ✓ S

$\therefore$ K is the orthocentre (Altitudes concurrent)

OR

$\hat{C}_2 = \hat{A}_1$ (tan-chord) ✓ S

$\hat{C}_2 = \hat{G}_1$ ($\angle$'s in same segm.) ✓ S/R

$\therefore\ \hat{A}_1 = \hat{G}_1$ ✓ S/R

$\therefore$ GN || AD (alt. $\angle$'s equal) ✓ S/R

$\therefore\ \hat{N}_2 = \hat{A}_3 = 90°$ ✓ S (alt. $\angle$'s, //)

$\therefore$ GN and BC are altitudes of Δ AGC ✓ S ✓ S

$\therefore$ Altitudes concurrent

$\therefore$ K is the orthocentre

(6)

OR

$\hat{B}_2 = 90°$ ✓ S (tan-chord) ✓ R

In Δ KNC and Δ KBG

$\hat{K}_3 = \hat{K}_1$ (vert.opp $\angle$'s)

$\hat{C}_2 = \hat{G}_1$ ($\angle$'s in same seg.) } ✓ S/R

$\hat{N}_1 = \hat{B}_1 = 90°$ ✓ S (sum $\angle$'s Δ)

✓ S ✓ S

GN and BC are altitudes of Δ AGC

$\therefore$ Altitudes are concurrent

K is the orthocentre

$\hat{B}_2 = 90°$ - 2 marks $\hat{N}_1 = 90°$ - 2 marks altitudes – 2 marks

9.2.2	In ΔABC and ΔKBG ✓ S $\hat{B}_2 = \hat{B}_1 = 90°$ (proved) ✓ S $\hat{C}_2 = \hat{G}_1$ ($\angle$'s in same segm.) ✓ S/R $\hat{A}_2 = \hat{K}_1$ ($\angle$s of Δ) $\Delta ABC \mid\mid\mid \Delta KBG$ ($\angle, \angle, \angle$) ✓ R $\frac{BC}{BG} = \frac{AC}{KG}$ ✓ S $BC.KG = AC.BG$ (5)	Choosing the correct triangles, either at beginning or end. Reason mark allocated for either the third $\angle$ or equiangular or $\angle\angle\angle$ or $\angle\angle$.
9.2.3	In ΔBCG and ΔEFG $\hat{B}_1 = \hat{E}_1 = 90°$ ✓ S $\hat{G}_1 + \hat{G}_2 = \hat{G}_1 + \hat{G}_2$ ✓ S $\hat{C}_1 = \hat{F}_1$ ($\angle$s of Δ) $\Delta BCG \mid\mid\mid \Delta EFG$ ($\angle, \angle, \angle$) ✓ R $\frac{BC}{EF} = \frac{BG}{EG}$ $\therefore \frac{BC}{BG} = \frac{EF}{EG}$ ....(1) ✓ S $\frac{BC}{BG} = \frac{AC}{KG}$ ✓ S ....(2) (proven in 9.2.2.) (1) x (2) $\frac{BC}{BG} \cdot \frac{BC}{BG} = \frac{EF.AC}{EG.KG}$ ✓ S **OR** $\frac{BC}{BG} = \frac{EF}{EG}$ $\therefore BC = \frac{EF.BG}{EG}$ (FE // BC) and $BG = \frac{BC.KG}{AC}$ (from 9.2.2) $\therefore \frac{BC^2}{BG^2} = \frac{BC.EF.BG}{EG} \div \frac{BG.BC.KG}{AC}$ $= \frac{BC.EF.BG}{EG} \times \frac{AC}{BG.BC.KG}$ $= \frac{EF.AC}{EG.KG}$ (6)	or $\hat{B}_1 = \hat{E}_1 = 90°$ ✓ S EF // BC (corresp. $\angle$'s =) ✓ S $\hat{C}_1 = \hat{F}_1$ (corresp. $\angle$'s, //) substitution

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<tr><td>9.2.4</td><td>

$\frac{EF.AC}{EG.KG}+1 = \frac{BC^2}{BG^2}+1$ ✓ S (from 9.2.3)

$= \frac{BC^2+BG^2}{BG^2}$ ✓ S ✓ S

$= \frac{GC^2}{BG^2}$ ✓ S (Pyth.) (4)

</td><td>

OR

$\frac{BC^2}{BG^2} = \frac{EF.AC}{EG.KG}$ ✓ S (from 9.2.3)

Pythagoras: ✓ S

$\frac{GC^2 - BG^2}{BG^2} = \frac{EF.AC}{EG.KG}$

$\frac{GC^2}{BG^2} - 1 = \frac{EF.AC}{EG.KG}$ ✓ S

$\frac{GC^2}{BG^2} = \frac{EF.AC}{EG.KG} + 1$ ✓ S

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OR

From Δ BCG:

$GC^2 = BG^2 + BC^2$ ✓ S (Pyth.)

$\frac{GC^2}{BG^2} = 1+\frac{BC^2}{BG^2}$ ✓ S ✓ S

$= 1 + \frac{EF.AC}{EG.GK}$ ✓ S (from 9.2.3)

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OR Δ BCG EF // BC

$\therefore \frac{GE}{GB} = \frac{GF}{GC}$ ✓ S (line // 1 side Δ)

$(\frac{GE}{GB})^2 = (\frac{GF}{GC})^2$ ✓ S

$(\frac{GC}{GB})^2 = (\frac{GF}{GE})^2$

$= \frac{EG^2 + EF^2}{EG^2}$ ✓ S (Pyth.)

$= 1 + \frac{EF^2}{EG^2}$

but $\therefore \frac{EF}{EG} = \frac{AC}{KG}$ (9.2.3) ✓ S

$\therefore \frac{GC^2}{BG^2} = 1 + \frac{EF.AC}{EG.KG}$

working with both sides – penalty 1

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