NATIONAL
DEPARTMENT OF EDUCATION

NASIONALE
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DEPARTMENT OF EDUCATION

POSSIBLE ANSWERS FOR :

SENIOR SERTIFIKAAT-EKSAMEN / SENIOR CERTIFICATE EXAMINATION
WISKUNDE SG / MATHEMATICS SG
VRAESTEL II / PAPER II
NOVEMBER 2003

✓ A ≡ 1 mark for accuracy

✓ CA ≡ 1 mark for consistent accuracy

✓ M ≡ 1 mark for correct method

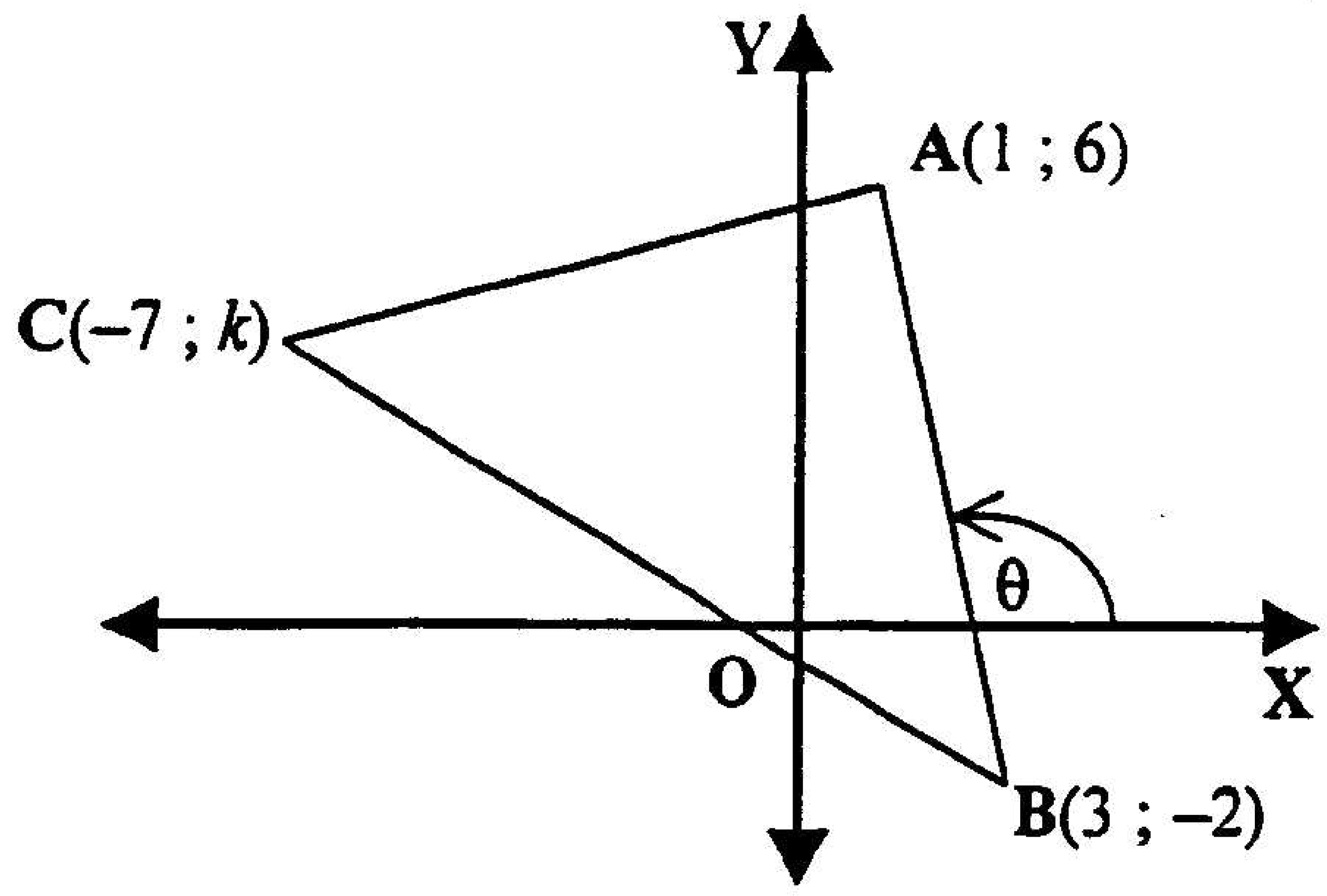
✓ S ≡ 1 mark for the correct statement

✓ R ≡ 1 mark for the correct reason

✓ S/R ≡ 1 mark for the correct statement with the correct reason

Penalise candidate once only in entire paper for rounding off. (Possible questions: 1.1.2; 4.3; 5.2.2; 6.2)

QUESTION 1



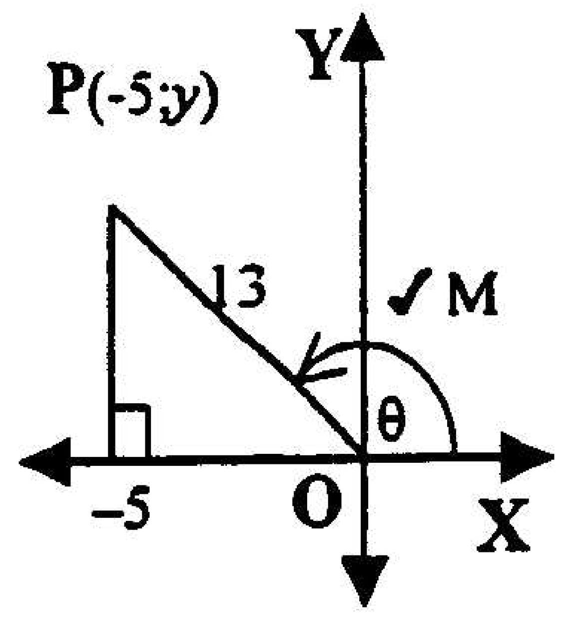
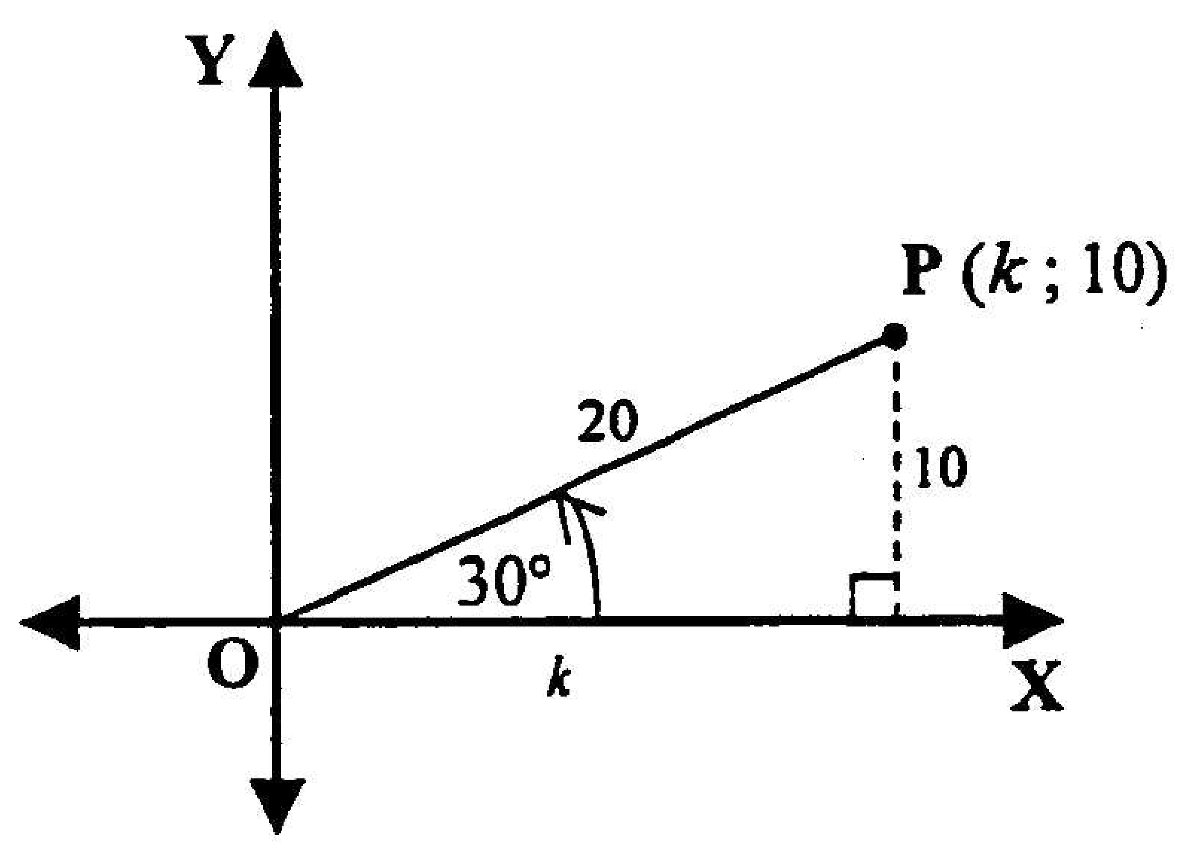
Solution	Marks	Comments
		Wrong formula ⇒ no marks
1.1.1 $m_{AB} = \dfrac{y_A - y_B}{x_A - x_B}$ ✓M $= \dfrac{6-(-2)}{1-3}$ $= -4$ ✓A	(2)	Correct use of formula for gradient Calculating value correctly
1.1.2 $\tan\theta = m_{AB} = -4$ ✓M ref. angle = 76° or − 76° ✓CA $\theta = 180° - 76°$ $= 104°$ ✓CA	(3)	Using formula for ∠ of inclination correctly Calculating reference ∠ correctly – in case of ref ∠ = – 76 and stops answer, max 1 mark Calculating ∠ of inclination correctly – must be obtuse If gradient is positive – maximum 2 marks Correct answer only – full marks
1.1.3 $AB = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$ ✓M $= \sqrt{(1-3)^2 + (6-(-2))^2}$ ✓A $= \sqrt{4+64}$ $= \sqrt{68}$ or $2\sqrt{17}$ ✓CA	(3)	$AB^2 = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} \Rightarrow$ 1mark penalty Using distance formula correctly Substituting correct values in formula correctly Calculating AB correctly
1.1.4 $AC = \sqrt{(x_A - x_C)^2 + (y_A - y_C)^2}$ ✓CA $\sqrt{68} = \sqrt{(1+7)^2 + (6-k)^2}$ ✓A $68 = 64 + 36 - 12k + k^2$ ✓CA $k^2 - 12k + 32 = 0$ ✓CA $(k-8)(k-4) = 0$ ✓CA $k = 8$ or $k = 4$ ✓CA		Substituting correct values into correct formula correctly; AC = AB Expanding correctly Simplifying correctly into standard form Factorising correctly / Using formula correctly Both values of *k* correct
OR $AC = \sqrt{(x_A - x_C)^2 + (y_A - y_C)^2}$ ✓CA $\sqrt{68} = \sqrt{(1+7)^2 + (6-k)^2}$ ✓A $68 = 64 + (6-k)^2$ ✓CA $4 = (6-k)^2$ ✓CA $6 - k = -2$ or $6 - k = 2$ ✓CA $k = 8$ or $k = 4$ ✓CA		Substituting correct values into correct formula correctly; AC = AB Squaring correctly Simplifying correctly Taking square root correctly Correct values of *k*
OR $k = 4$ ✓✓✓A or $k = 8$ ✓✓✓A	(6)	Correct answer only by translation ⇒ full marks One answer only – max 3 marks

1.2.1 $M\left(\frac{x_D + x_H}{2};\frac{y_D + y_H}{2}\right)$ ✓M $M\left(\frac{-1+3}{2};\frac{-1+(-5)}{2}\right)$ ✓A ✓A M(1 ; –3) (3)	Using correct formula for coordinates of midpoint correctly – do not have to write down formula Calculating coordinates correctly – not necessary to be in brackets
1.2.2 $m_{DH} = \frac{y_D - y_H}{x_D - x_H}$ $= \frac{-1-(-5)}{-1-3}$ ✓A $= -1$ ✓CA $\therefore$ $m_{\perp} = 1$ ✓CA subst. (1;–3) in $y = x + c$ $\therefore$ $-3 = 1 + c$ OR $y + 3 = (x - 1)$ ✓CA $c = -4$ $y = x - 4$ ✓CA	Substituting correct values correctly into the correct formula for gradient Calculating gradient correctly Writing down correct gradient for perp line Substituting coordinates of M correctly Simplifying equation correctly
OR PD = PH ✓M $PD^2 = PH^2$ ✓A ✓A $(x+1)^2 + (y+1)^2 = (x-3)^2 + (y+5)^2$ $x^2 + 2x + 1 + y^2 + 2y + 1 = x^2 - 6x + 9 + y^2 + 10y + 25$ ✓CA $8x - 8y - 32 = 0$ ✓CA $x - y - 4 = 0$ (5)	Equating correct distances anywhere in proof Substituting correct values correctly into distance formulae Expanding completely correct Simplifying correctly – any equivalent form
1.3.1 W(0 ; 5) or $x = 0; y = 5$ ✓A (1)	Must have both coordinates correct
1.3.2 $m_{TW} = 2$ ✓A $\therefore$ $m_{WP} = -\frac{1}{2}$ ✓CA $y = mx + c$ ✓M $\therefore$ $y = -\frac{1}{2}x + 5$ ✓CA	Calculating gradient of TW correctly Deducing gradient of WP correctly Using correct version of straight line formula Substituting correct values into formula
OR $m_{TW} = 2$ ✓A $m_{WP} = -\frac{1}{2}$ ✓CA $y - 5 = -\frac{1}{2}(x - 0)$ ✓M $\therefore$ $y = -\frac{1}{2}x + 5$ ✓CA (4)	Calculating gradient of TW correctly Deducing gradient of WP correctly Substituting coordinates of W into correct formula for straight line Simplifying equation correctly
1.3.3 $0 = -\frac{1}{2}x + 5$ ✓M $\therefore$ $0 = -x + 10$ $\therefore$ $x = 10$ ✓CA $\therefore$ P(10 ; 0) (2)	Substituting $y = 0$ into equation of WP Solving x
1.3.4 Area of ΔWOP = $\frac{1}{2}$WO.OP or $\frac{1}{2}b.h$ ✓M $= \frac{1}{2} \times 5 \times 10$ ✓CA $= 25$ units^2 ✓CA (3) [32]	Using correct formula for area Substitute correct values according to 1.3.1 and 1.3.3 Calculate area correctly – ignore units

QUESTION 2

2.1 $x^2 + y^2 = r^2$ ✓M substitute A(−3;−2) $(-3)^2 + (-2)^2 = r^2$ ✓A $r^2 = 9 + 4$ $= 13$ ✓CA $\therefore\ x^2 + y^2 = 13$ (3)	Using correct equation for circle – not necessary to write formula down Substituting coordinates of A correctly Correctly calculating value of r^2 $x^2 + y^2 = \sqrt{13} \Rightarrow$ penalty of 1 mark
2.2 $x = 5 - y$ $x^2 + y^2 = 13$ $\therefore\ (5-y)^2 + y^2 = 13$ ✓CA $\therefore\ 25 - 10y + y^2 + y^2 = 13$ ✓CA $\therefore\ 2y^2 - 10y + 12 = 0$ ✓CA $\therefore\ y^2 - 5y + 6 = 0$ ✓CA $\therefore\ (y-2)(y-3) = 0$ or $(2y-4)(y-3) = 0$ $\therefore\ y = 2$ or $y = 3$ ✓CA $\therefore\ x = 3$ or $x = 2$ ✓CA $\therefore$ B(2 ; 3) and C(3 ; 2) ✓CA	Substituting correctly into equat of 2.1 Expanding correctly Writing into standard form correctly Factorising correctly Correct values for y Calculating values for x correctly Giving coordinates of B and C correctly as number pairs with smallest x coordinate to B
OR	
$y = 5 - x$ $x^2 + y^2 = 13$ $\therefore\ x^2 + (5-x)^2 = 13$ ✓CA $\therefore\ x^2 + 25 - 10x + x^2 = 13$ ✓CA $\therefore\ 2x^2 - 10x + 12 = 0$ $\therefore\ x^2 - 5x + 6 = 0$ ✓CA $\therefore\ (x-2)(x-3) = 0$ ✓CA $\therefore\ x = 2$ or $x = 3$ ✓CA $\therefore\ y = 3$ or $y = 2$ ✓CA $\therefore$ B(2 ; 3) and C(3 ; 2) ✓CA (7) [10]	Substituting correctly Expanding correctly Writing into standard form correctly Factorising correctly Correct values for x Calculating values for y correctly Giving coordinates of B and C correctly as number pairs Correct answer only 4 marks max $\Rightarrow$ 2 per point

QUESTION 3

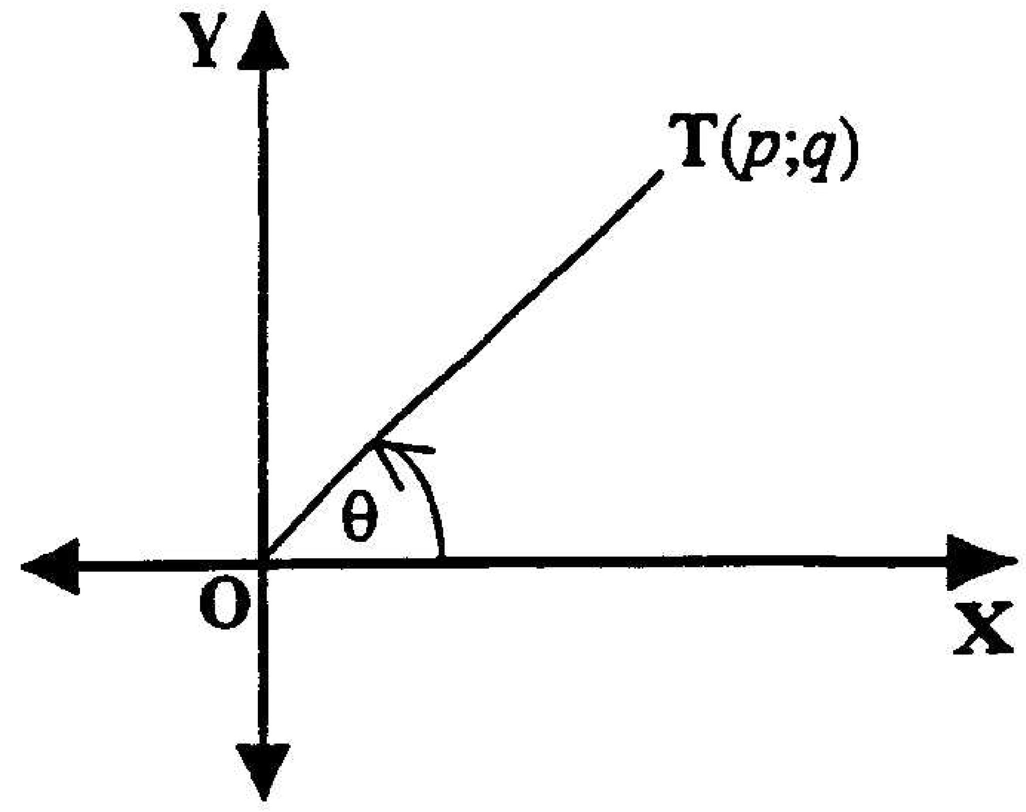
3.1 $\cos\theta = -\frac{5}{13}$ $x^2 + y^2 = r^2$ ✓M $\therefore (-5)^2 + y^2 = 13$ $\therefore y^2 = 144$ $\therefore y = 12$ ✓A  $\therefore \operatorname{cosec}\theta = \frac{13}{12}$ ✓CA		No diagram – looses M mark for diagram Radius vector in wrong quadrant – looses M mark for diagr. Using Pyth correctly to calculate y Calculating y correctly – correct value of y only Writing down correct value for cosec θ
OR $\sin\theta = \sqrt{1-\cos^2\theta}$ $= \sqrt{1-\left(\frac{-5}{13}\right)^2}$ ✓M $= \sqrt{\frac{144}{169}}$ $= \frac{12}{13}$ ✓A $\therefore \operatorname{cosec}\theta = \frac{13}{12}$ ✓CA	(4)	Substituting correct values Calculating correctly Writing down correct value for cosec θ
3.2  		
3.2.1 $\sin 30° = \frac{1}{2} = 0{,}5$ ✓A	(1)	Any version of 0,5
3.2.2 $\sin 30° = \frac{10}{OP}$ $\therefore \frac{1}{2} = \frac{10}{OP}$ $\therefore OP = 20$ units ✓CA	(1)	Correctly calculating the value of OP
3.2.3 $20^2 = k^2 + 10^2$ ✓M $400 - 100 = k^2$ $k^2 = 300$ $k = \sqrt{300}$ ✓CA $= 10\sqrt{3}$ ✓CA		Using correct values from 3.2.2 in correct form of Pyth Taking square root on both sides Correct simplification of square root – only the positive square root acceptable in final answer
OR $\tan 30° = \frac{10}{k}$ ✓M ✓A $\therefore \frac{1}{\sqrt{3}} = \frac{10}{k}$ $\therefore k = 10\sqrt{3}$ ✓CA		Using tan or cot definition correctly Correct value for tan 30° or cot 30° Solving k correctly

	OR $\frac{k}{OP} = \cos 30°$ ✓M $k = r\cos 30°$ $= 20 \times \frac{\sqrt{3}}{2}$ ✓A $= 10\sqrt{3}$ ✓CA	(3)	Using cos or sec definition correctly Correct value for cos 30° or sec 30° Solving k correctly Correct answer in simplified form only
3.3	$\frac{\sin(180°+\theta).\sec(90°-\theta)}{\tan(180°-\theta)}$ $= \frac{(-\sin\theta).\text{cosec}\,\theta}{(-\tan\theta)}$ ✓A ✓A ✓A $= \frac{1}{\tan\theta}$ ✓CA $= \cot\theta$	(4)	Correct reduction – including sign shown Omitting θ in interim steps ⇒ no penalty sin θ.cosec θ = 1 and sign correct Omitting θ in final answer ⇒ 1 mark penalty
3.4	$2\cos 210°.\cot 60°.\tan 315°$ $= 2(-\cos 30°)(\cot 60°)(-\tan 45°)$ ✓A ✓A $= 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} \times 1$ ✓CA ✓CA ✓CA $= 1$ ✓CA	(6) [19]	Correct reduction – including sign Changing tan 60° to cot 30° is fine – does not carry any mark however Correct values for special angles without calculator Correct simplification Answer only – no mark at all

QUESTION 4

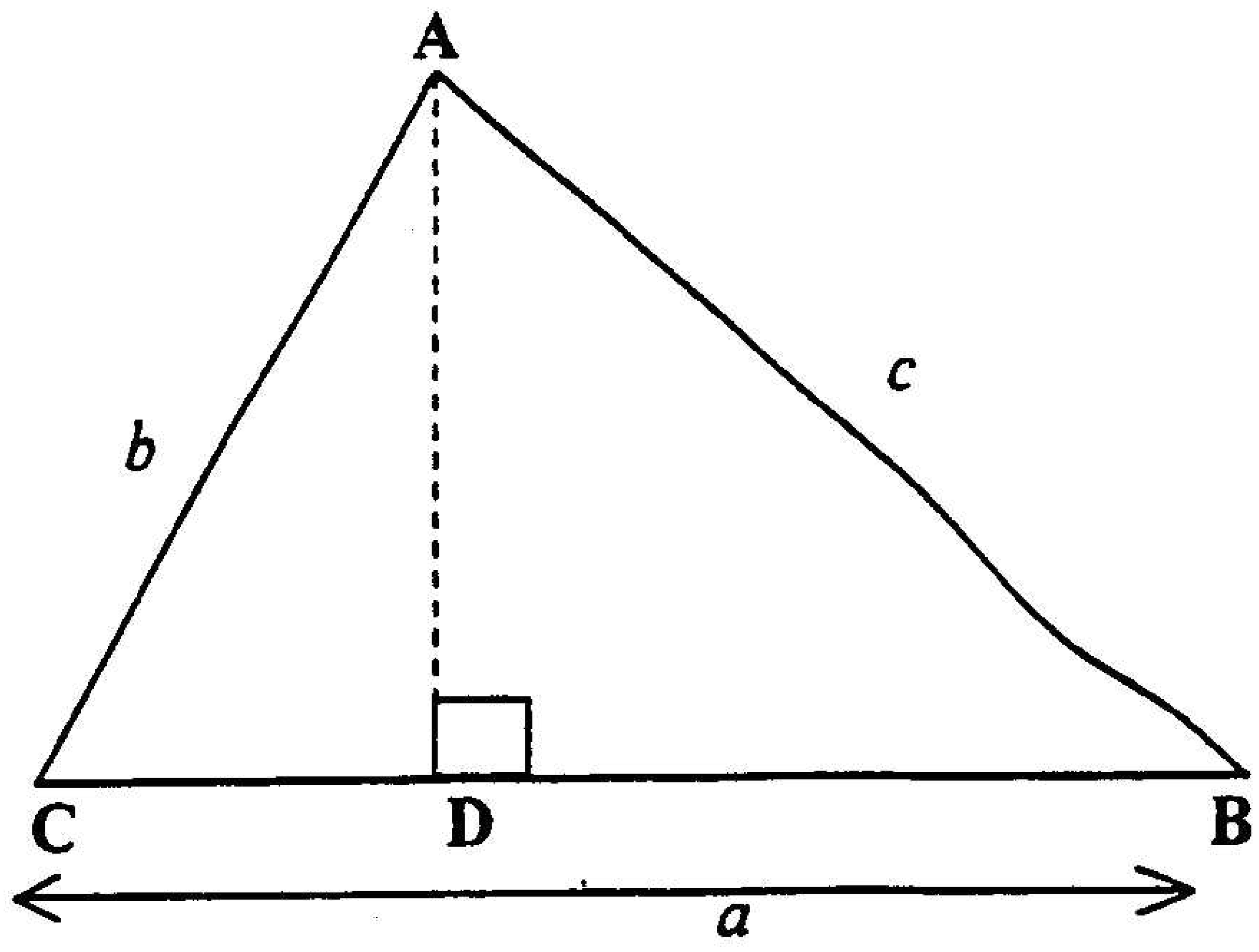
4.1.1	$f(x) = 2\cos x$ ✓A ✓A	(2)	Correct coefficient Correct trigonometric ratio
4.1.2	$g(x) = \sin x$ ✓A ✓A	(2)	Correct coefficient Correct trigonometric ratio 1 mark penalty once for using θ in stead of x or omitting x in either 4.1.1 or 4.1.2 or both
4.2	Range: $y \in [-2\,;2]$ ✓A ✓M OR $\{y : -2 \leq y \leq 2\}$ ✓A ✓M	(2)	Correct interval and in correct order Correct notation: $y \in$ / set notation not required Using x instead of $y \Rightarrow$ no mark at all
4.3	$p = \sin 63{,}4°$ $= 0{,}9$ ✓CA OR $p = 2\cos 63{,}4°$ ✓CA $= 0{,}9$	(1)	Calculating p correctly consistent with 4.1
4.4	B$(243{,}4°\,;-0{,}9)$ ✓A ✓CA	(2)	Correct value for x coordinate Correct value for y coordinate with respect to 4.3
4.5	$63{,}4° \leq x \leq 243{,}4°$ ✓CA ✓M OR $63{,}4° \leq x \leq x_B$ ✓A ✓M OR $x \in [63{,}4°\,;\ 243{,}4°]$ ✓CA ✓M OR $x \in [63{,}4°\,; x_B\,]$ ✓A ✓M	(2) [11]	Correct interval M mark for correct notation⟹ correct order of numbers as well as correct form of inequalities with equal signs

QUESTION 5

5.1	 Using different variables without connecting them to p and $q \Rightarrow$ penalty of 1 mark
5.1.1 LHS $= 1 + \tan^2\theta$	Working with LHS and RHS simultaneous $\Rightarrow$ penalty of 1 mark
$= 1 + \left(\frac{q}{p}\right)^2$ ✓A	Using the correct values for tan ratio
$= \frac{p^2 + q^2}{p^2}$ ✓A	Correctly combine terms on LCM
$= \frac{OT^2}{p^2}$ ✓A (Pyth)	Applying Pyth correctly to numerator Using r for OT acceptable
RHS $= \sec^2\theta = \left(\frac{OT}{p}\right)^2$ ✓A	Using correct values for sec ratio
OR	
$p^2 + q^2 = OT^2$ ✓A (Pyth)	Applying Pyth correctly
$\frac{p^2}{p^2} + \frac{q^2}{p^2} = \frac{r^2}{p^2}$ ✓A	Dividing with p^2
✓A ✓A $1 + \left(\frac{q}{p}\right)^2 = \left(\frac{r}{p}\right)^2$	Correctly writing as square of a ratio
$1 + \tan^2\theta = \sec^2\theta$ (4)	
5.1.2 LHS $= \cot\theta \cdot \sec^2\theta$	$\sec^2\theta = 1 + \tan^2\theta$
$= \cot\theta(1 + \tan^2\theta)$ ✓A	
✓A ✓A $= \cot\theta + \cot\theta\tan^2\theta$	Expanding correctly
✓A $= \cot\theta + 1.\tan\theta$	$\cot\theta.\tan\theta = 1$ Working with LHS and RHS simultaneously$\Rightarrow$ 1 mark penalty unless already penalised for it in 5.1.1
OR	
RHS $= \cot\theta + \tan\theta$	
$= \frac{1}{\tan\theta} + \tan\theta$ ✓A	$\cot\theta = \frac{1}{\tan\theta}$
$= \frac{1 + \tan^2\theta}{\tan\theta}$ ✓A	correctly add on common denominator
$= \frac{\sec^2\theta}{\tan\theta}$ ✓A	$1 + \tan^2\theta = \sec^2\theta$
$= \cot\theta\sec^2\theta$ ✓A = LHS	$\cot\theta = \frac{1}{\tan\theta}$
OR	

	LHS $= \frac{\cos\theta}{\sin\theta}.\frac{1}{\cos^2\theta} = \frac{1}{\sin\theta.\cos\theta}$ ✓A		Correctly converted into sin θ and cos θ
	RHS $= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$ ✓A		Correctly converted into sin θ and cos θ
	$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta.\cos\theta}$ ✓A		Correctly combine terms on LCM
	$= \frac{1}{\sin\theta.\cos\theta}$ ✓A	(4)	$\cos^2\theta + \sin^2\theta = 1$
5.2.1	$\sin 2x = 0{,}562 \quad 2x = [90°;270°]$ ref. ∠ = 34,2° ✓A ∴ $2x = 180° - 34{,}2°$ ✓CA $= 145{,}8°$ ∴ $x = 72{,}9°$ ✓CA	(3)	Calculating reference ∠ correctly Recognising ∠ in 2nd quadrant – only possible mark if dividing by 2 in 1st line Dividing reference ∠ by 2 – maximum 2 1 mark for ref ∠ and 1 mark for 2nd quad Correctly calculating x Extra solution is penalised by 1 mark
5.2.2	$x = 7\sec^2 142{,}5° + \tan 301{,}5°$ ✓A ✓A $= 11{,}1215... - 1{,}6318...$ $= 9{,}5$ ✓A	(3) [14]	Correct answer only – full marks

QUESTION 6



6.1 Draw AD ⊥ BC ✓M

Let CD = x, then DB = $a - x$ and $AD^2 = b^2 - x^2$

$AB^2 = AD^2 + DB^2$ ✓M (Pyth)

$c^2 = b^2 - x^2 + (a - x)^2$ ✓A

$= b^2 - x^2 + a^2 - 2ax + x^2$ ✓A

$= a^2 + b^2 - 2ax$ ✓A

but $\cos C = \frac{x}{b}$ $\therefore$ $x = b\cos C$ ✓A

$c^2 = a^2 + b^2 - 2ab\cos C$

If proven $a^2 = b^2 + c^2 - 2bc\cos A$ or $b^2 = a^2 + c^2 - 2ac\cos B$ without stating similarly $c^2 = a^2 + b^2 - 2ab\cos C$ maximum 4 marks

if an obtuse Δ is drawn – no penalty

OR

Draw AD ⊥ CB ✓M

In ΔACD: ✓A ✓A

CD = $b\cos C$; $\therefore$ DB = $a - b\cos C$; AD = $b\sin C$

$AB^2 = AD^2 + DB^2$ (Pyth)

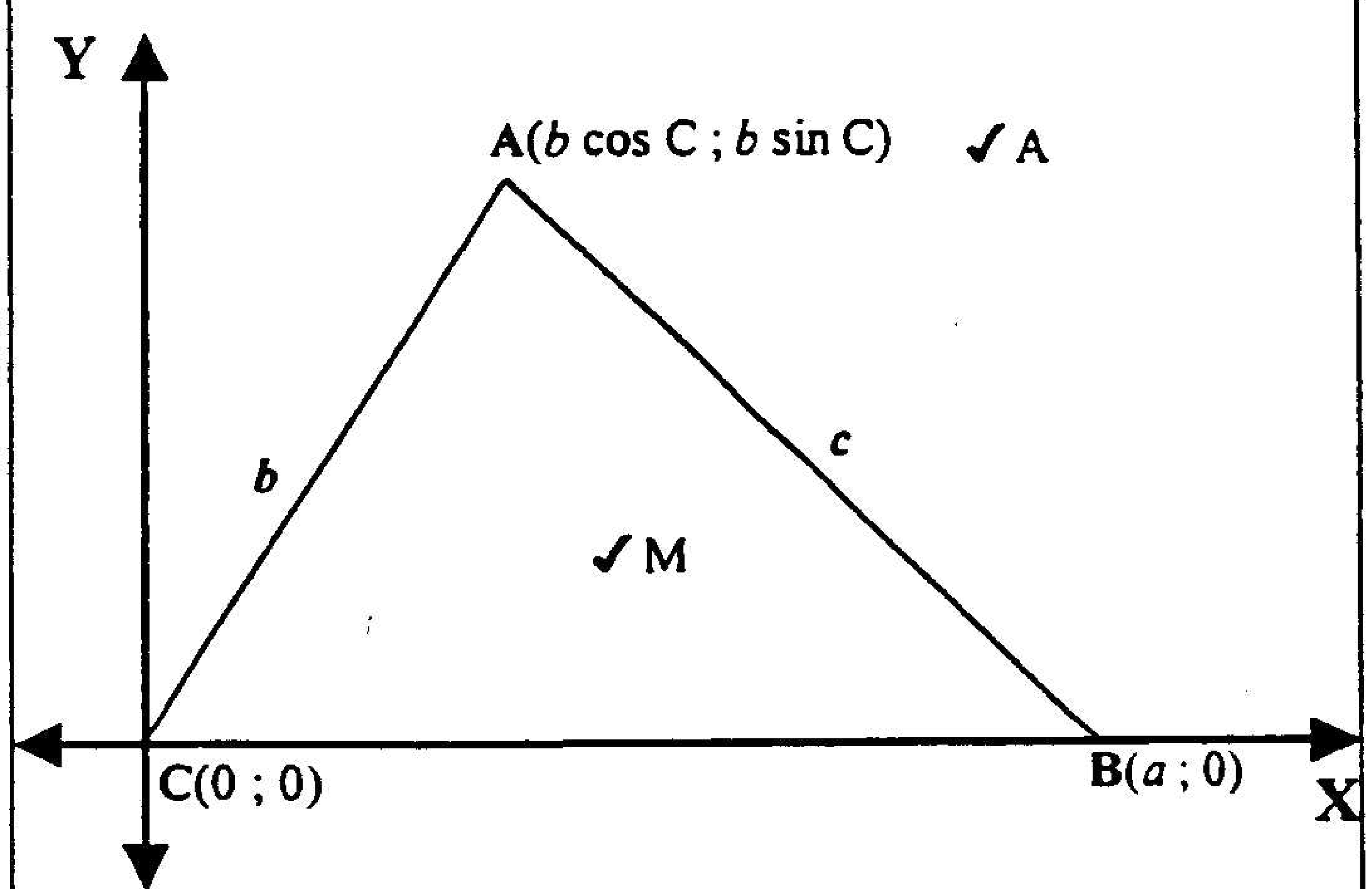
$c^2 = (b\sin C)^2 + (a - b\cos C)^2$ ✓A

$= b^2\sin^2 C + a^2 - 2ab\cos C + b^2\cos^2 C$ ✓A

$= a^2 + b^2(\sin^2 C + \cos^2 C) - 2ab\cos C$ ✓A

$= a^2 + b^2 - 2ab\cos C$

OR



Correct coordinates for A

M mark goes for diagram in correct position

$AB^2 = (x_A - x_B)^2 + (y_A - y_B)^2$ (distance formula)

$c^2 = (b\cos C - a)^2 + (b\sin C - 0)^2$ ✓A

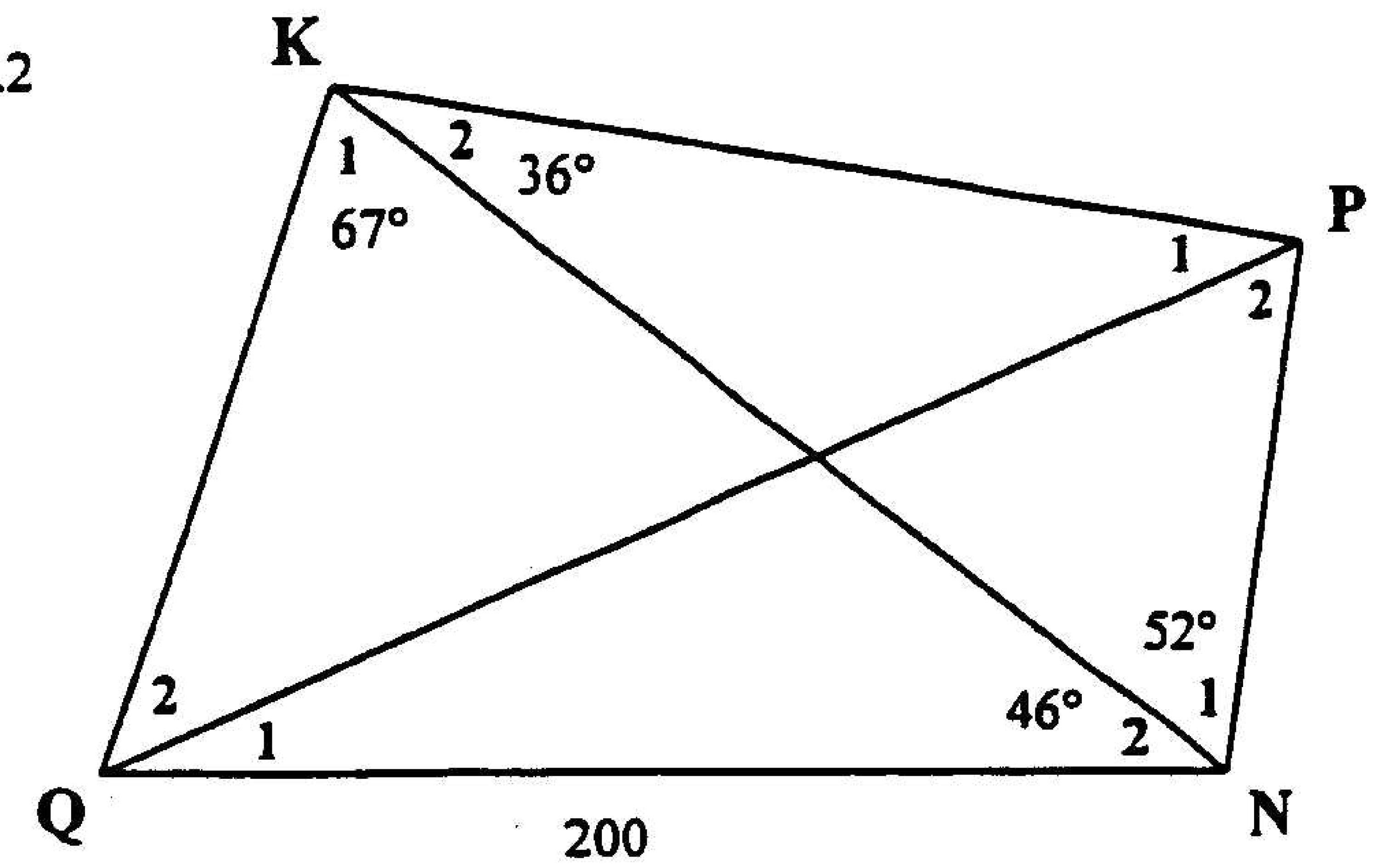
$= b^2\cos^2 C - 2ab\cos C + a^2 + b^2\sin^2 C$ ✓A

$= a^2 + b^2(\cos^2 C + \sin^2 C) - 2ab\cos C$ ✓A

$= a^2 + b^2 - 2ab\cos C$ ✓A (6)

Substituting correct coordinates correctly
Expanding correctly
Factorising correctly
$\sin^2 C + \cos^2 C = 1$

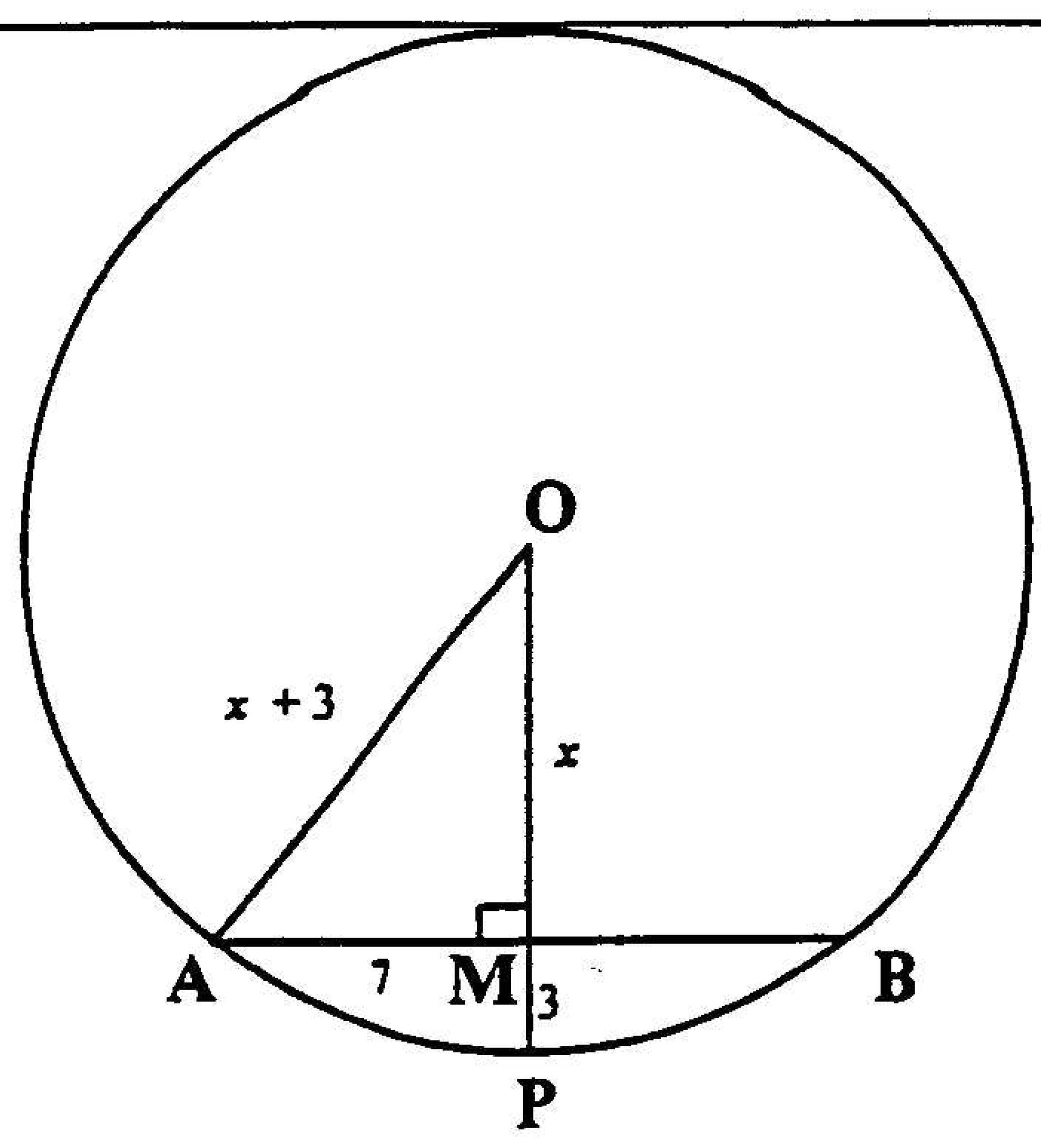
6.2



6.2.1	$K\hat{P}N = 92°$ ✓A	(1)	
6.2.2	$\dfrac{PN}{\sin K_2} = \dfrac{KN}{\sin K\hat{P}N}$ ✓M $\dfrac{PN}{\sin 36°} = \dfrac{KN}{\sin 92°}$ ✓CA $\therefore$ $PN = \dfrac{200 \sin 36°}{\sin 92°}$ $= 118$ m ✓CA	(3)	Using sine rule correctly Correct substitution Calculating PN correctly – units not necessary Correct answer only $\Rightarrow$ full marks
6.2.3	$K\hat{Q}N = 67°$ ✓A	(1)	
6.2.4	✓CA $QN = KN = 200$ m (sides opp = ∠ˢ) ✓R **OR** $\dfrac{QN}{\sin 76°} = \dfrac{KN}{\sin K\hat{Q}N}$ ✓A $\therefore$ $QN = 200$ m ✓CA	(2)	
6.2.5	In Δ PQN, $PQ^2 = PN^2 + QN^2 - 2\,PN.ON \cos Q\hat{N}P$ ✓M $= 118^2 + 200^2 - 2 \times 118 \times 200 \cos 98°$ ✓CA $= 60492{,}970....$ ✓CA $\therefore$ $PQ \approx 246$ m ✓CA	(4) [17]	Correct application of cos rule Incorrect formula $\Rightarrow$ no marks at all Correct substitution Correct calculation of PQ^2 Correct calculation of PQ No penalty if 2nd last step is skipped

QUESTION 7

7.1



$AM = MB = 7$ cm ✓S (line from centre of ⊙ ⊥ chord) ✓R

$OA = x + 3$ ✓S

$OA^2 = OM^2 + AM^2$ (Pythagoras)

$(x+3)^2 = x^2 + 7^2$ ✓CA

$x^2 + 6x + 9 = x^2 + 49$ ✓A

$6x = 40$

$x = \frac{40}{6} = \frac{20}{3}$ ✓CA

$\therefore$ radius $= \frac{29}{3}$ cm ✓CA

OR

$AM = MB = 7$ cm ✓S (line from centre of ⊙ ⊥ chord) ✓R

$OA = OP = r$ ✓S

$OM = r - 3$ ✓CA

$OA^2 = AM^2 + OM^2$

$r^2 = (r-3)^2 + 7^2$ ✓CA

$r^2 = r^2 - 6r + 9 + 49$ ✓A

$\therefore \quad 6r = 58$

$\therefore \quad r = \frac{58}{6} = \frac{29}{3}$ cm ✓CA (7)

Comments:

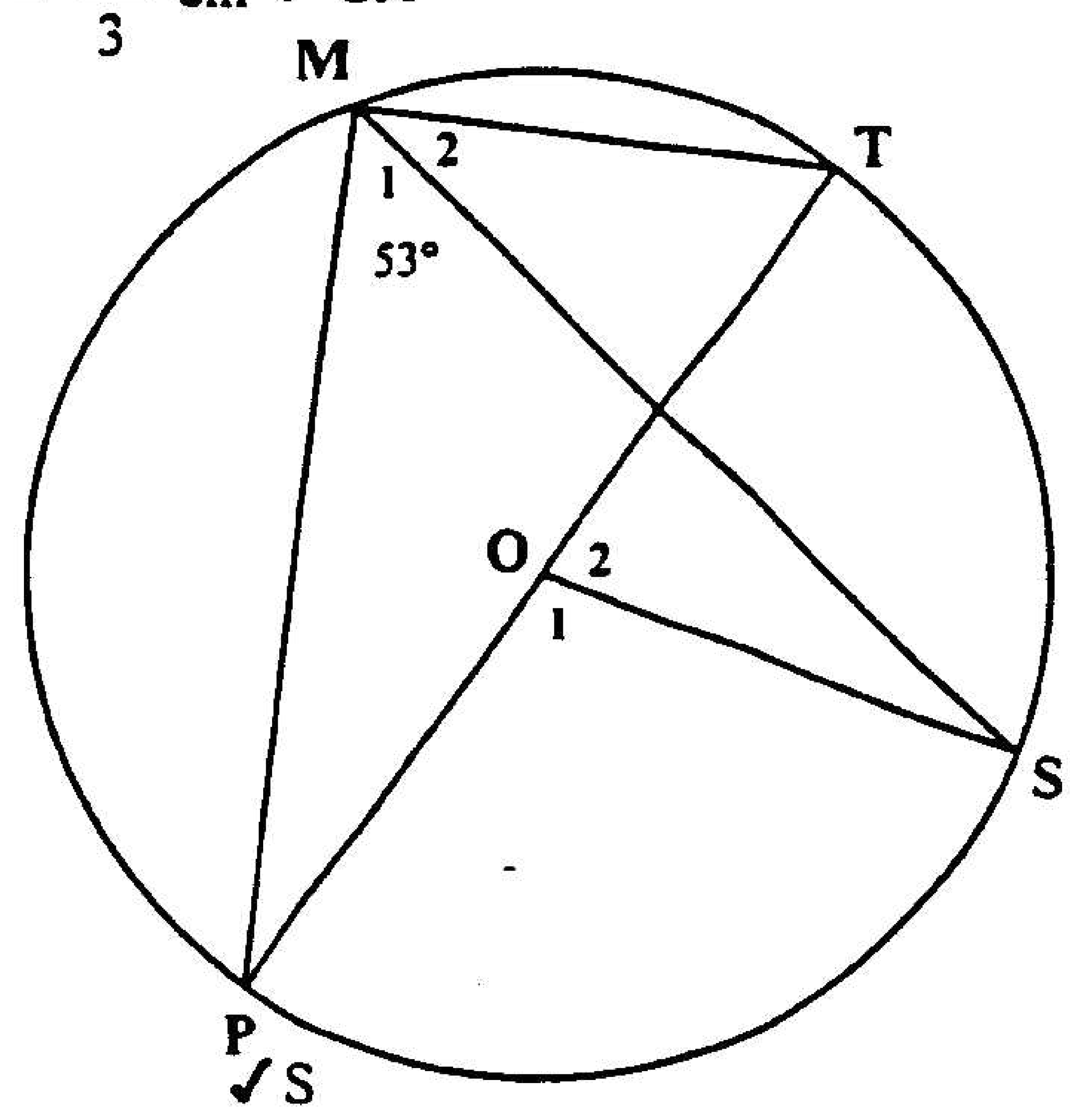
or specific reason – OM ⊥ AB
knowing that radius is $x + 3$

correct substitution is CA mark
squaring correctly (no squaring of binomial ⇒ no mark)

finding x correctly

accept fraction or decimal form (9,67 or 9,7 or rounding off to 10)

7.2



✓S

$\hat{O}_1 = 2\hat{M}_1 = 106°$ (∠ at centre = 2 ∠ on ⊙) ✓R

$\therefore \quad \hat{O}_2 = 74°$ ✓S (sum of adj ∠^s on straight line) ✓R

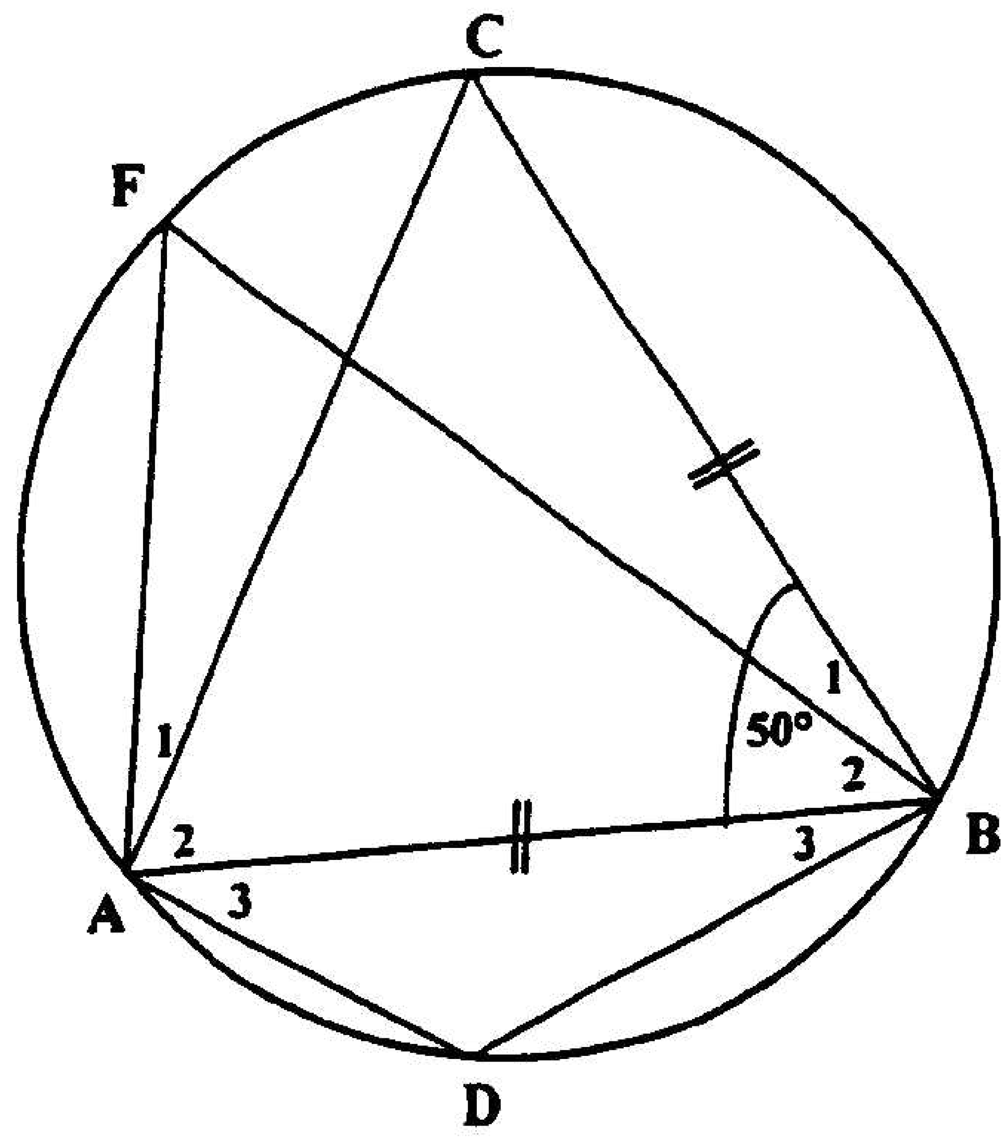
OR

$P\hat{M}T = 90°$ (∠ in semi ⊙)

$\therefore \quad \hat{M}_2 = 37°$ ✓S (adj compl ∠^s) ✓R

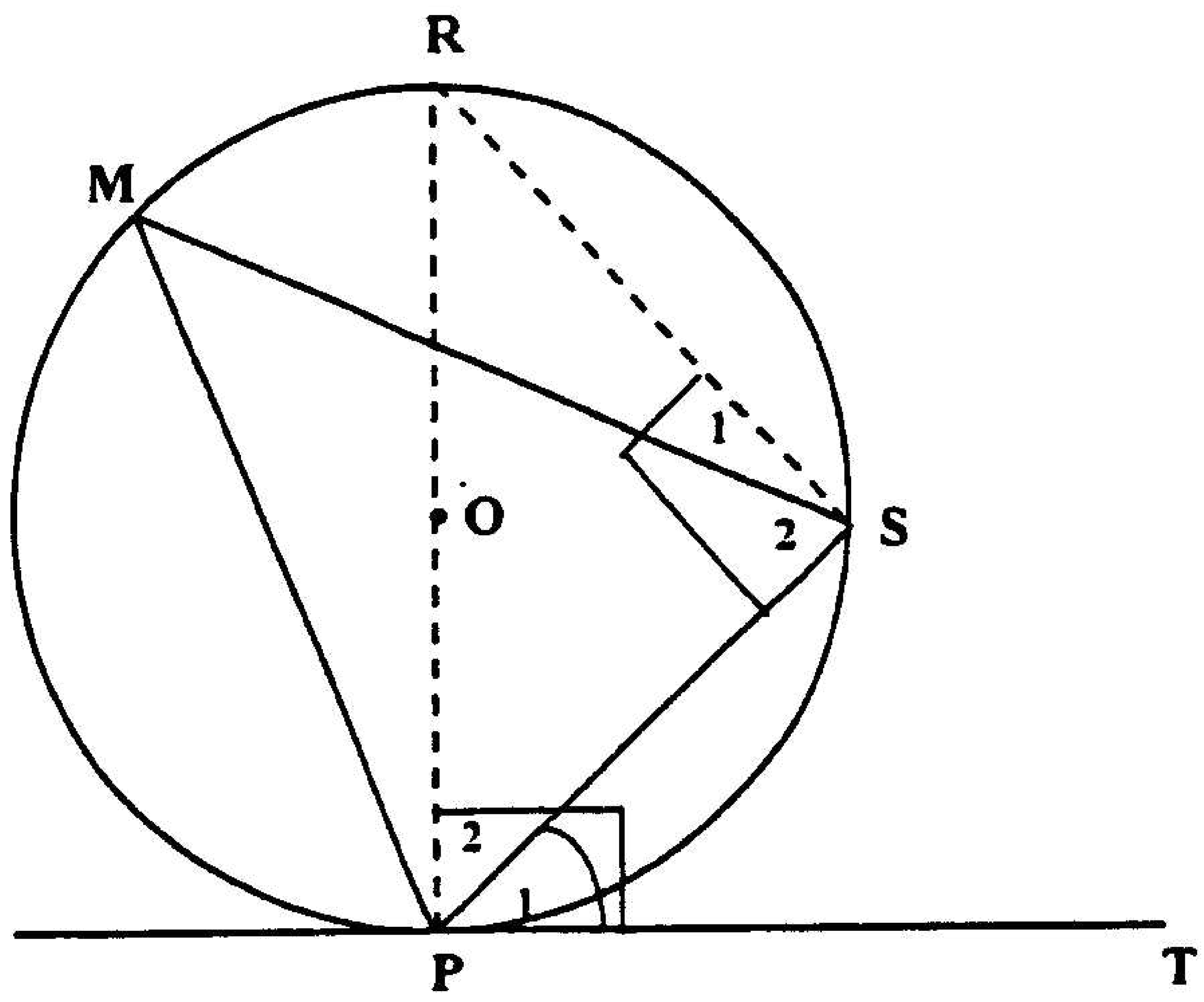
$\therefore \quad \hat{O}_2 = 74°$ ✓S (∠ at centre = 2 ∠ at circ) ✓R

(4)

7.3 	
7.3.1 In Δ ABC $\hat{A}_2 = \hat{C} = 65°$ ✓S (int $\angle$s of Δ; $\angle^s$ opp = sides of Δ) ✓R $\hat{C} = \hat{F} = 65°$ ✓S ($\angle^s$ in same segment) ✓R (4)	at least one of the reasons needed here or $\hat{A}_2 = \hat{F}$ (subt by = chords)
7.3.2 $\hat{D} = 115°$ ✓CA (opp $\angle^s$ of cyclic quad) ✓R (2) [17]	

QUESTION 8

8.1

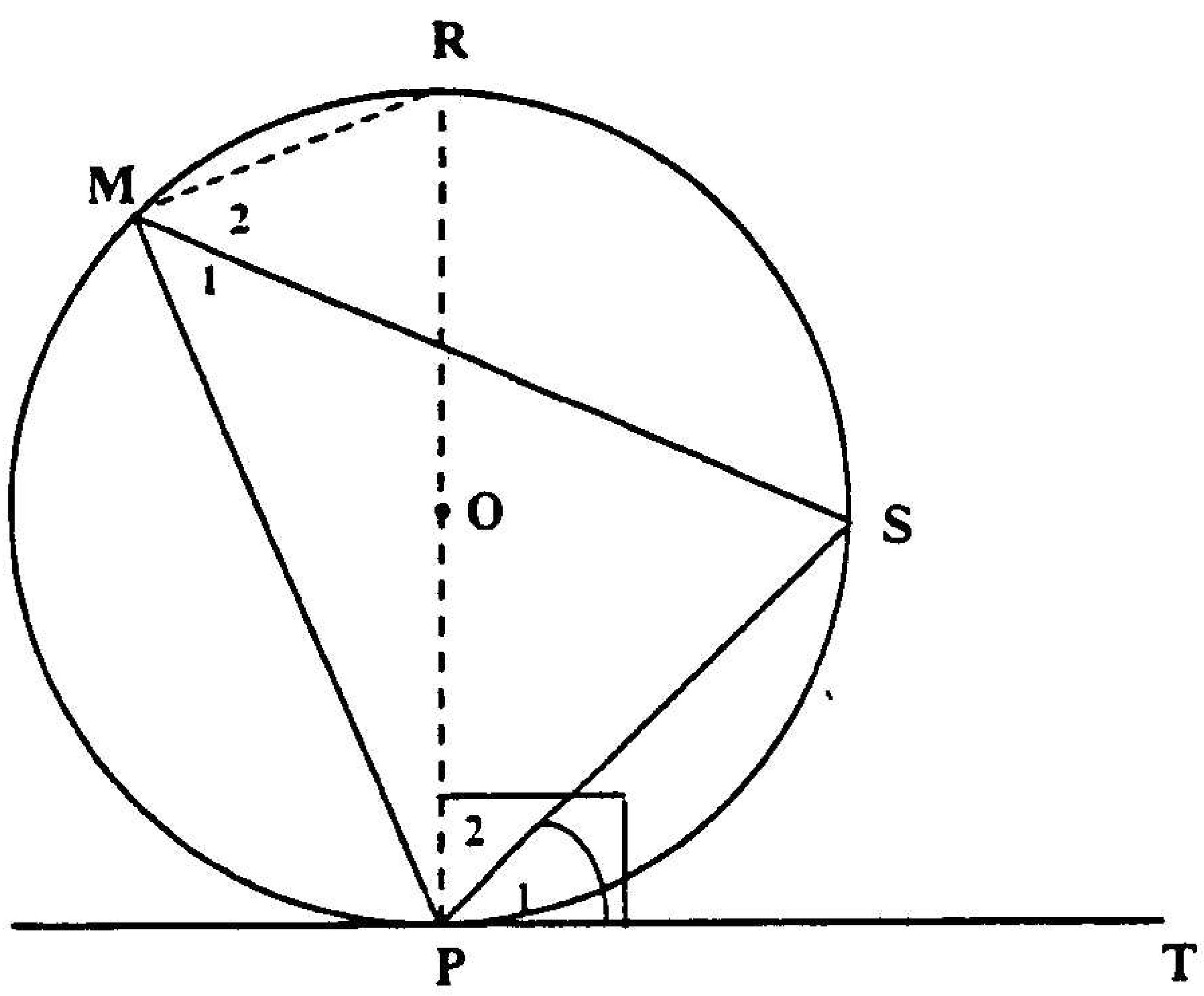


Draw diameter PR and draw RS ✓M

Construction may also only be shown on diagram

$\hat{P}_1 + \hat{P}_2 = 90°$	(rad ⊥ tang) ✓S/R	or diam ⊥ tang
$R\hat{S}P = 90°$ ✓S	(∠ in semi ⊙) ✓R	
∴ $\hat{P}_1 = \hat{R}$	(sum ∠ˢ of Δ) ✓S/R	
and $\hat{R} = \hat{M}$	(∠ˢ in same segment) ✓S/R	
∴ $S\hat{P}T = \hat{M}$		Penalise 1 mark if final statement not shown

OR

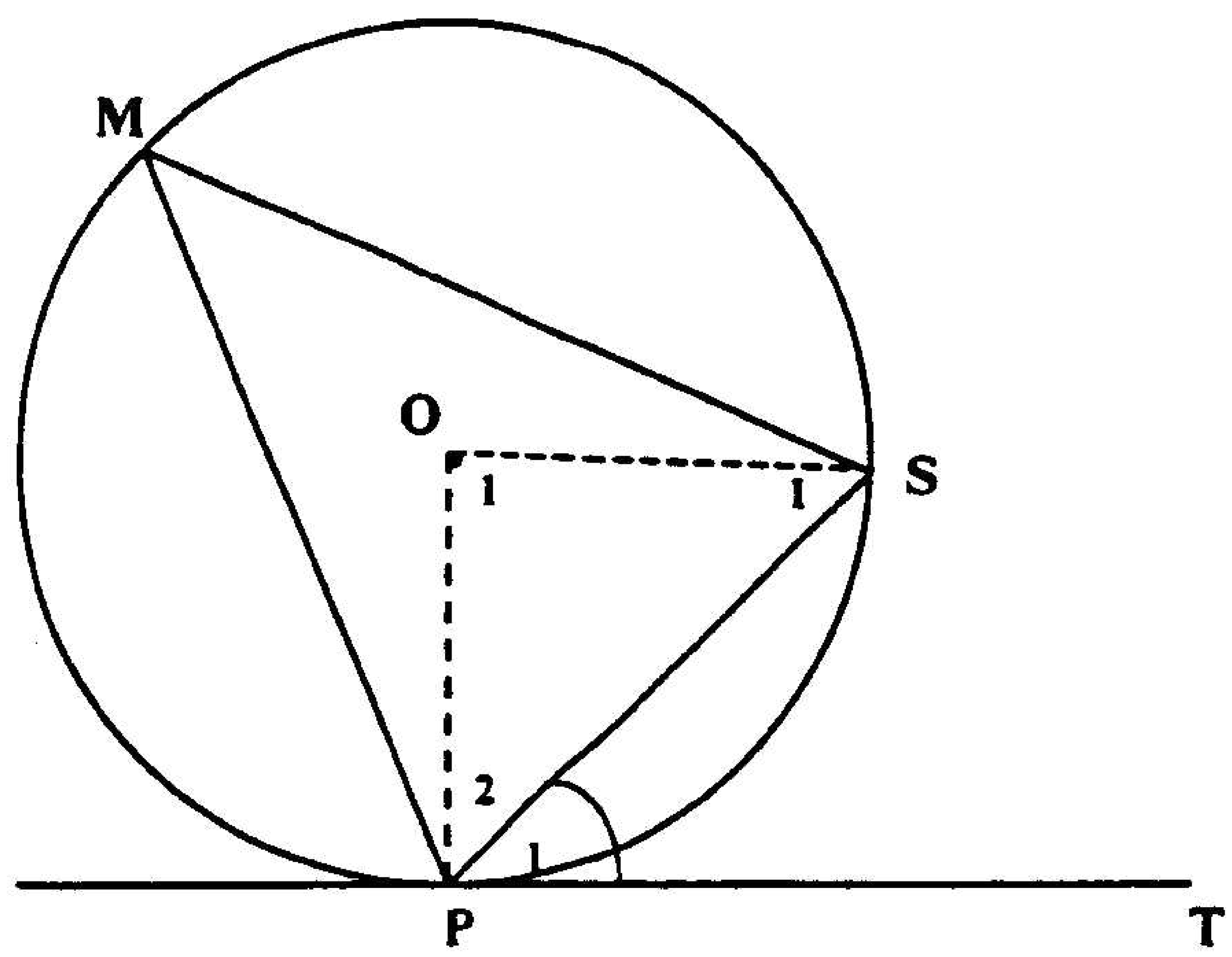


Draw diameter PR and draw RM ✓M

Construction may also be shown on diagram only

$\hat{P}_1 + \hat{P}_2 = 90°$	(rad ⊥ tang) ✓S/R	Or diam ⊥ tang
$R\hat{M}P = 90°$ ✓S	(∠ in semi ⊙) ✓R	
and $\hat{P}_2 = \hat{M}_2$ ✓S	(∠ˢ in same segment) ✓R	
∴ $S\hat{P}T = \hat{M}$		

OR



Draw OP and OS ✓M

$\hat{P}_1 + \hat{P}_2 = 90°$ (rad ⊥ tang) ✓S/R

$\hat{O}_1 = 2\hat{M}$ ✓S (∠ at centre = 2 × ∠ on circle) ✓R

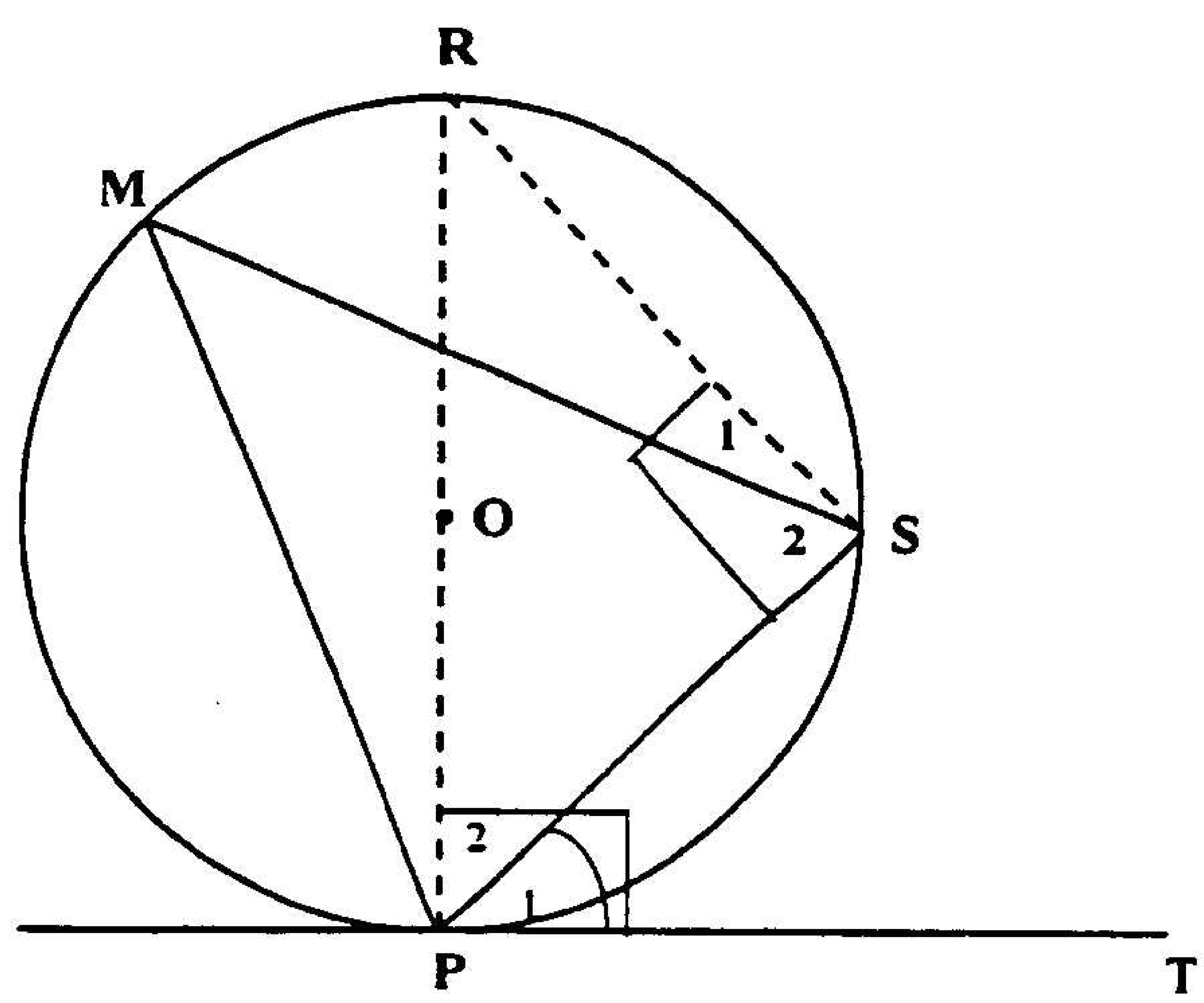
$\hat{S}_1 + \hat{P}_2 = 180° - 2\hat{M}$ (sum of ∠s in Δ) ✓S/R

$\hat{S}_1 = \hat{P}_2 = 90° - \hat{M}$ (equal ∠s opp equal sides) ✓S/R

$\therefore S\hat{P}T = \hat{M}$

Construction can be shown on diagram only
Or diam ⊥ tang

OR



Draw PR ⊥ PT ✓M

Or Draw diameter PR ⊥ PT

∴ PR a diameter (line ⊥ tangent) ✓S/R

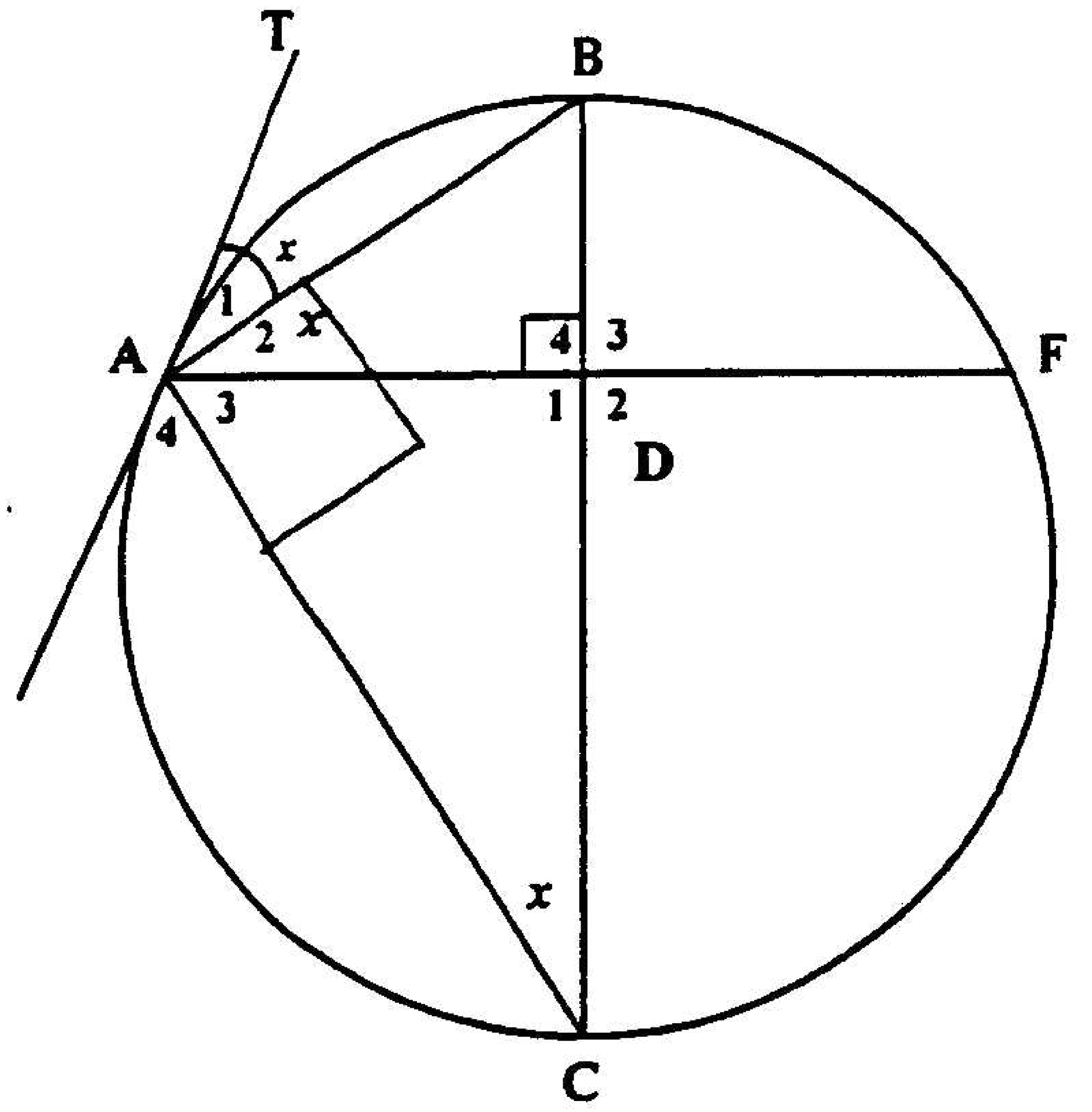
$R\hat{S}P = 90°$ ✓S (∠ in semi ⊙) ✓R

∴ $\hat{P}_1 = \hat{R} = 90° - \hat{P}_2$ (sum ∠s of Δ) ✓S/R

and $\hat{R} = \hat{M}$ (∠s in same segment) ✓S/R

∴ $S\hat{P}T = \hat{M}$ (6)

8.2



8.2.1 $\hat{A}_1 = \hat{C} = x$ ✓S (∠ betw tangent and chord) ✓R

∴ $\hat{A}_3 = 90° - x$ (sum of ∠ˢ of Δ) ✓S/R

$C\hat{A}B = 90°$ ✓S (∠ in semi ⊙) ✓R

∴ $\hat{A}_2 = x$ (5)

8.2.2 In Δ ADB and Δ CDA

(i) $\hat{A}_2 = \hat{C} = x$ (proved) ✓S

(ii) $\hat{D}_4 = \hat{D}_1 = 90°$ (AD ⊥ BC) ✓S/R

(iii) $\hat{B} = \hat{A}_3$ (sum of int. ∠ˢ of Δ) } ✓S/R

∴ Δ ADB ||| Δ CDA (equiangular)

Mark goes either for equiangular or proving the third set of angles equal

OR

$C\hat{A}B = 90°$ ✓S and AD ⊥ BC ✓S (proven / given) ✓S/R

∴ Δ ADB ||| Δ CDA (line from right angle perp to hyp)

(3)

8.2.3 $\frac{AD}{CD} = \frac{DB}{DA}$ ✓S (Δˢ |||) ✓R

AD = DF (Diameter ⊥ chord) ✓S/R

∴ $DF^2 = DB.CD$ (3)

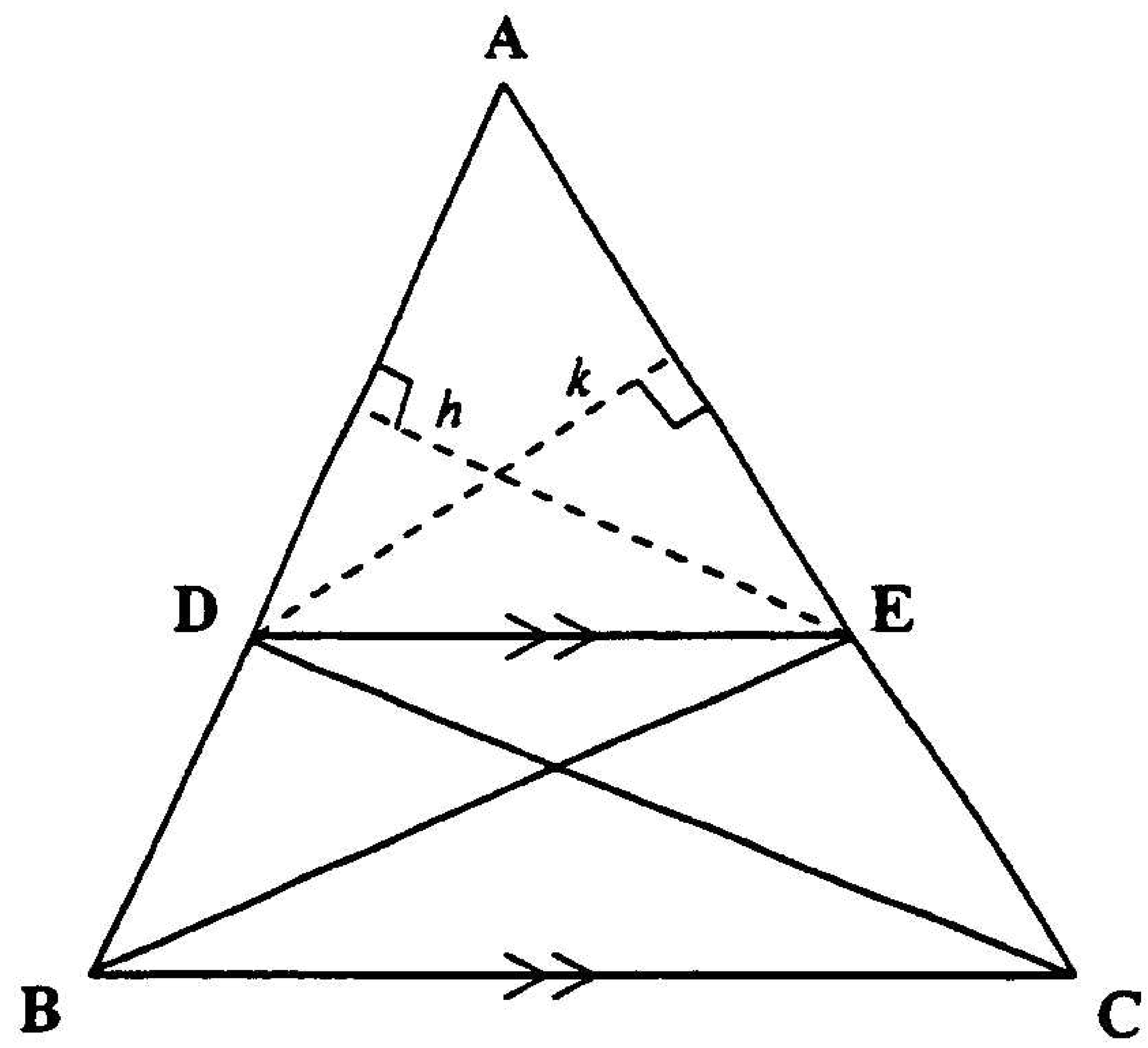
[17]

If candidates start statement with "therefore" after proving similarity in 8.2.2 can get R mark

Or can use as reason altitude from rt ∠ to hypotenuse in rt angled Δ

QUESTION 9

9.1



Join DC and BE and draw perpendiculars h and k. ✓M

$$\frac{\text{Area } \Delta ADE}{\text{Area } \Delta BDE} = \frac{\frac{1}{2} \times AD \times h}{\frac{1}{2} \times DB \times h} = \frac{AD}{DB}$$ ✓S ✓S

$$\frac{\text{Area } \Delta ADE}{\text{Area } \Delta CDE} = \frac{\frac{1}{2} \times AE \times k}{\frac{1}{2} \times EC \times k} = \frac{AE}{EC}$$ ✓S ✓S

Area Δ BDE = Area Δ CDE (same base and betw same || lines) ✓R

$$\therefore \frac{\text{Area } \Delta ADE}{\text{Area } \Delta BDE} = \frac{\text{Area } \Delta ADE}{\text{Area } \Delta CDE}$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

(6)

Don't need to draw heights on diagram
Comparing Δ's with different (heights) ⇒ breakdown ⇒ mark for correct construction only

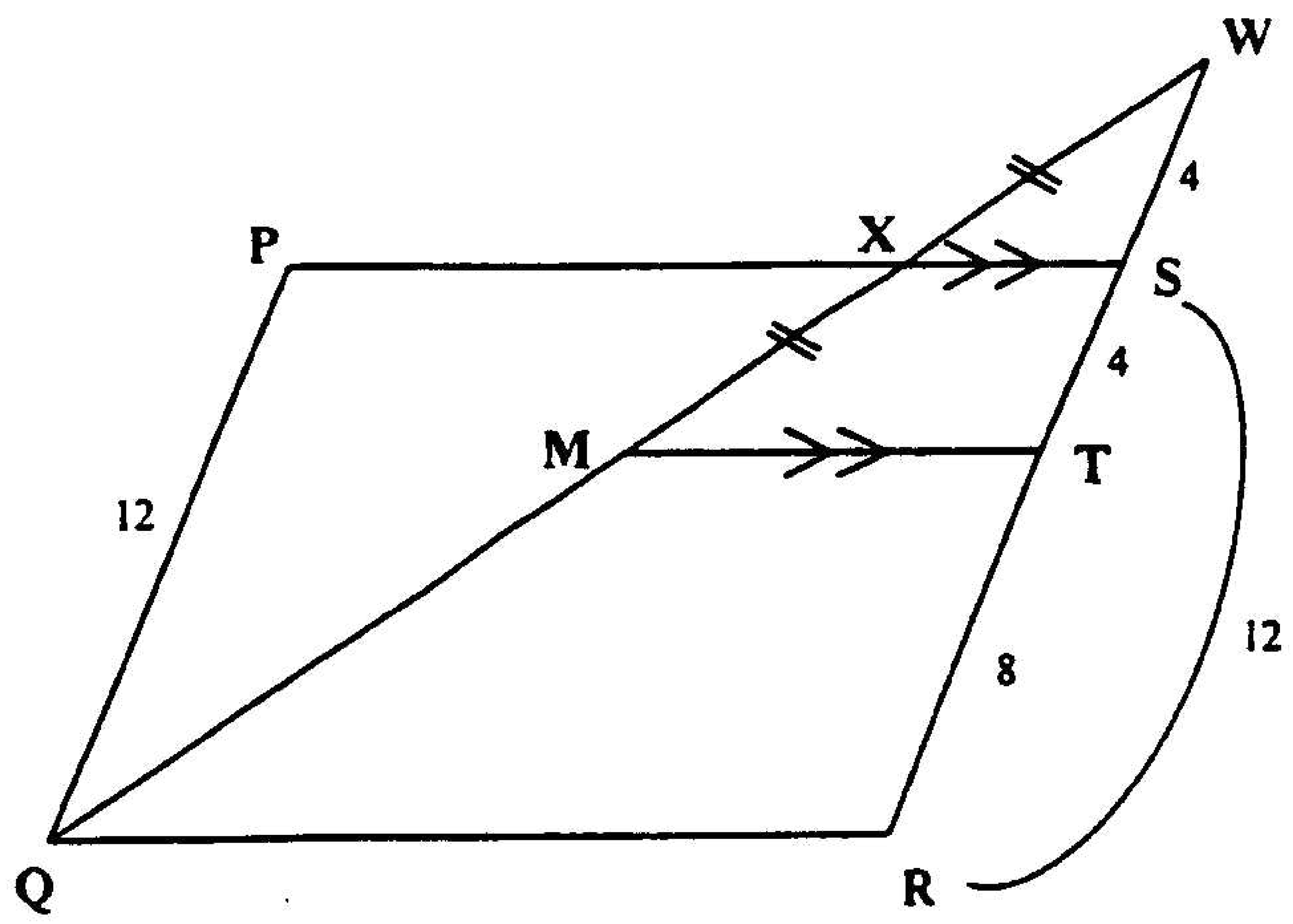
Using the same h in both ratios ⇒ penalty of 1 mark
Using the wrong altitude when comparing Δ's ⇒ penalty of 1 mark

Don't need to indicate altitudes if not used in proof. Construction of DC and BE can be shown on diagram only

No penalty if word "area" is omitted

Same altitude accepted as reason instead of showing $\frac{1}{2}$.base.height.

Must give final conclusion or penalised by 1 mark.

9.2 	
9.2.1 ST = 4 ✓S (line || to one side of Δ cuts other in prop.) ✓R SR = 12 (opp sides parm) ✓S/R ∴ TR= 8 cm ✓S (4)	Or line through mid.pt of one side of Δ parallel to second side
9.2.2 $\frac{WX}{XQ} = \frac{WS}{SR}$ (XS || QR , opp sides of parm ||) ✓S/R $= \frac{4}{12}$ $= \frac{1}{3}$ ✓CA WX = XM $\frac{XM}{XQ} = \frac{WX}{XQ} = \frac{1}{3}$ ✓S	Or any equivalent ratio
OR $\frac{XM}{XQ} = \frac{ST}{SR}$ ✓S (XS∥ MT∥QR) ✓R $= \frac{4}{12}$ $= \frac{1}{3}$ ✓CA (3) [13] **TOTAL : 150**	Or equivalent ratio Answer only scores 2 marks