POSSIBLE ANSWERS FOR :

MATHEMATICS/SG/P1 **FINAL COPY** NOV 2003 **1**

DOE/

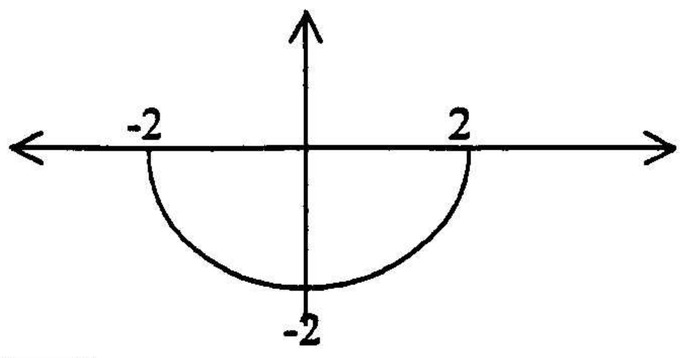
1	1.1	$x^2 - 8x = 0$ $x(x-8) = 0$ $x = 0 \text{ or } x = 8$	3	✓ standard form ✓ factorization ✓ both answers **[If divide by x: max $\frac{1}{3}$]** **[penalty –1 if ≠ 0]**
	1.2	$(k^2-1)^2 - 8(k^2-1) = 0$ $\Rightarrow k^2 - 1 = 0 \text{ or } k^2 - 1 = 8$ $k^2 = 1 \text{ or } k^2 = 9$ $k = \pm 1 \text{ or } k = \pm 3$	4	✓ deductions from 1.1 ✓ values for k^2 ✓ ±; ✓ answer **[If multiply out max $\frac{3}{4}$]** **[only one answer: $k^2 - 1 =$ 8 max $\frac{2}{4}$]**
	OR	$k^4 - 2k^2 + 1 - 8k^2 + 8 = 0$ $k^4 - 10k^2 + 9 = 0$ $(k^2-9)(k^2-1) = 0$ $k = \pm 3 \text{ or } k = \pm 1 \quad Max\frac{3}{4}$		
	1.3	$x^2 - 4x - 4 = 0$ $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-4)}}{2}$ $= \frac{4 \pm \sqrt{32}}{2}$ $= 4{,}83 \text{ or } -0{,}83$	5	**[Wrong formula: 0]** ✓ formula ✓ substitution ✓ simplification ✓✓ correct answers **[-1 Penalty for incorrect rounding off]** **[if simplification error leads to $x = \frac{4 \pm \sqrt{0}}{2}$ max $\frac{2}{5}$]**
	1.4	$b^2 - 4ac = (0)^2 - 4(16) = -64$ *Roots non – real*	3	✓ delta ✓ correct substitution ✓ answer
	OR	$x^2 = -16$ *roots non – real …(since $x^2 \geq 0$)*		✓ ✓✓ statement **[Correct answer without explanation: full marks]**

	1.5.1	$b^2-4ac=(-2)^2-4(1)(k-1)$ $=4-4k+4$ $=8-4k$ For real roots $\Delta \geq 0$ $8-4k \geq 0$ $-4k \geq -8$ $k \leq 2$ $8 \geq 4k$ $2 \geq k$	5	✓ substitution ✓ simplification ✓ $\Delta \geq 0$ ✓ transfer 8 or $-4k$ ✓ answer [BD if $b^2-4ac=0$: max 2/5] if $\Delta > 0$ max $\frac{4}{5}$]
	1.5.2	Put x = 3 into the equation : $3^2-2(3)+(k-1)=0$ $k=-2$	2	✓ substitution ✓ answer

	1.6	Substitute $y=2-x$ in equation 2: $x^2+(2-x)^2=34$ $x^2+4-4x+x^2=34$ $2x^2-4x-30=0$ $(x+3)(x-5)=0$ $x=-3$ or $x=5$ $y=+5$ or $y=-3$	8	✓ $y=2-x$ ✓ substitution ✓simplify/expanding) ✓ standard form ✓ factorisation ✓ x – values ✓✓ corresponding y - values
	OR	$x=2-y$ $(2-y)^2+y^2=34$ $4-4y+y^2+y^2=34$ $2y^2-4y-30=0$ $(y-5)(y+3)=0$ $y=5$ or $y=-3$ $x=-3$ or $x=5$	[30]	✓ $x=2-y$ ✓ substitution ✓simplification (expanding) ✓ standard form ✓ factorisation ✓ y – values ✓✓ corresponding x - values

2	2.1	$f(x)=x^3+mx^2+nx+6$ $f(1)=0$ $1+m+n+6=0$ $m+n=-7$.....(1) $f(-1)=8$ $-1+m-n+6=8$ $m-n=3$.....(2) $m=-2$ and $n=-5$	8	✓ f(1) ✓ = 0 ✓ simplify(eq 1) ✓ f(-1) ✓ = 8 ✓ simplify(eq 2) ✓ m, ✓ n [$f(-1)=0$ and $f(1)=8$: m = -2; n = 3: BD max $\frac{6}{8}$], apply CA]

	2.2	$x^3-2x^2-5x+6=(x-1)(x^2-x-6)$ $=(x-1)(x-3)(x+2)$	3 [11]	✓ $-x$ ✓ -6 ✓ factors
3	3.1.1	$-x^2-4x+5=0$ $(-x-5)(x-1)=0$ *or* $(x+5)(x-1)=0$ $x=-5$ or $x=1$ y intercept is 5 $E(-5;0)$ and $P(1;0)$ and $M(0;5)$	5	✓ $f(x)=0$ ✓ factors ✓ x values ✓ y value ✓ coordinate notation **[Max of 1 penalty mark for not displaying coordinates]**
	3.1.2	$N=\left(-\frac{b}{2a};\frac{-\Delta}{4a}\right)=\left(\frac{-(-4)}{-2};\frac{-36}{-4}\right)=(-2;9)$ *or* $y=-(-2)^2-4(-2)+5=9$ **OR** $f'(x)=-2x-4=0$ $-2x=4$ $x=-2$ 3 marks for x 2 marks for y	5	✓ formula $-\frac{b}{2a}$ or $x=\frac{-5+1}{2}$ ✓substitution to determine x- ✓correct x value ✓formula or substitution of x value ✓ correct y-value
	3.1.3	$y=mx+5$ $0=m(-5)+5$ $m=1; c=5$ (*accept* $y=x+5$) **[Answer by inspection: full marks]**	3	✓ $c=5$ ✓ substitution ✓$m=1$
	OR	$c=5,\quad m=\frac{5}{5}=1$ (*accept* $y=x+5$) $c=5,\quad m=\frac{5}{-5}=-1$ Max $\frac{2}{3}$ $\therefore y=-x+5$		✓ $c=5$ ✓ substitution ✓$m=1$
	3.1.4	$y_T=mx+c$ $=1(-2)+5$ $=3$ $T(-2;3)$ $\therefore NT=9-3$ $=6$ *units*	4	✓ substitution ✓ DT = 3 ✓ ND = 9 ✓ NT = 9 – 3 **[answer only full marks]**
	OR	$(-x^2-4x+5)-(x-5)$ $NT=-x^2-5x$ *by* $x=-2$ $=-(-2)^2-5(-2)$ $=-4+10$ $NT=6$		✓ difference (y_f-y_g) ✓ NT ✓ substitute –2 ✓ answer
	3.1.5	coordinates of S = (–4;5)	2	✓✓one for each coordinate

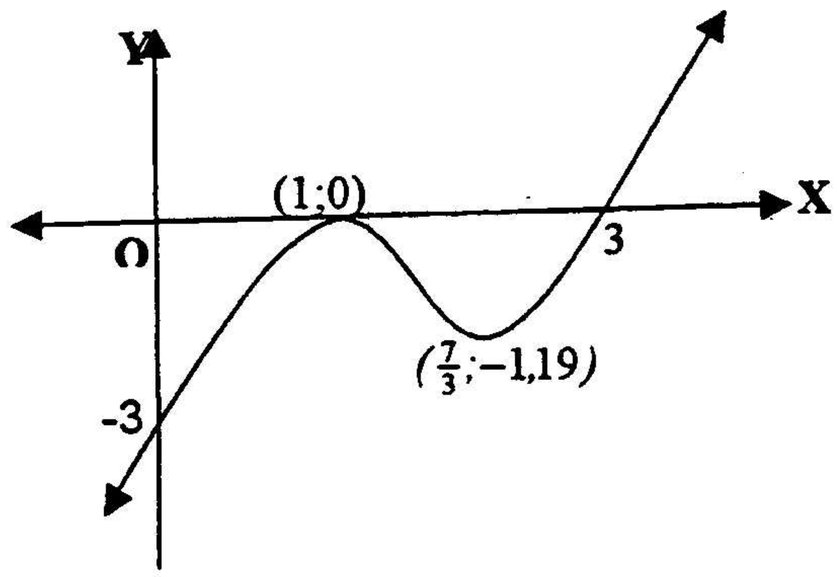
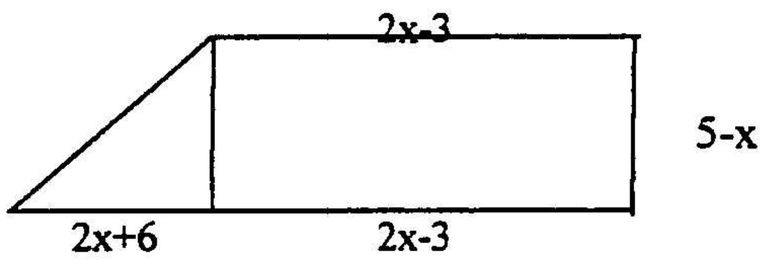
	3.2		3	✓ form (semi circle) ✓ below x-axis ✓ intercepts or radius
	3.3	C	2 **[24]**	✓✓ **[If D is answer: only 1]**

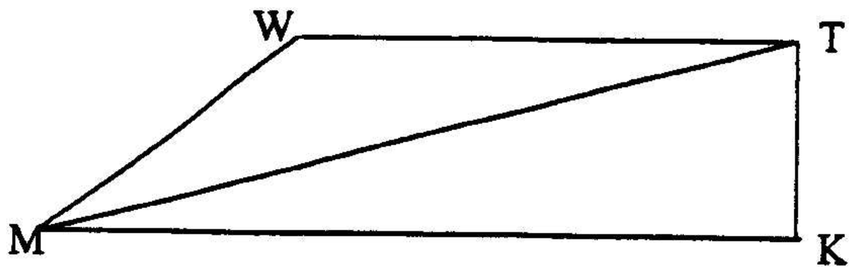
	4.1.1	$\frac{3^{n+1}-3^n}{3^{n-1}}$ $=\frac{3^n.3-3^n}{3^n.3^{-1}}$ $=\frac{3^n(3-1)}{3^n.3^{-1}}$ $=\frac{2}{\frac{1}{3}}$ Or $\frac{3-1}{\frac{1}{3}}$ $=6$	4	✓common factor/ split factors ✓correct terms in brackets ✓$\frac{1}{3}$ ✓simplification
	OR	$\frac{3^{n-1}(3^2-3)}{3^{n-1}}$ $=9-3=6$		✓✓✓ ✓
	OR	$\frac{3^{n+1}}{3^{n-1}}-\frac{3^n}{3^{n-1}}$ $=3^2-3^1$ $=9-3=6$		✓ ✓✓ (one each) ✓
	4.1.2	$\frac{\sqrt{98}-\sqrt{50}}{\sqrt{2}}=\frac{\sqrt{49\times2}-\sqrt{25\times2}}{\sqrt{2}}=\frac{7\sqrt{2}-5\sqrt{2}}{\sqrt{2}}=2$	3	✓ 98 = 2.49; 50 = 2.25 ✓ $7\sqrt{2}$ *and* $5\sqrt{2}$ ✓answer
	OR	$\frac{\sqrt{98}-\sqrt{50}}{\sqrt{2}}=\frac{\sqrt{98}.\sqrt{2}-\sqrt{50}.\sqrt{2}}{2}=\frac{\sqrt{196}-\sqrt{100}}{2}=\frac{14-10}{2}=2$		
	4.1.3	$log_3 9-log_5\sqrt{5}=log_3 3^2-log_5 5^{\frac{1}{2}}$ $=2log_3 3-\frac{1}{2}log_5 5$ $=2-\frac{1}{2}$ $=1\frac{1}{2}$	5	✓✓ each term ✓ log law ✓ simplification ✓answer

	4.2.1	$x^{-\frac{3}{4}} = 8$ $x = 8^{-\frac{4}{3}}$ $= \left(2^3\right)^{-\frac{4}{3}}$ $= 2^{-4}$ $= \frac{1}{16}$	4	✓to the power $-\frac{4}{3}$ ✓ $8 = 2^3$ ✓ 2^{-4} ✓ answer
	OR	$-\frac{3}{4}\log x = \log 8$ $\log x = -\frac{4}{3}\log 8$ $= \log(2^3)^{-\frac{4}{3}}$ $= \log 2^{-4}$ $x = 2^{-4}$ $= \frac{1}{16}$ $\left(x^{\frac{-3}{4}}\right)^{-4} = (2)^{-4}$ $x^3 = 2^{-12}$ $x = 2^{-4}$ $x = \frac{1}{16}$		✓ ✓ ✓ ✓
	4.2.2	$5^x = 0{,}20 = \frac{1}{5} = 5^{-1}$ $x = -1$	3	✓ $\frac{1}{5}$ ✓ 5^{-1} ✓answer
	OR	$x \log 5 = \log 0{,}2$ $x = \frac{\log 0{,}2}{\log 5}$ $= \frac{\log \frac{1}{5}}{\log 5}$ $= \frac{-\log 5}{\log 5}$ $= -1$ **IF** $x \log 5 = \log 0{,}2$ $x = \frac{\log 0{,}2}{\log 5}$ $= -1$ $max\ \frac{1}{3}$		✓ log law ✓ $0{,}2 = \frac{1}{5}$ ✓ answer
	4.2.3	(i) $x = (\frac{1}{8})^3$ $= \frac{1}{512}$ (ii) $\frac{1}{8} = x^3$ $x = \frac{1}{2}$ $\log x = 3\log \frac{1}{8}$ $x = (\frac{1}{8})^3 = \frac{1}{512}$ $\log \frac{1}{8} = 3\log x$ $\frac{1}{8} = x^3 \Rightarrow x = \frac{1}{2}$	2 2	(i) ✓ $x = (\frac{1}{8})^3$ ✓answer (ii) ✓exponential form ✓answer.
	4.3	$3^x = 15$ $x = \log_3 15$ *or* $x \log 3 = \log 15$ $x = \frac{\log 15}{\log 3}$ $= 2{,}46$	3 [26]	✓log form ✓ $\frac{log\,15}{log\,3}$ ✓answer

5	5.1	$\sum_{n=1}^{4} 3.2^{n-1} = 3.2^0 + 3.2^1 + 3.2^2 + 3.2^3$ $= 3 + 6 + 12 + 24$ $= 45$	$S_n = \frac{a(r^n - 1)}{r - 1}$ $= \frac{3(2^4 - 1)}{2 - 1}$ $= 45$	4	✓T_1 ✓T_4 ✓ terms added ✓answer
	5.2	$T_n = a + (n-1)d$ $901 = 4 + (n-1)3$ $3n - 3 = 901 - 4$ $3n = 900$ $n = 300$ terms $S_n = \frac{n}{2}[a + l]$ $= \frac{300}{2}[4 + 901]$ $= 135\,750$	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{300} = \frac{300}{2}[2(4) + 299(3)]$ $= 150(305)$ $= 135750$	6	✓ formula ✓substitution ✓ answer ✓ formula ✓substitution ✓ answer
	5.3.1	$T_n = ar^{n-1}$ $T_{20} = 1(2)^{19}$ $= 524\,288$ cents $= R5\,243$	$1 \times 2 \times 2 \times 2 \ldots\ldots 19$ *times* $= 524288$	4	✓ formula ✓ substitution ✓ 524 288 c or R5 242,88 ✓ rounding off **[max 1 mark penalty for not rounding off]**
	5.3.2	$S_n = \frac{a(r^n - 1)}{r - 1}$ $S_{20} = \frac{1(2^{20} - 1)}{2 - 1}$ $= R10\,486$ (accept R10 485,75)	$\times 2 = R10486$	3	✓ formula ✓ substitution ✓ answer
	5.4	$A = P\left(1 + \frac{r}{100}\right)^n$ $16\,000 = P\left(\frac{100 + 18}{100}\right)^4$ or $P(1{,}18)^4$ $P = 8\,253$ (*accept* 8 252,62) rand **[(NB: B/D: Max of $\frac{2}{5}$ if A $\leftrightarrow$ P) (Answer R 31020)]**		5 [22]	✓ formula **[If use formula with "-" sign: max $\frac{3}{5}$ ($R35389$)]** ✓ A correctly substituted ✓ r = 18 & n = 4 ✓ P subject of formula ✓ answer

6	6.1	$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ $= \lim_{h\to 0} \frac{-(x+h)^2 + x^2}{h}$ $= \lim_{h\to 0} \frac{-x^2 - 2xh - h^2 + x^2}{h}$ $= \lim_{h\to 0} \frac{-2xh - h^2}{h}$ $= \lim_{h\to 0} (-2x - h)$ $= -2x$	5	✓formula ✓substitution ✓expansion ✓simplification ✓answer **[Penalize once only one mark for notation in question 6]**
	OR	$f(x+h) = -(x+h)^2$ $= -x^2 - 2xh - h^2$ $f(x+h) - f(x) = -2xh - h^2$ $\frac{f(x+h)-f(x)}{h} = -2x - h$ $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ $= \lim_{h\to 0} (-2x - h)$ $= -2x$		
	6.2.1	$f(x) = x^4 + x^{\frac{1}{4}}$ $f'(x) = 4x^3 + \frac{1}{4}x^{-\frac{3}{4}}$	3	✓exponential form ✓✓derivative of each term **[Attempt to write with pos exponents fail:ignore]**
	6.2.2	$y = \frac{x(x^2-1)}{x-1} = \frac{x(x+1)(x-1)}{(x-1)} = x^2 + x$ $\frac{dy}{dx} = 2x + 1$	4	✓factorisation ✓factorisation ✓simplification ✓derivative
	6.3	Average gradient $= \frac{f(4)-f(1)}{4-1}$ $f(1) = -5$ $f(4) = 40$ $\therefore$ *average gradient* $= 15$ **[For stating general formula ($\frac{f(b)-f(a)}{b-a}$) one mark]**	4 [16]	✓formula ✓✓each y-value ✓answer

7.1	$y-intercept=-3$ $x-intercepts:(x-1)(x-3)(x-1)=0$ $x=1$ or $x=3$ $Turning\ points: 3x^2-10x+7=0$ $(3x-7)(x-1)=0$ $x=\frac{7}{3}$ or $x=1$ Coordinates of turning points: If $x=\frac{7}{3}, then\ y=(\frac{7}{3})^3-5(\frac{7}{3})^2+7(\frac{7}{3})-3=-1{,}19 \quad (-\frac{32}{27})$ If $x=1, then\ y=0$  	y int ✓✓ x int ✓✓✓ TP ✓✓✓✓✓✓✓✓ Graph: ✓ shape ✓ Tpts	14	✓y- intercept ✓✓ x- intercepts ✓derivative ✓= 0 ✓factors ✓solutions ✓subst in correct equation ✓✓correct y-values ✓shape ✓x-intercepts ✓ y-intercept ✓TPs shown **[If graph is completely correct, with all information showed on graph: full marks]**
7.2	7.2.1 $A=\frac{1}{2}(WT+MK).TK$ $=\frac{1}{2}(2x-3+4x+3).(5-x)$ $=\frac{1}{2}(6x).(5-x)$ $=15x-3x^2$	$3x(5-x)$ $=15x-3x^2$	2	✓formula/ substitution ✓simplification
	OR  $A=(2x-3)(5-x)+\frac{1}{2}(2x+6)(5-x)$ $=10x-2x^2-15+3x+5x-x^2+15-3x$ $=-3x^2+15x$			

	OR	 $Area\ of\ trap = area(\Delta WTM + \Delta TMK)$ $=\frac{1}{2}(2x-3)(5-x)+\frac{1}{2}(4x+3)(5-x)$ $=\frac{1}{2}\left[-2x^2+13x-15+(-4x^2+17x+15\right]$ $=\frac{1}{2}\left[-2x^2+13x-15-4x^2+17x+15\right]$ $=\frac{1}{2}\left[-6x^2+30x\right]$ $-3x^2+15x$		
	7.2.2	$A'=0$ $15-6x=0$ $6x=15$ $x=2,5\ meter$	3	✓derivative = 0 ✓derivative ✓correct answer
	7.2.3	$\text{maximum area} = 15(2,5)-3(2,5)^2$ $=18,75m^2$ **[7.2.2 and 7.2.3 can be treated as a unit for marking purposes]**	2 [21]	✓substitution ✓answer

TOTAL: 150