DEPARTMENT OF EDUCATION

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DEPARTEMENT VAN ONDERWYS

POSSIBLE ANSWERS FOR :

SENIOR SERTIFIKAAT-EKSAMEN / SENIOR CERTIFICATE EXAMINATION
WISKUNDE HG / MATHEMATICS HG
VRAESTEL II / PAPER II
NOVEMBER 2003

✓ A ≡ 1 mark for accuracy

✓ CA ≡ 1 mark for consistent accuracy

✓ M ≡ 1 mark for correct method

✓ S ≡ 1 mark for the correct statement

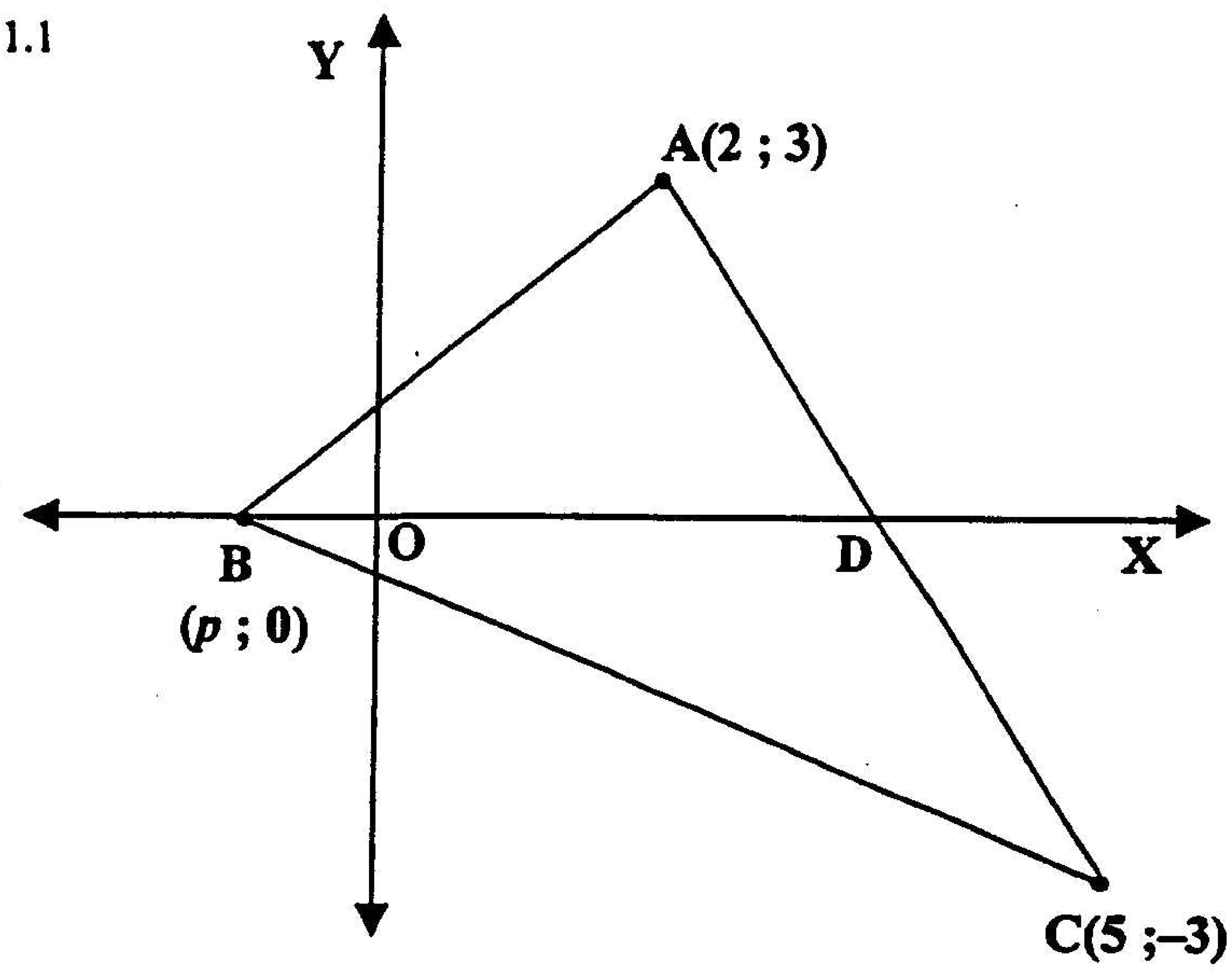
✓ R ≡ 1 mark for the correct reason

✓ S/R ≡ 1 mark for the correct statement with the correct reason

Penalise candidate once only in entire paper for rounding off. (Possible questions: 1.1.3; 1.1.4; 5.3; 6.2.4)

QUESTION 1

1.1

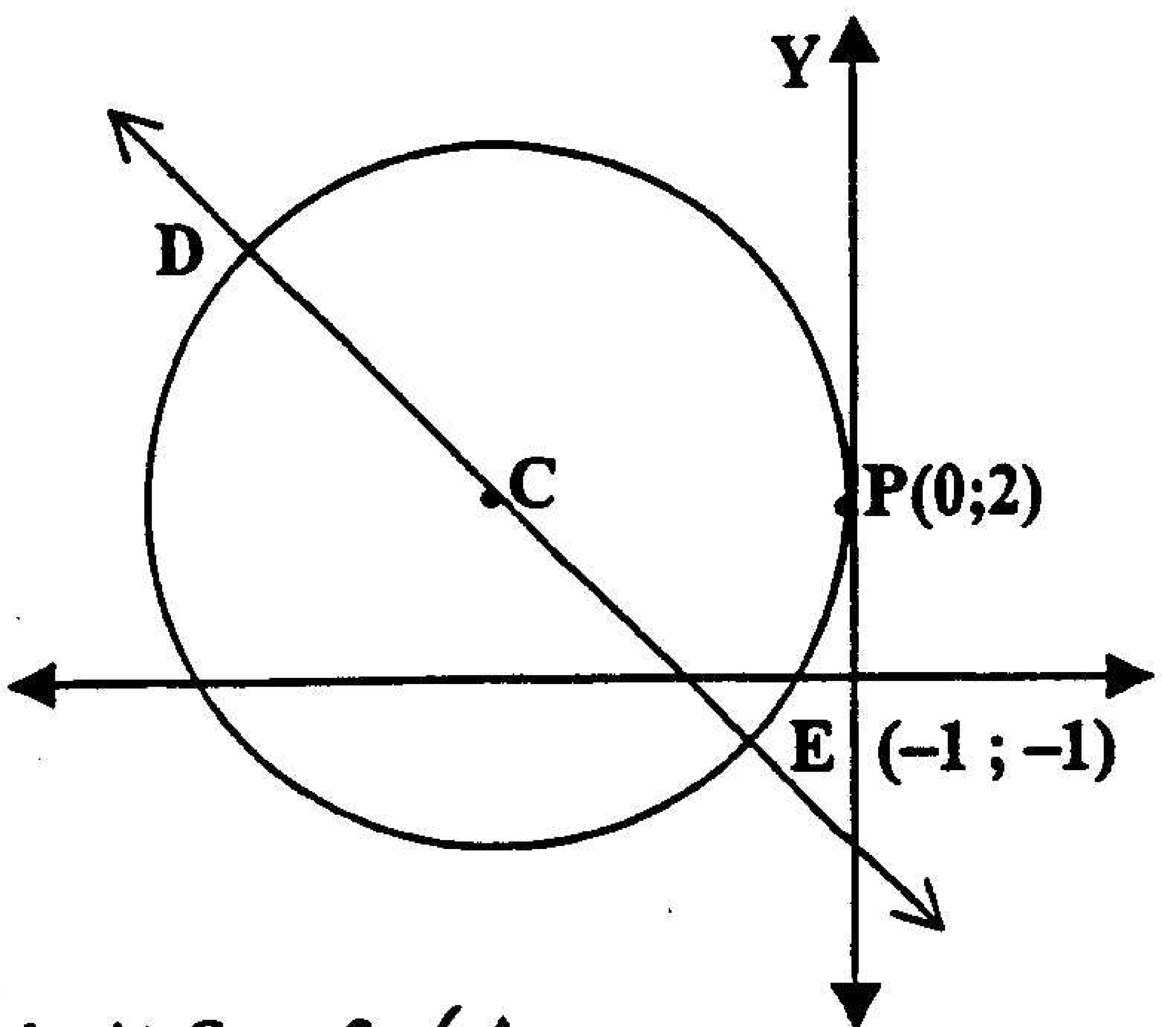


1.1.1 $m_{AC} = \frac{-3-3}{5-2}$ or $\frac{3-(-3)}{2-5} = -\frac{6}{3} = -2$ ✓ A	Finding the gradient correctly at any of the steps
$y-3=-2(x-2)$ **OR** $y+3=-2(x-5)$ ✓ M ✓ A **OR** $-3=-2(5)+c$ **OR** $3=-2(2)+c$	Using an appropriate formula correctly Substituting a point in the line correctly
$c=7$ $y=-2x+7$ ✓ CA	Simplifying correctly
at D, $y=0$ ✓ A $\therefore$ $x=3,5$ ✓ CA D(3,5 ; 0)	Substituting $y=0$ Calculating x correctly
OR	
$m_{AC} = \frac{-3-3}{5-2}$ or $\frac{3-(-3)}{2-5} = -\frac{6}{3} = -2$ ✓ A	Finding the gradient correctly at any of the steps
✓ ✓ ✓ ✓ CA ✓ A $\therefore$ By displacement $x=3,5$ and $y=0$	Full marks provided gradient is correct
OR	
D the midpoint of AC $\therefore$ $x = \frac{x_A + x_C}{2}$ } ✓ M	Stating that D is midpoint of AC or Using correct formula for midpoint
$= \frac{2+5}{2} = 3,5$ ✓ ✓ ✓ ✓ A	Calculating x correctly
$y=0$ ✓ A	Value of y
OR ✓ ✓ ✓ ✓ A ✓ A Answer only $x=3,5$ and $y=0$	Maximum 5 marks
OR	
$m_{AD} = m_{AC}$ ✓ M	Equating correct gradients
✓ A ✓ A $\therefore$ $\frac{0-3}{x-2} = \frac{3+3}{2-5} = \frac{2}{-1}$	Substituting correct values correctly
$\therefore$ $2x-4=3$ ✓ CA $\therefore$ $x=3,5$ ✓ CA $y=0$ ✓ A	Simplifying correctly Calculating x correctly Value of y
OR	

$m_{AD} = m_{dC}$ ✓ M	Equating correct gradients
✓ A ✓ A $\therefore \dfrac{0-3}{x-2} = \dfrac{0+3}{x-5}$	Substituting correct values correctly
$\therefore \dfrac{-3}{x-2} = \dfrac{3}{x-5}$	
$\therefore 3x-6 = -3x+15$ ✓ CA $\therefore x = 3{,}5$ ✓ CA $y = 0$ ✓ A (6)	Simplifying correctly Calculating x correctly Value of y
1.1.2 BC = AC $\sqrt{(p-5)^2+(0+3)^2} = \sqrt{(5-2)^2+(3+3)^2}$ ✓ M ✓ A	Using distance formula correctly Correct substitution
$(p-5)^2+9 = 45$ $(p-5)^2 = 36$ OR $p^2-10p-11=0$ ✓ CA $p-5=-6$ $(p+1)(p-11)=0$ ✓ CA $p=-1$ ✓ CA (5)	Simplifying correctly $-\sqrt{\ }$ or factorising correctly Value of p – must be negative Both answers – not last mark Answer only – no mark at all Using BC = AB or AC = AB – maximum 1 mark providing dist formula is used
1.1.3 $\tan A\hat{D}X = m_{AC} = -2$ ✓ M ✓ CA $A\hat{D}X = 116{,}6°$ ✓ CA (3)	Using correct formula for inclination ∠ Substituting correct gradient from 1.1.1 Calculating inclination ∠ correctly If $m_{AC} < 0$, ∠ must be obtuse If $m_{AC} > 0$, ∠ must be acute
1.1.4 $\tan A\hat{B}D = 1$ ✓ CA $A\hat{B}D = 45°$ ✓ CA $\hat{A} = 116{,}6° - 45° = 71{,}6°$ ✓ CA	Subst. correct gradient into correct formula Calculating ∠ of incl. correctly Calculating A correctly
OR In ΔABC: $\cos A = \dfrac{18+45-45}{2\sqrt{18}\sqrt{45}}$ ✓ M ✓ A $\therefore A = 71{,}6°$ ✓ CA	Using cos rule correctly Substituting correctly into cos rule Calculating A correctly
OR In ΔABD: $\cos A = \dfrac{11{,}25+18-20{,}25}{2\sqrt{11{,}25}\sqrt{18}}$ ✓ M ✓ A $\therefore A = 71{,}6°$ ✓ CA	Using cos rule correctly Substituting correctly into cos rule Calculating A correctly
OR $\dfrac{\sin A}{BD} = \dfrac{\sin A\hat{B}D}{AD}$	Using sin rule correctly
$\therefore \sin A = \dfrac{4{,}5 \sin 45°}{\dfrac{\sqrt{45}}{2}}$ ✓ M ✓ A	Substituting correctly into sin rule
$\therefore A = 71{,}6°$ ✓ CA (3)	Calculating A correctly
OR $\tan A = \dfrac{-2-1}{1+(-2).1}$ ✓ M ✓ A $\therefore A = 71{,}6°$ ✓ CA	Using tan expansion Correct substitution Calculating A correctly

1.2 1.2.1 $x^2+y^2+6y=7$ $x^2+y^2+6y+3^2=7+9$ ✓ A ✓ CA $x^2+(y+3)^2=16$ ✓ CA M$(0\,;-3)$✓ CA (4)	Completing square correctly Changing LHS into midpt form correctly Correct value for x coordinate Correct value for y coordinate Correct answer only – full marks
1.2.2 Subst. $y=x+1$ in circle equation $x^2+(x+1)^2+6(x+1)-7=0$ ✓ M $x^2+x^2+2x+1+6x+6-7=0$ ✓ CA $2x^2+8x=0$ ✓ CA $2x(x+4)=0$ OR $\Delta=64$ ✓ CA $x=0$ or $x=-4$, TWO SOLUTIONS ✓ CA thus NOT a tangent (OR ∴ line a secant) ✓ CA	Substituting correct equation correctly into any correct form of equation of circle Correct multiplication Correctly into standard form Factorising correctly or Calculating Δ correctly Correct justification Correct conclusion Answer only – no mark at all
OR $x=y-1$ $(y-1)^2+y^2+6y-7=0$ ✓ M $y^2-2y+1+y^2+6y-7=0$ ✓ CA $2y^2+4y-6=0$ ✓ CA $y^2+2y-3=0$ ✓ CA $(y+3)(y-1)=0$ ✓ CA thus NOT a tangent (OR ∴ line a secant) ✓ CA	Substituting correct equation correctly into any correct form of equation of circle Correct multiplication Correctly into standard form Factorising correctly or Calculating Δ correctly Correct justification Correct conclusion Answer only – no mark at all
OR Centre of ⊙ is $(0\,;-3)$ ✓ A y-intercept of ⊙ is $(0\,;1)$ ✓ A y-intercept of line is $(0\,;1)$ ✓ A ∴ line and ⊙ intersect at $(0\,;1)$ ✓ A ∴ at this point line is not perpendicular to radius ✓ A ∴ line not a tangent / line a secant ✓ A (6) [27]	

QUESTION 2

	**OR** Further alternative for 2.1.1 Find equation of perp bisector of PE $x = 1 - 3y$ and subst into $3x + 4y + 7 = 0$ 1 M mark for the substitution then 1 C A mark for $y = 2$ 1 CA mark for $x = -5$ rest of marks as in first solution.
2.1 2.1.1 At C, $y = 2$ ✓ A	Determining correct value for y coordinate of C
$\therefore 3x + 4(2) + 7 = 0$ ✓ M $\therefore 3x = -15$ $\therefore x = -5$ ✓ CA $\therefore$ C(–5 ; 2)	Substituting y-co ordinate of C in eq. of line Calculating x coordinate of C correctly
$\therefore r = 5$ OR $r^2 = 25$ ✓ CA $\therefore (x+5)^2 + (y-2)^2 = 5^2$ [OR 25] ✓ CA	Calculating radius correctly Correctly writing equation for circle in correct form
OR	
$CP^2 = CE^2$ ✓ M $\therefore (x-0)^2 + (y-2)^2 = (x+1)^2 + (y+1)^2$ ✓ A	Equating correct line segments Applying distance formula correctly
$\therefore (x-0)^2 + (2-2)^2 = (x+1)^2 + (2+1)^2$ $\therefore x^2 = x^2 + 2x + 1 + 9$ $\therefore 2x = -10$ $\therefore x = -5$ ✓ CA $\therefore$ C(–5 ; 2)	Calculating value of x correctly
$\therefore r = 5$ OR $r^2 = 25$ ✓ CA $\therefore (x+5)^2 + (y-2)^2 = 5^2$ [OR 25] ✓ CA (5)	Calculating radius correctly Correctly writing equation for circle in correct form
2.1.2 DE = $2r$ = 10 units ✓ CA (1)	Calculating DE correctly
2.1.3 mid.pt. of PE is $(-\frac{1}{2} ; \frac{1}{2})$ ✓ A ✓ A	Calculating coordinates of midpt correctly
$m_{PE} = \frac{3}{1}$ ✓ A	Calculating gradient of PE correctly
$\therefore m_{perp} = -\frac{1}{3}$ ✓ CA $y - \frac{1}{2} = -\frac{1}{3}(x + \frac{1}{2})$ OR $\frac{1}{2} = -\frac{1}{3}(-\frac{1}{2}) + c$ ✓ M $c = \frac{1}{3}$	Deducing gradient of perp bisector correct Substituting coordinates of midpt correctly
$\therefore y = -\frac{1}{3}x + \frac{1}{3}$ OR $x + 3y - 1 = 0$ OR any equiv form ✓ CA	Simplifying equation correctly
OR	
$m_{PE} = \frac{3}{1}$ ✓ A	Calculating gradient of PE correctly
$\therefore m_{perp} = -\frac{1}{3}$ ✓ CA ✓ A ✓ A perp bisector passes through C(-5 ; 2) ✓ M $y - 2 = -\frac{1}{3}(x+5)$ OR $2 = -\frac{1}{3}(-5) + c$	Deducing gradient of perp bisector correct Substituting coordinates of C correctly
$c = \frac{1}{3}$	Simplifying equation correctly
$\therefore y = -\frac{1}{3}x + \frac{1}{3}$ OR $x + 3y - 1 = 0$ OR any equiv form ✓ CA	
OR	

$CP^2 = CE^2$	
$(x-0)^2 + (y-2)^2 = (x+1)^2 + (y+1)^2$ ✓A ✓A ✓M	Using distance formula correctly Substituting coord.s of P and E correctly
$x^2 + y^2 - 4y + 4 = x^2 + 2x + 1 + y^2 + 2y + 1$ ✓CA ✓CA	Expanding correctly
$2x + 6y - 2 = 0$ ✓CA	Simplifying equation correctly
$x + 3y - 1 = 0$ (6)	
2.1.4 subst. C(–5 ; 2) into $x + 3y - 1 = 0$ ✓M	Subst coord.s of C into equation correctly
LHS = 2(–5) + 6(2) – 2 ✓A = 0 = RHS	Proving LHS = RHS
∴ C is on perp bisector of PE and given C is on DE ✓CA ∴ the lines intersect at C	Drawing correct conclusion
OR	
subst $y = -\frac{1}{3}x + \frac{1}{3}$ into $3x + 4y + 7 = 0$	
$\therefore 3x + 4(-\frac{1}{3}x + \frac{1}{3}) + 7 = 0$ ✓M ✓A	Substituting one equation into another Correct substitution
$\therefore 5x = -25$ $\therefore x = -5$ $\therefore y = 2$ ✓CA	Solving for x and y correctly
OR	
subst $x = 1 - 3y$ into $3x + 4y + 7 = 0$	
$\therefore 3(1-3y) + 4y + 7 = 0$ ✓M ✓A ✓	Substituting one equation into another Correct substitution
$\therefore 3 - 9y + 4y + 7 = 0$ $\therefore 5y = 10$ $\therefore y = 2$ ✓CA $\therefore x = -5$ (3)	Solving for x and y correctly
2.2	
2.2.1 $M\hat{P}N = 90° \therefore m_{MP} \times m_{NP} = -1$ ✓M	Product of correct gradients = – 1
$\therefore \frac{y}{x+1} \times \frac{y+2}{x-3} = -1$ ✓A ✓A	Correct values for correct gradients
$\therefore y(y+2) = -1(x+1)(x-3)$	
$\therefore y^2 + 2y = -x^2 + 2x + 3$ ✓CA ✓CA	Simplifying correctly
$\therefore x^2 + y^2 - 2x + 2y - 3 = 0$ ✓CA	Writing into correct form correctly
OR	
$MN^2 = MP^2 + PN^2$ ✓M	Using Pythagoras theorem correctly
$20 = (x+1)^2 + y^2 + (x-3)^2 + (y+2)^2$ ✓A ✓A	Using correct value for MN^2 Substituting MP^2 and PN^2 correctly
$= x^2 + 2x + 1 + y^2 + x^2 - 6x + 9 + y^2 + 4y + 4$ ✓CA	Expanding correctly
$2x^2 + 2y^2 - 4x + 4y - 6 = 0$ ✓CA	Simplifying correctly
$x^2 + y^2 - 2x + 2y - 3 = 0$ ✓CA	Writing into correct form correctly
OR	
$MN = \sqrt{20}$ $\therefore r = \frac{\sqrt{20}}{2}$ ✓CA	Calculating the radius correctly
midpt of MN is (1 ; – 1) ✓A	Calculating the midpt correctly
$(x-1)^2 + (y+1)^2 = \left(\frac{\sqrt{20}}{2}\right)^2$ ✓M ✓A	Using the equation for circle correctly Substituting the correct values for midpt correctly
$x^2 - 2x + 1 + y^2 + 2y + 1 = \frac{20}{4} = 5$ ✓CA	Expanding correctly
$x^2 + y^2 - 2x + 2y - 3 = 0$ ✓CA (6)	Writing into correct form correctly

2.2.2 $x^2 - 2x - 3 = 0$ ✓ M $(x + 1)(x - 3) = 0$ ✓ CA $x = -1$ or $x = 3$ ✓ CA $(-1 ; 0)$ or $(3 ; 0)$	(3) [24]	Substituting $y = 0$ Factorising correctly or using formula correctly Answers for x Coordinate form not necessary Answer only – full marks If 2.2.1 results in a linear equation, a maximum of 2 marks can be given for 2.2.2

QUESTION 3

3.1 $\text{cosec}(-225^\circ)[\cos 750^\circ.\sec(-30^\circ) - \tan(360^\circ - \theta).\cos(-\theta)]$

✓A ✓A ✓A ✓A ✓A

$= \text{cosec}\, 45^\circ[\cos 30^\circ.\ \sec 30^\circ - (-\tan\theta).\cos\theta]$

✓CA ✓CA ✓CA

$= \sqrt{2}\,[1 + \sin\theta]$ or $\frac{2}{\sqrt{2}}[1 + \sin\theta]$ (8)

Sign and reduced angle; cosec 45° = sec 45°
Correct values for specific angles
cos 30° sec 30° = 1
tan θ.cos θ = sin θ

3.2

3.2.1

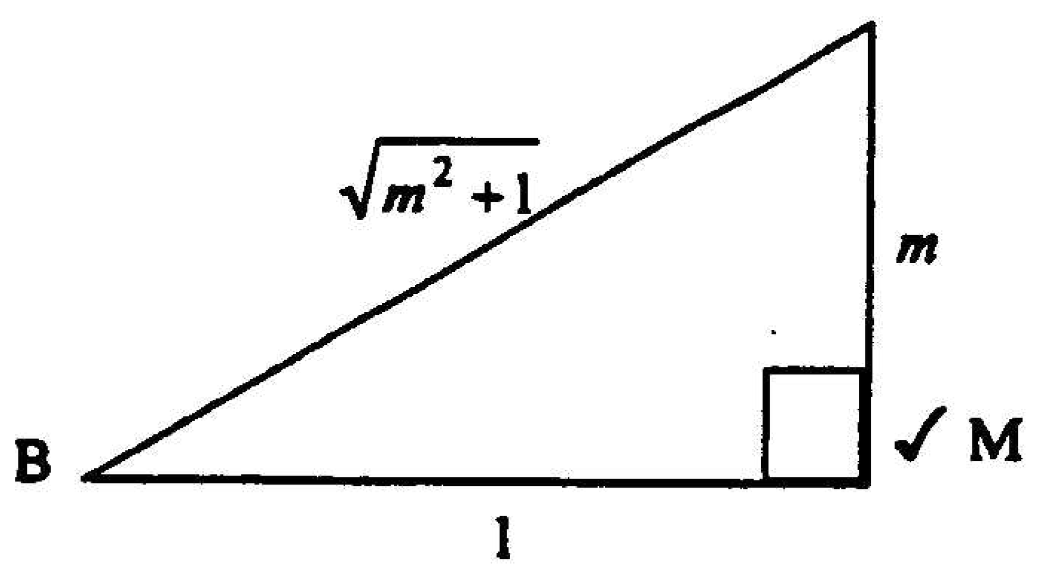


Diagram correct
Penalise 1 mark if omitted

$\text{cosec}^2(180^\circ + B) = \text{cosec}^2 B$ ✓A

$= \left(\frac{\sqrt{m^2+1}}{m}\right)^2$ ✓CA

$= \frac{m^2+1}{m^2}$

Reducing correctly
Substituting correctly

OR

$\text{cosec}^2(180^\circ + B) = \text{cosec}^2 B$ ✓A

$= 1 + \cot^2 B$

$= 1 + \frac{1}{m^2}$ ✓CA

$= \frac{m^2+1}{m^2}$

Reducing correctly
Substituting correctly

OR

$\text{cosec}^2(180^\circ + B) = 1 + \cot^2(180^\circ + B)$

$= 1 + \cot^2 B$ ✓A

$= 1 + \frac{1}{m^2}$ ✓CA

$= \frac{m^2+1}{m^2}$ (3)

Reducing correctly
Substituting correctly

3.2.2 $\tan(90^\circ - B) + \tan(45^\circ + B)$

✓A ✓A

$= \cot B + \frac{\tan 45^\circ + \tan B}{1 - \tan 45^\circ.\tan B}$

✓CA ✓A

$= \frac{1}{m} + \frac{1+m}{1-m}$ ✓CA (5)

$= \frac{1+m^2}{m(1-m)}$

Reducing correctly
Expanding correctly
tan 45° = 1 - both
Substituting m correctly in each term

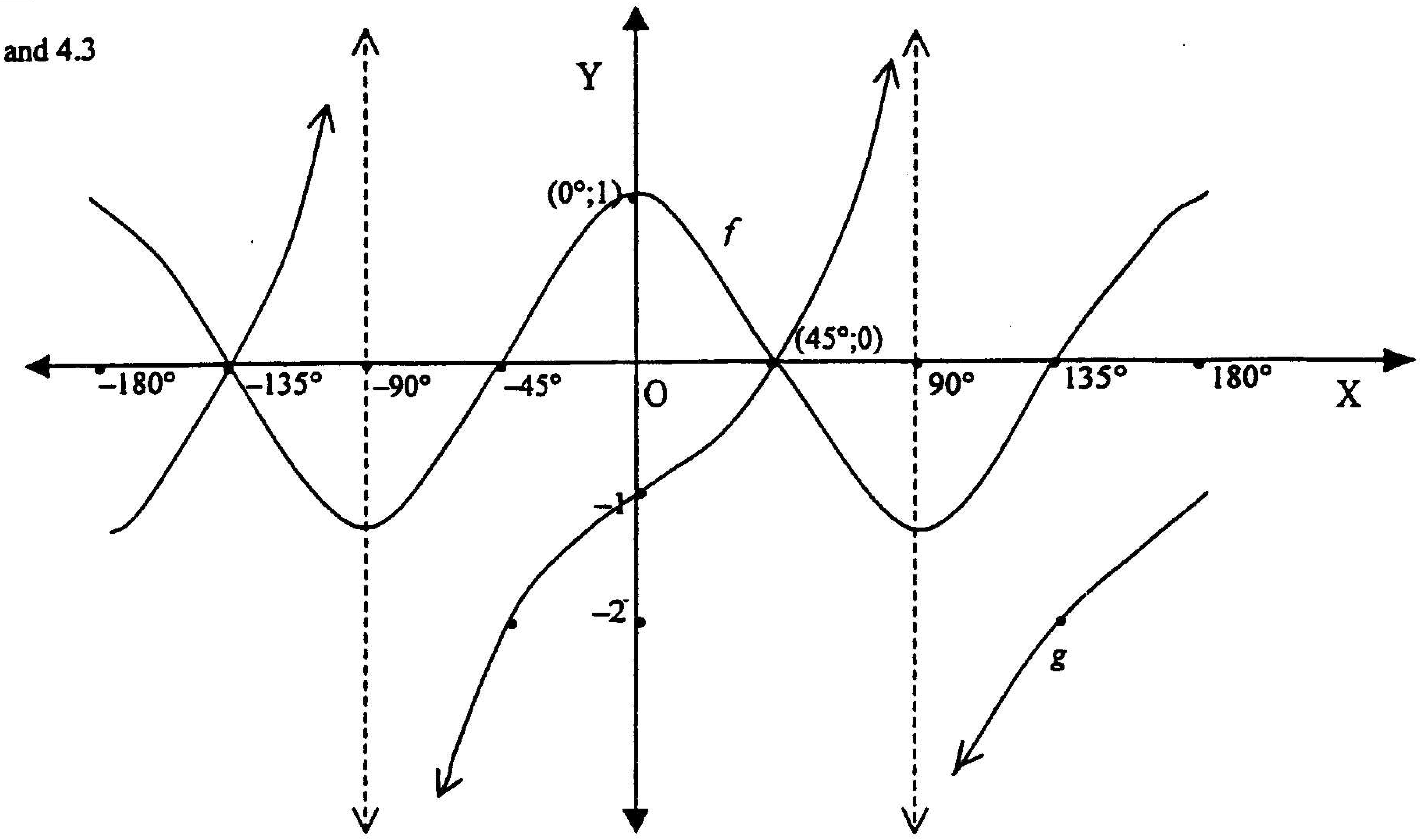
3.3 LHS $= \dfrac{(\sin\theta - \cos\theta)^2}{1-\sin^2\theta}$	
$= \dfrac{\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta}{\cos^2\theta}$ ✓A ✓A	Expanding correctly $1 - \sin^2\theta = \cos^2\theta$
$= \dfrac{1-2\sin\theta\cos\theta}{\cos^2\theta}$ ✓A	$\sin^2\theta + \cos^2\theta = 1$
$= \dfrac{1}{\cos^2\theta} - \dfrac{2\sin\theta}{\cos\theta}$ ✓A ✓A	Separating and simplifying terms correctly
$= \sec^2\theta - 2\tan\theta = \text{RHS}$	Can only get full marks if final line is included
OR	
RHS $= \sec^2\theta - 2\tan\theta$	
$= \dfrac{1}{\cos^2\theta} - \dfrac{2\sin\theta}{\cos\theta}$ ✓A ✓A	Correct identities
$= \dfrac{1-2\sin\theta\cos\theta}{\cos^2\theta}$ ✓A	One term on LCM correctly
$= \dfrac{\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta}{1-\sin^2\theta}$ ✓A ✓A	$1 = \sin^2\theta + \cos^2\theta$ $\cos^2\theta = 1 - \sin^2\theta$
$= \dfrac{(\sin\theta - \cos\theta)^2}{1-\sin^2\theta}$	
OR	
LHS $= \dfrac{(\sin\theta - \cos\theta)^2}{1-\sin^2\theta}$	
$= \dfrac{(\sin\theta - \cos\theta)^2}{\cos^2\theta}$ ✓A	$1 - \sin^2\theta = \cos^2\theta$
$= \left(\dfrac{\sin\theta}{\cos\theta} - \dfrac{\cos\theta}{\cos\theta}\right)^2$ ✓A	Separating terms correctly in bracket
$= (\tan\theta - 1)^2$ ✓A ✓A	$\left(\dfrac{\sin\theta}{\cos\theta} = \tan\theta\right)$; simplifying correctly
$= \tan^2\theta - 2\tan\theta + 1$ ✓A	Expanding correctly
$= \sec^2\theta - 2\tan\theta$	Can only get full marks if final line is included
OR	
RHS $= \sec^2\theta - 2\tan\theta$	
$= \tan^2\theta - 2\tan\theta + 1$ ✓A	$\sec^2\theta = 1 + \tan^2\theta$
$= (\tan\theta - 1)^2$ ✓A	factorising correctly
$= \left(\dfrac{\sin\theta}{\cos\theta} - 1\right)^2$ ✓A	$\left(\dfrac{\sin\theta}{\cos\theta} = \tan\theta\right)$
$= \left(\dfrac{\sin\theta - \cos\theta}{\cos\theta}\right)^2$ ✓A	Joining two terms into one term on the same denominator
$= \dfrac{(\sin\theta - \cos\theta)^2}{\cos^2\theta}$ ✓A	One term on LCM correctly $\cos^2\theta = 1 - \sin^2\theta$
$= \dfrac{(\sin\theta - \cos\theta)^2}{1-\sin^2\theta} = \text{LHS}$ (5)	Note final conclusion must be shown to earn full marks If candidates work with both sides, two marks per side are allocated plus a mark for the conclusion

Note: $2\sin\theta\cos\theta = \sin 2\theta$

3.4 $\tan 3x \cdot \cot 24° - 1 = 0$ $\therefore \tan 3x = \frac{1}{\cot 24°}$ ✓ A $\therefore \tan 3x = \tan 24°$ (or 0,45) ✓ A $\therefore 3x = 24° + 180°k$ ✓ A [OR $3x = 24° + 360°k$ or $204° + 360°k$] ✓ CA $\therefore x = 8° + 60°k$ [OR $x = 8° + 120°k$ or $68° + 120°k$] $k \in Z$ ✓ A OR $\frac{\sin 3x}{\cos 3x} \times \frac{\cos 24°}{\sin 24°} - 1 = 0$ ✓ A $\sin 3x \cos 24° - \cos 3x \sin 24° = 0$ $\sin(3x - 24°) = 0$ ✓ A ✓ A $\therefore 3x - 24° = 0° + 360°.k$ or $3x - 24° = 180° + 360°.k$ $\therefore 3x = 24° + 360°.k$ or $3x = 204° + 360°.k$ $\therefore x = 8° + 120°.k$ or $x = 68° + 120°.k$ ✓ A $k \in Z$ ✓ A OR $\therefore 3x = 24° + 180°k$ ✓ A $\therefore x = 8° + 60°k$ ✓ A OR $3x - 24° = 360° + 360°.k$ $\therefore 3x = 384° + 360°.k$ $\therefore x = 128° + 120°.k$ (5) [26]	Correctly getting one term on each side Correct identity or value/ can skip first two lines without penalty Correct general solution Dividing by 3 correctly $k \in Z$ no general solution – max 3 marks $\tan 3x \cdot \cot 24° - 1 = 0$ $\therefore \tan 3x = \frac{1}{\cot 24°}$ ✓ A $\therefore \tan 3x = \tan 24°$ (or 0,45) ✓ A $\therefore 3x = 24°$ $\therefore x = 8°$ ✓ A

QUESTION 4

4.2 and 4.3



4.1 $a = 1$ ✓A $b = 2$ ✓A	(2)	
4.2 graph of f: period ✓A; amplitude ✓A; shape ✓A	(3)	
4.3 graph of g: asymptotes ✓A; x-intercepts ✓A; y-intercepts ✓A; shape ✓A	(4)	must label x-intercept angles if curve crosses asymptote cannot get shape mark
4.4 4.4.1 $-90° < x \leq -45°$ ✓CA ; $90° < x \leq 135°$ ✓CA ; $x = -135°$ ✓CA ; $x = 45°$ ✓CA notation ✓A	(5)	1 mark per interval, 1 mark per point, 1 mark notation
OR $x \in (-90°; -45°]$ ✓CA ; $x \in (90°; 135°]$ ✓CA $x = -135°$;✓CA $x = 45°$✓CA notation ✓CA		brackets must be correct for notation mark (don't penalise for the $x \in$)
4.4.2 $-180°$ ✓CA ; $180°$ ✓CA ; $0°$ ✓CA ; $-45°$; $135°$	(3) [17]	1 mark for 0° and one mark for any two of the others Incorrect graphs – one mark for any two correct answers to a maximum of 3 marks

QUESTION 5

5.1 5.1.1 $\cos(A-B) = \cos A \cos B + \sin A \sin B$ ✓A (1)	
5.1.2 $\sin(A+B)$ $= \cos[90° - (A+B)]$ ✓A $= \cos[(90° - A) - B]$ ✓A $= \cos(90° - A)\cos B + \sin(90° - A)\sin B$ ✓A $= \sin A \cos B + \cos A \sin B$ (3)	can skip first step without penalty must use 5.1.1
5.2	
5.2.1 $\sin(45° + x).\sin(45° - x)$ $= (\sin 45° \cos x + \cos 45° \sin x)(\sin 45° \cos x - \cos 45° \sin x)$ ✓A ✓A $= \sin^2 45° \cos^2 x - \cos^2 45° \sin^2 x$ ✓CA $= \frac{1}{2}\cos^2 x - \frac{1}{2}\sin^2 x$ ✓CA $= \frac{1}{2}(\cos^2 x - \sin^2 x)$ ✓CA $= \frac{1}{2}\cos 2x$ (5)	a mark for each expansion. Could subst. $\frac{1}{\sqrt{2}}$ for ratios at this stage multiplying substituting numbers for ratios taking out common factor. (can omit this line without penalty)
• alternate method. 5.2.2 maximum value of $\cos 2x$ is 1 ✓A $\therefore$ max. value of $\sin(45° + x).\sin(45° - x)$ is $\frac{1}{2}$ ✓A (2)	answer only scores full marks.
5.3 $\sin 2x + 2\sin x + \cos^2 x + \cos x = 0$ $\therefore 2\sin x \cos x + 2\sin x + \cos^2 x + \cos x = 0$ ✓A $\therefore 2\sin x(\cos x + 1) + \cos x(\cos x + 1) = 0$ ✓A $\therefore (\cos x + 1)(2\sin x + \cos x) = 0$ ✓CA $\therefore \cos x = -1$ ✓CA or $2\sin x = -\cos x$ ✓CA $\therefore x = 180°$ ✓CA $\tan x = -\frac{1}{2}$ ✓CA reference $\angle = 26{,}6°$ ✓CA $x = 153{,}4° + 180°k, k \in Z$ [OR $-26{,}6° + 180°k, k \in Z$] $x = -26{,}6°$ or $153{,}4°$ ✓CA ✓CA (10)	expansion grouping factors OR $4\sin^2 x = \cos^2 x = 1 - \sin^2 x$ $\therefore \sin x = \pm\frac{1}{\sqrt{5}}$ ✓CA If say $\tan x = \frac{1}{2}$ can score maximum 8 marks. If additional wrong solutions given, penalise by a mark. Dividing by $\cos x + 1$ – maximum 7 marks
[21]	

QUESTION 6

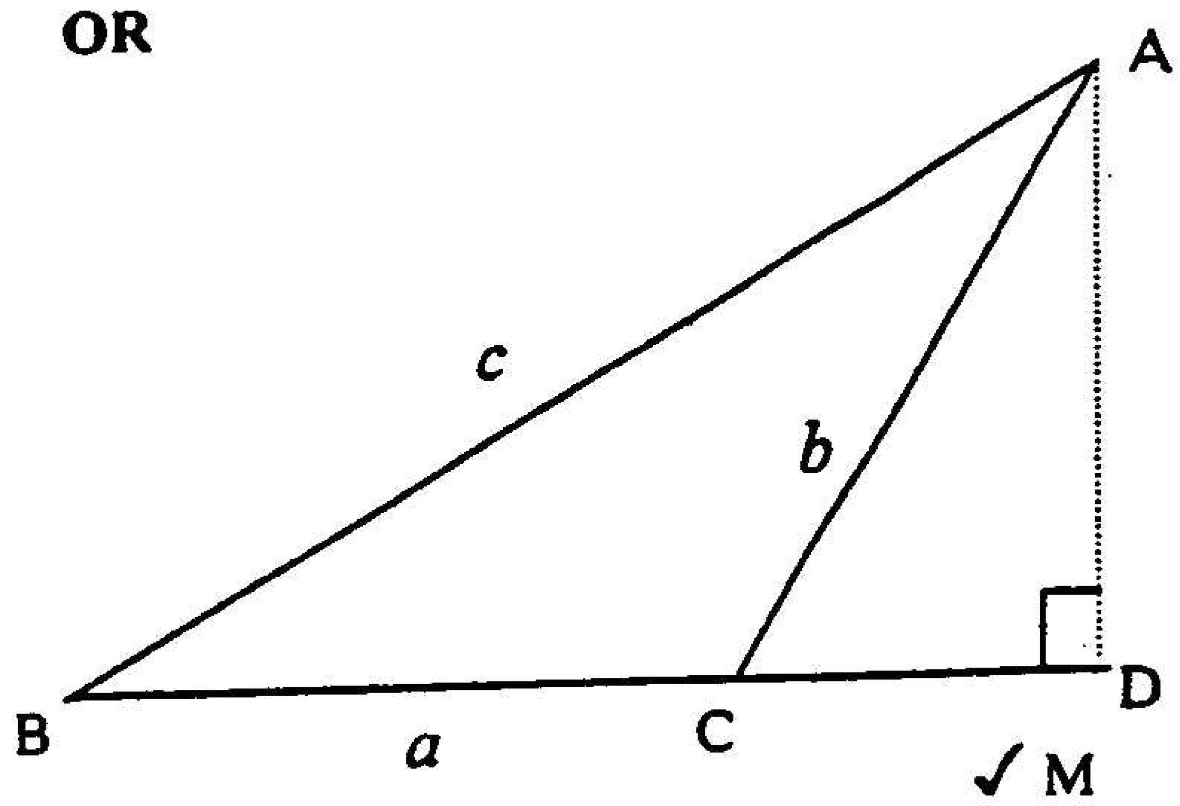
6.1 By Area formula,

Area Δ ABC $= \frac{1}{2}$ ac sin B $= \frac{1}{2}$ ab sin C ✓A ✓A ✓A

Dividing through by $\frac{1}{2}$abc yields ✓M

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

OR



✓M

Draw AD perpendicular to the extension of BC

AD = c sin B ✓A

and AD = b sin $(180° - A\hat{C}B)$ ✓A

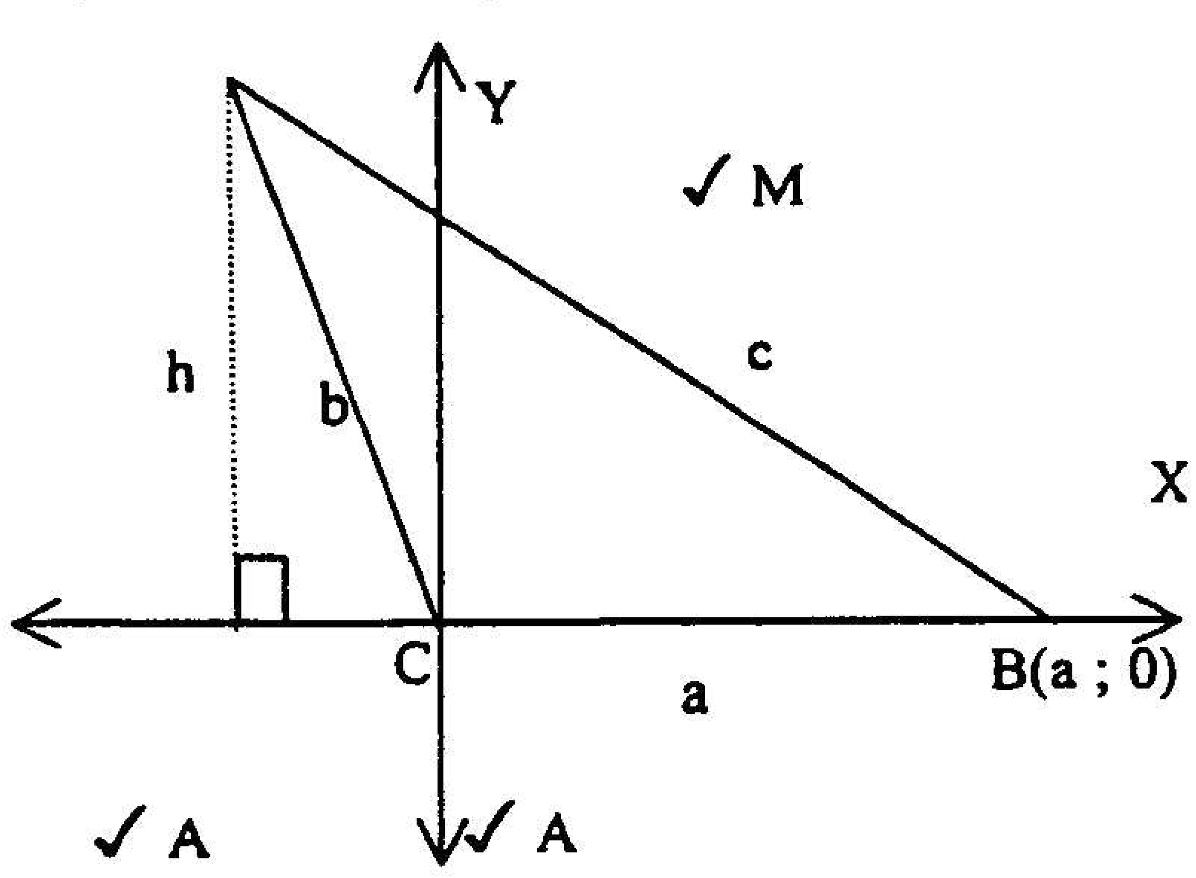
= b sin $A\hat{C}B$

$\therefore$ c sin B = b sin $A\hat{C}B$ ✓A

$$\therefore \frac{\sin B}{b} = \frac{\sin C}{c}$$

OR

A(b cos C ; b sin C) ✓A



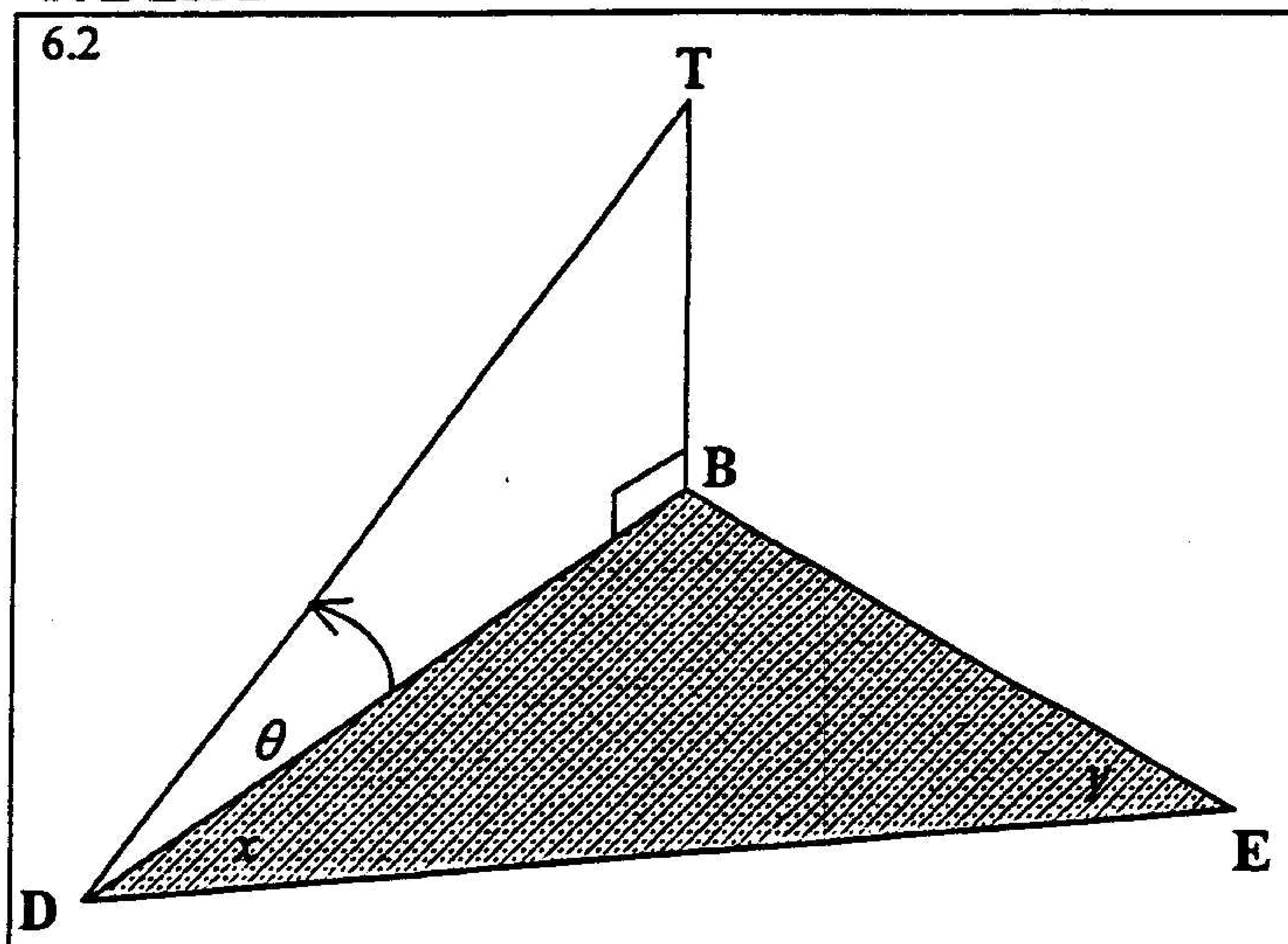
h = b sin $A\hat{C}B$ = c sin B

$$\therefore \frac{\sin B}{b} = \frac{\sin C}{c}$$ (4)

must mention use of Area.

the final conclusion line must occur in all the proofs. If omitted, 1 mark penalty.

6.2



6.2.1 $\tan\theta = \frac{TB}{DB}$ ✓ A

$\therefore TB = DB\tan\theta$ ✓ A **OR** $TB = \frac{DB}{\cot\theta}$

(2)

6.2.2 $\frac{DB}{\sin E} = \frac{DE}{\sin B}$ ✓ M

but $D\hat{B}E = 180° - (x + y)$ ✓ A

$\therefore \frac{DB}{\sin y} = \frac{10}{\sin[180° - (x+y)]}$ ✓ CA

$\therefore \frac{DB}{\sin y} = \frac{10}{\sin(x+y)}$ ✓ CA

$\therefore DB = \frac{10\sin y}{\sin(x+y)}$

(4)

6.2.1
OR $TB = DB\cot(90° - \theta)$

OR $TB = \frac{DB}{\tan(90° - \theta)}$

OR equivalent statement in terms of sines and cosines.

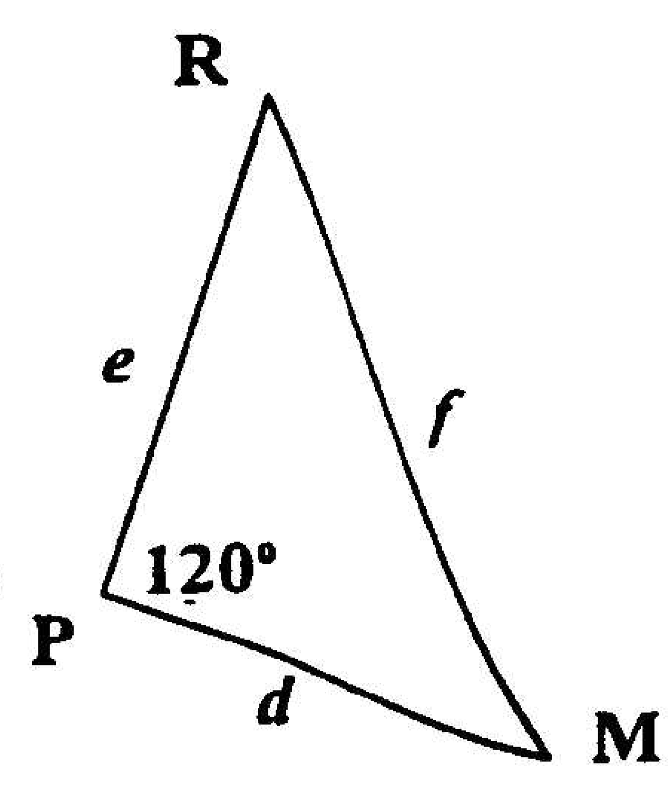
6.2.2
use of sine rule

determining $D\hat{B}E$

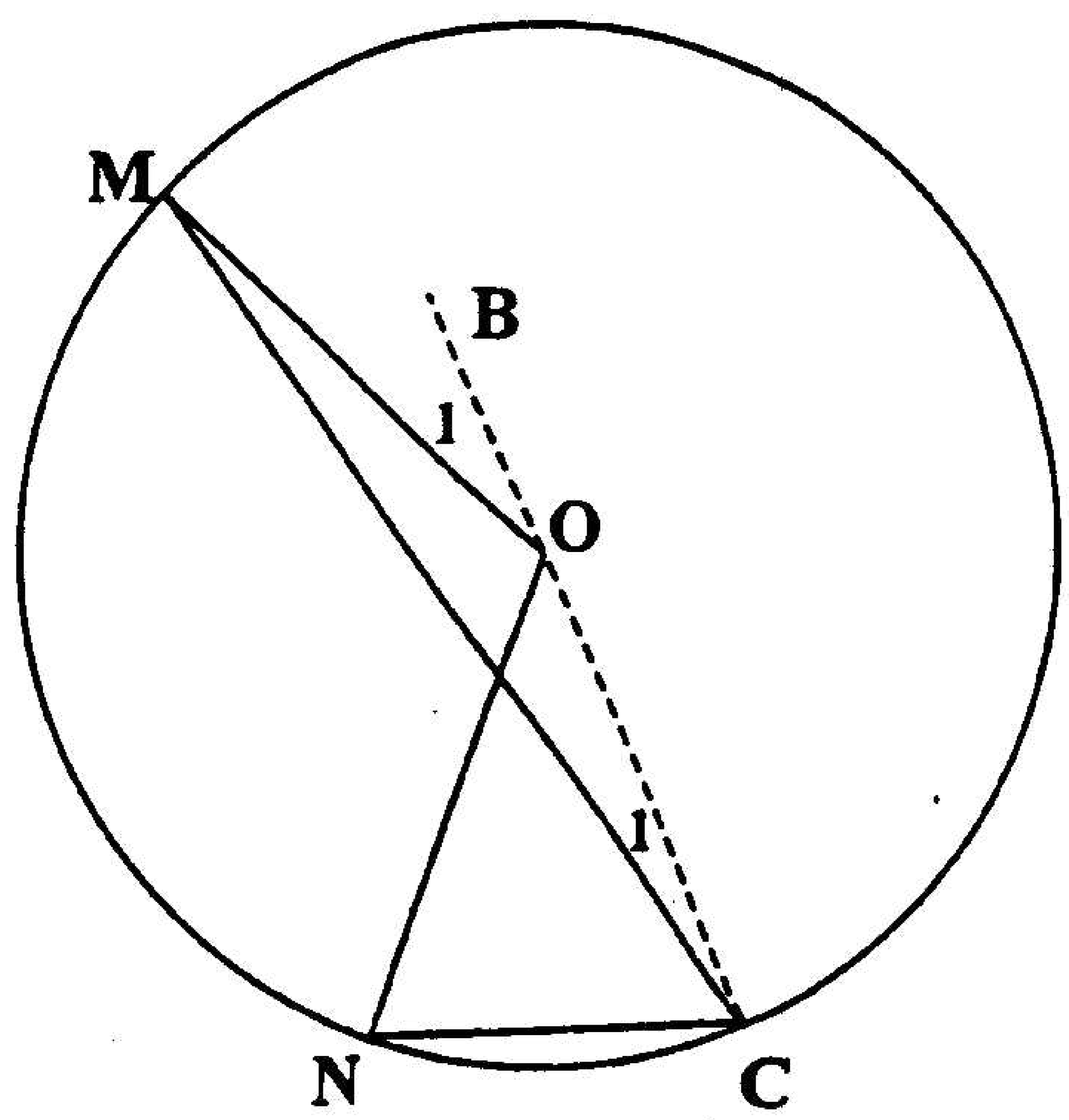
substitution

reduction

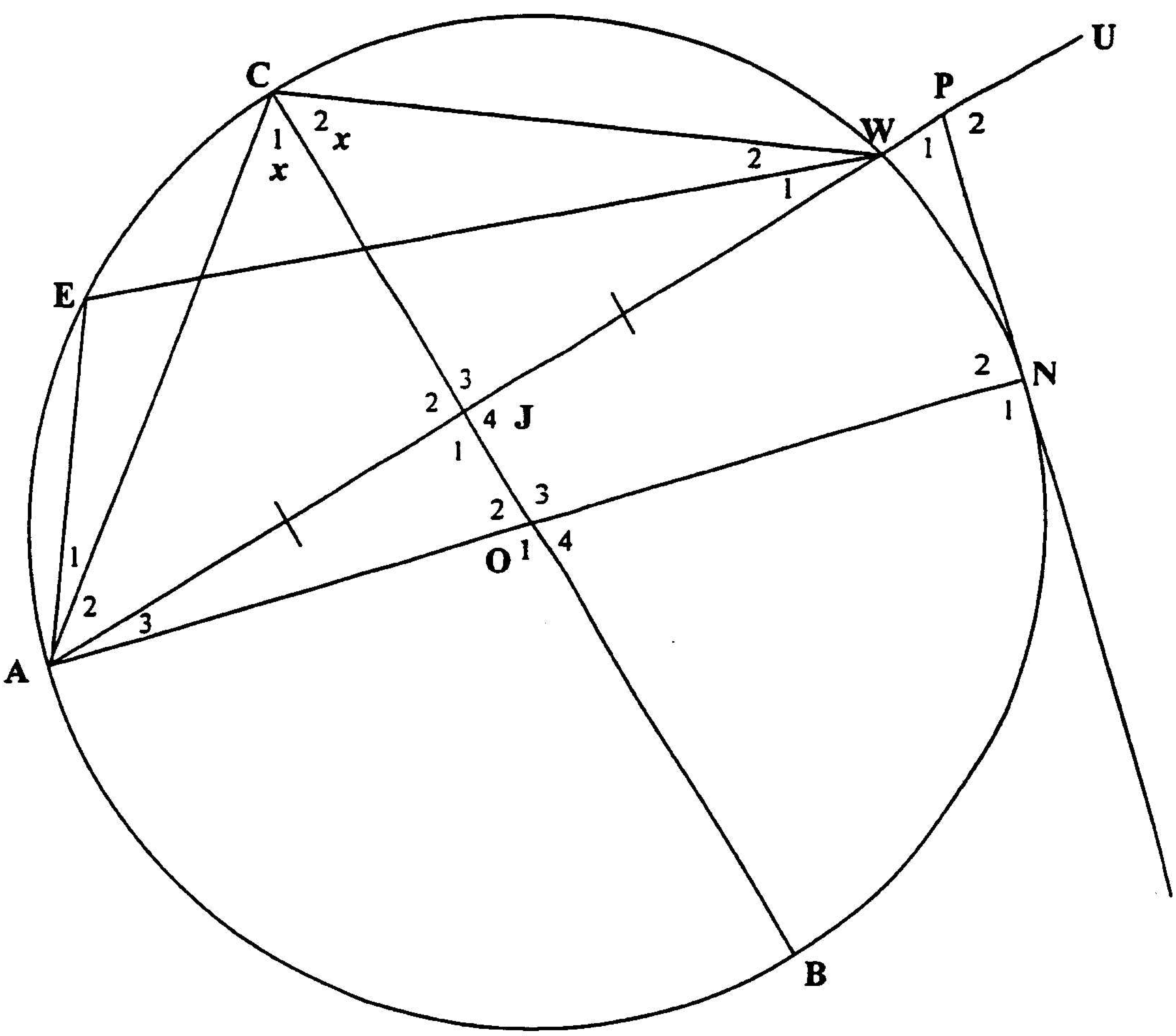
6.2.3 TB = DB tan θ	
$= \dfrac{10\sin y.\tan\theta}{\sin(y+y)}$ ✓ CA	Substitution – may skip this line without penalty
$= \dfrac{10\sin y\tan\theta}{\sin 2y}$ ✓ CA	simplification
$= \dfrac{10\ \sin y\ \tan\ \theta}{2\ \sin y\ \cos y}$ ✓ CA	expansion
$= \dfrac{5\tan\theta}{\cos y}$ ✓ CA	cancelling full marks if *y* is correctly substituted with an *x*
$= 5\sec y.\tan\theta$ (4)	
6.2.4 Area Δ BDE $= \dfrac{1}{2}$BD.DE.sin *x* ✓ M	use of appropriate area formula
$= \dfrac{1}{2}\times\dfrac{10\sin y}{\sin(x+y)}\times$ DE × sin *x* ✓ CA	substitution of BD (could also have 5 sec 35° for BD)
$= \dfrac{5\sin 35°.(10).\sin 35°}{\sin 70°}$ ✓ CA	substitution of values
= 17,5 m² ✓ CA	answer
OR	
Area Δ BDE $= \dfrac{1}{2}$BD.BE.sin 2*y* ✓ M	**OR** formula plus correct answer only earns full marks.
$= \dfrac{1}{2}\left(\dfrac{10\sin y}{\sin 2y}\right)^2 \sin 2y$ ✓ CA	**OR** $\dfrac{5\sin y.10\sin y}{2\sin y\cos y}$ ✓ CA for last 3 marks
$= \dfrac{50\sin^2 y}{2\sin y\cos y}$	= 25 tan *y*
= 25 tan 35° ✓ CA	= 25 tan 35° ✓ CA
= 17,5 m² ✓ CA (4)	= 17,5m² ✓ CA
6.3 $\therefore f^2 = d^2 + e^2 - 2de\cos 120°$ ✓ M ✓ A	using cosine rule substitution
$\therefore f^2 = d^2 + e^2 - 2de(-\cos 60°)$ ✓ A	reduction
$\therefore f^2 = d^2 + e^2 - 2de(-\frac{1}{2})$ ✓ CA	value of cos 60°
$\therefore de = f^2 - e^2 - d^2$ (4)	can skip second line
[22]	



QUESTION 7

7.1 	If simplified diagram is used: maximum of 4 marks – the first 4 Prove must make sense in terms of given diagram
Const.: Draw COB ✓ M OM = ON (radii) $\hat{M} = \hat{C}_1$ (∠s opp equal sides) ✓ S/R	Note that construction may just be shown on sketch.
✓ S *** $\hat{O}_1 = 2\hat{C}_1$ (ext. ∠ of Δ = sum int opp ∠s) ✓ R Similarly, $B\hat{O}N = 2\ O\hat{C}N$ ✓ S $\therefore M\hat{O}N = 2\ O\hat{C}N - 2\hat{C}_1$ $= 2(O\hat{C}N - \hat{C}_1)$ ✓ S $= 2\ M\hat{C}N$ $= 2\hat{C}$ (6)	If this statement *** is omitted, maximum of 3 marks

7.2



7.2.1 $\hat{N}_2 = 90°$ (diam ⊥ tangent) ✓S ✓R

$\hat{J}_4 = 90°$ (line from centre to midpt. of chord) ✓S ✓R

∴ONPJ is a cyclic quad. (opp. ∠s sum to 180°) ✓R (5)

7.2.2 Δ AJC ≡ Δ WJC ✓S (s, ∠, s) ✓R

∴ $\hat{C}_1 = \hat{C}_2$ ✓S

∴ OC bisects $A\hat{C}W$ (3)

7.2.3 $\hat{O}_1 = 2\hat{C}_1$ ✓S (∠ at centre = 2 ∠ on circle) ✓R

$= A\hat{C}W$ ($\hat{C}_1 = \hat{C}_2$) ✓S

∴ $\hat{O}_3 = A\hat{C}W$ (vert. opp. ∠'s) ✓S/R

∴ $\hat{P}_1 = 180° - A\hat{C}W$ ✓S (opp. ∠'s of cyclic quad.) ✓R (6)

7.2.4 $\hat{E} = A\hat{C}W$ ✓S (∠s in same segment) ✓R

$A\hat{C}W = \hat{O}_3$ (from 7.2.3) ✓S

$\hat{P}_2 = \hat{O}_3$ ✓S (ext. ∠ cyclic quad = int opp ∠) ✓R

∴ $\hat{P}_2 = \hat{E}$ (5)

[25]

OR 7.2.1 $\hat{N}_1 = 90°$ (diam ⊥ tangent) ✓S ✓R

$\hat{J}_4 = 90°$ (line from centre to midpt. of chord) ✓S ✓R

∴ONPJ is a cyclic quad. (ext ∠of quad = int opp ∠) ✓R

OR 7.2.1 use $\hat{N}_2$ and $\hat{J}_3$ above

OR 7.2.1 use $\hat{N}_2$ and $\hat{J}_1$ above

OR 7.2.3

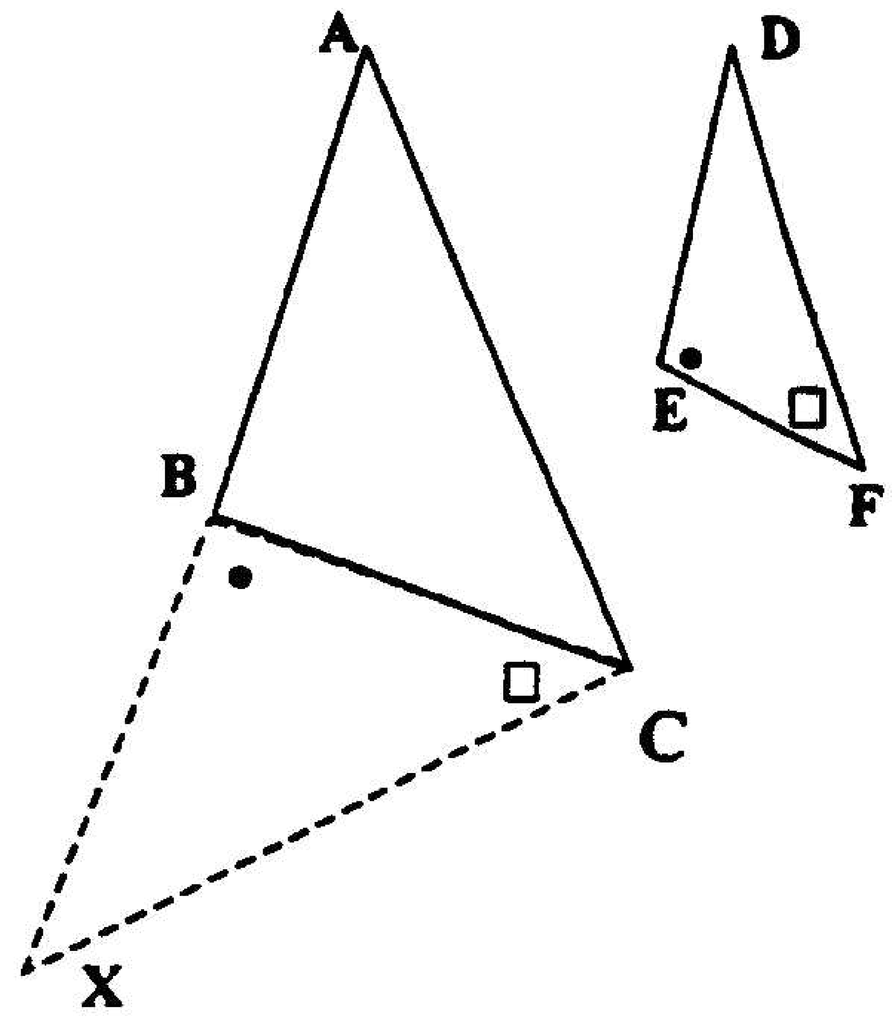
∴ $\hat{O}_2 = 180° - A\hat{C}W$ (adj. ∠s str. line) ✓S/R

∴ $\hat{P}_1 = 180° - A\hat{C}W$ ✓S (ext. ∠of cyclic quad= int.opp.∠) ✓R

OR
7.2.4
$\hat{P}_2 = A\hat{C}W$ ✓S (adj. ∠s str. line) ✓R

QUESTION 8

8.1



On BC construct Δ XBC such that $X\hat{B}C = \hat{E}$ and $X\hat{C}B = \hat{F}$ ✓ M

✓ S ✓ R

*** **Δ XBC ||| Δ DEF** (equiangular)

$\therefore \frac{XB}{DE} = \frac{BC}{EF}$ ✓ S (Δ s |||)

$= \frac{AB}{DE}$ ✓ S (given)

$\therefore$ XB = AB ✓ S

Similarly, XC = AC

BC = BC

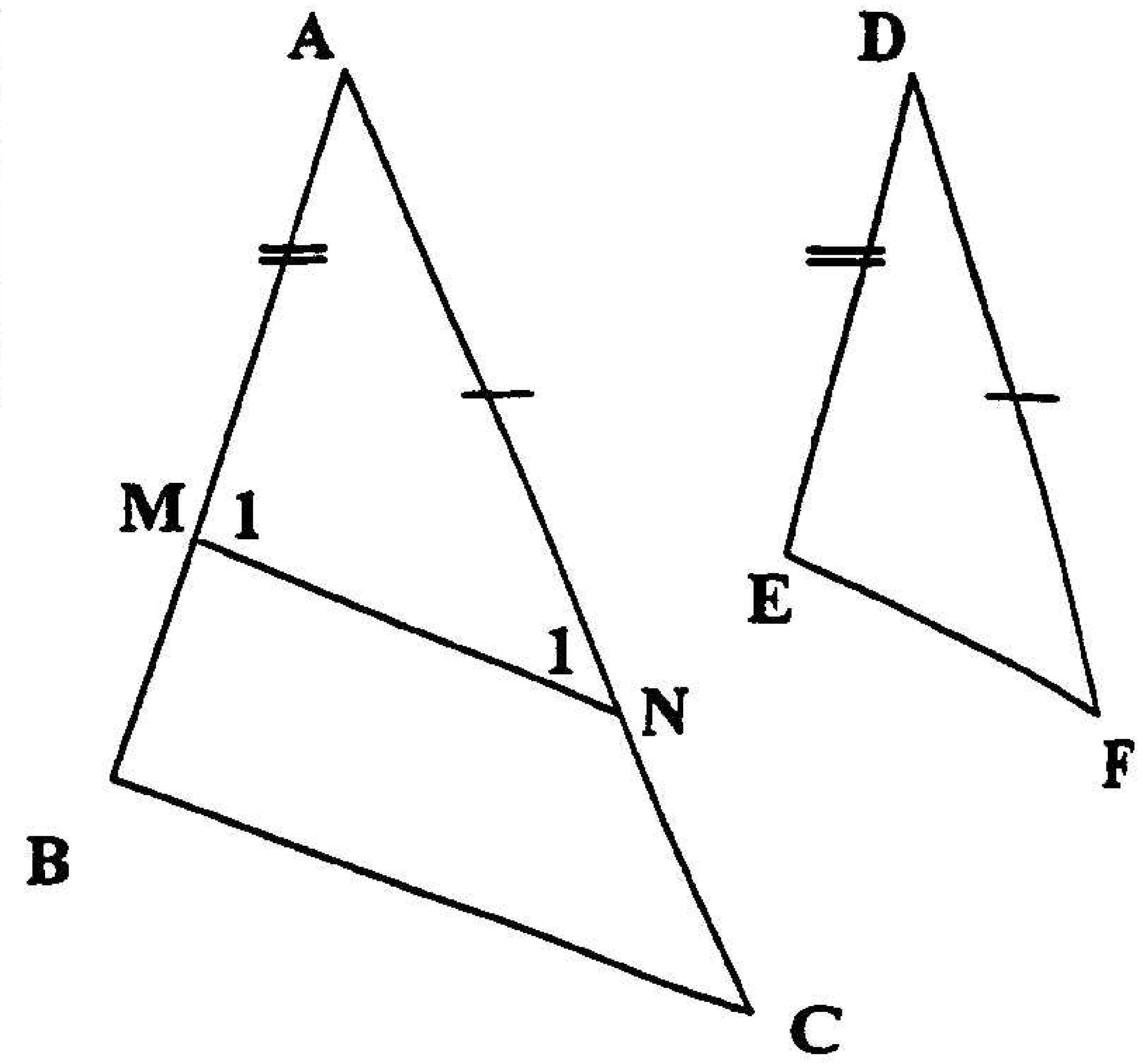
$\therefore$ Δ ABC ≡ Δ XBC (s, s, s) ✓ S

$\therefore$ Δ ABC and Δ XBC are equiangular

$\therefore$ Δ ABC and Δ DEF are equiangular ✓ S

(8)

Note: If similar triangles not used at all (see***), maximum of 1 mark (for const).
If line *** omitted, but reason in line below given, can get maximum of 7 marks.



Draw DE = AM on AB ✓ M
and DF = AN on AC

$\frac{AB}{DE} = \frac{AC}{DF}$ (given)

$\frac{AB}{AM} = \frac{AC}{AN}$ (construction)

$\therefore$ MN || BC (line dividing sides in prop.) ✓ S/R

$\therefore \hat{M}_1 = \hat{B}$ & $\hat{N}_1 = \hat{C}$ (|| lines; corresponding ∠s)

*** $\therefore$ **Δ AMN ||| Δ ABC** (equiangular) ✓ S/R

$\frac{AB}{AM} = \frac{BC}{MN}$ (Δ s |||) ✓ S

but $\frac{AB}{AM} = \frac{AB}{DE}$

$= \frac{BC}{EF}$ (given)

$\therefore \frac{BC}{MN} = \frac{BC}{EF}$ ✓ S

MN = EF ✓ S

$\therefore$ Δ AMN ≡ Δ DEF (s, s, s) ✓ S/R

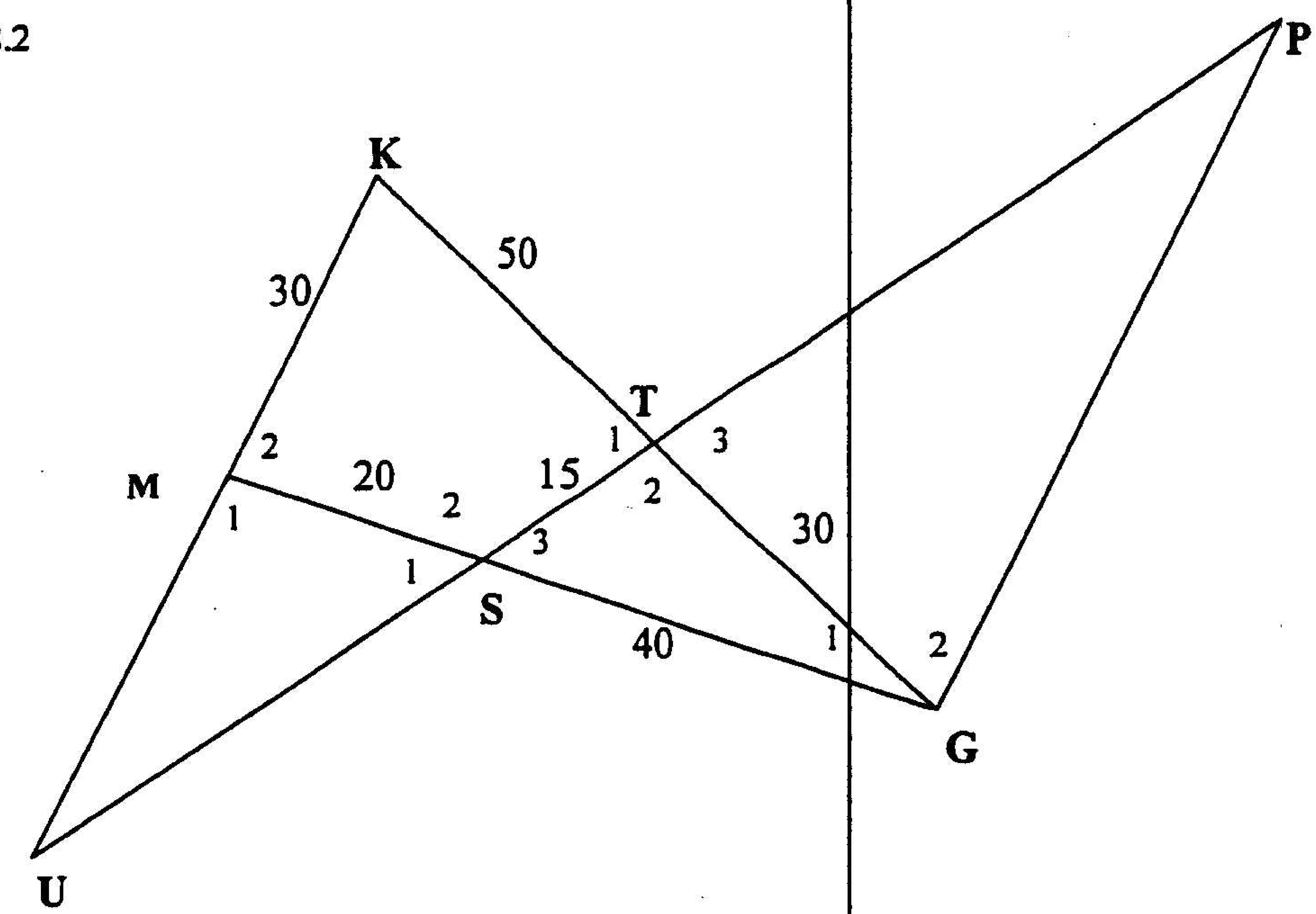
$\therefore$ Δ AMN and Δ DEF are equiangular ✓ S

$\therefore$ Δ ABC and Δ DEF are equiangular

(8)

Note: If line *** omitted, maximum of 2 marks
If line *** omitted, but reason in line below given, can get maximum of 7 marks.

8.2



8.2.1 $\frac{GS}{GK}=\frac{40}{80}=\frac{1}{2}$ ✓ S

$\frac{ST}{KM}=\frac{15}{30}=\frac{1}{2}$ ✓ S

$\frac{GT}{GM}=\frac{30}{60}=\frac{1}{2}$ ✓ S

$\therefore \frac{GS}{GK}=\frac{ST}{KM}=\frac{GT}{GM}$

$\therefore \Delta$ GST ||| Δ GKM (sides in prop.) ✓ R

Last mark can go with any of last two steps
2 ratios only – maximum 2 marks – loose last 2 marks

OR

✓ S

$\frac{GT}{TS}=\frac{GM}{MK}=\frac{2}{1}$ ✓ S

$\frac{TS}{SG}=\frac{KM}{KG}=\frac{3}{8}$

$\frac{GS}{GT}=\frac{KG}{GM}=\frac{4}{3}$ ✓ S

Δ GST ||| Δ GKM (ratio between sides of one Δ is same as that between sides of the other Δ) ✓ R

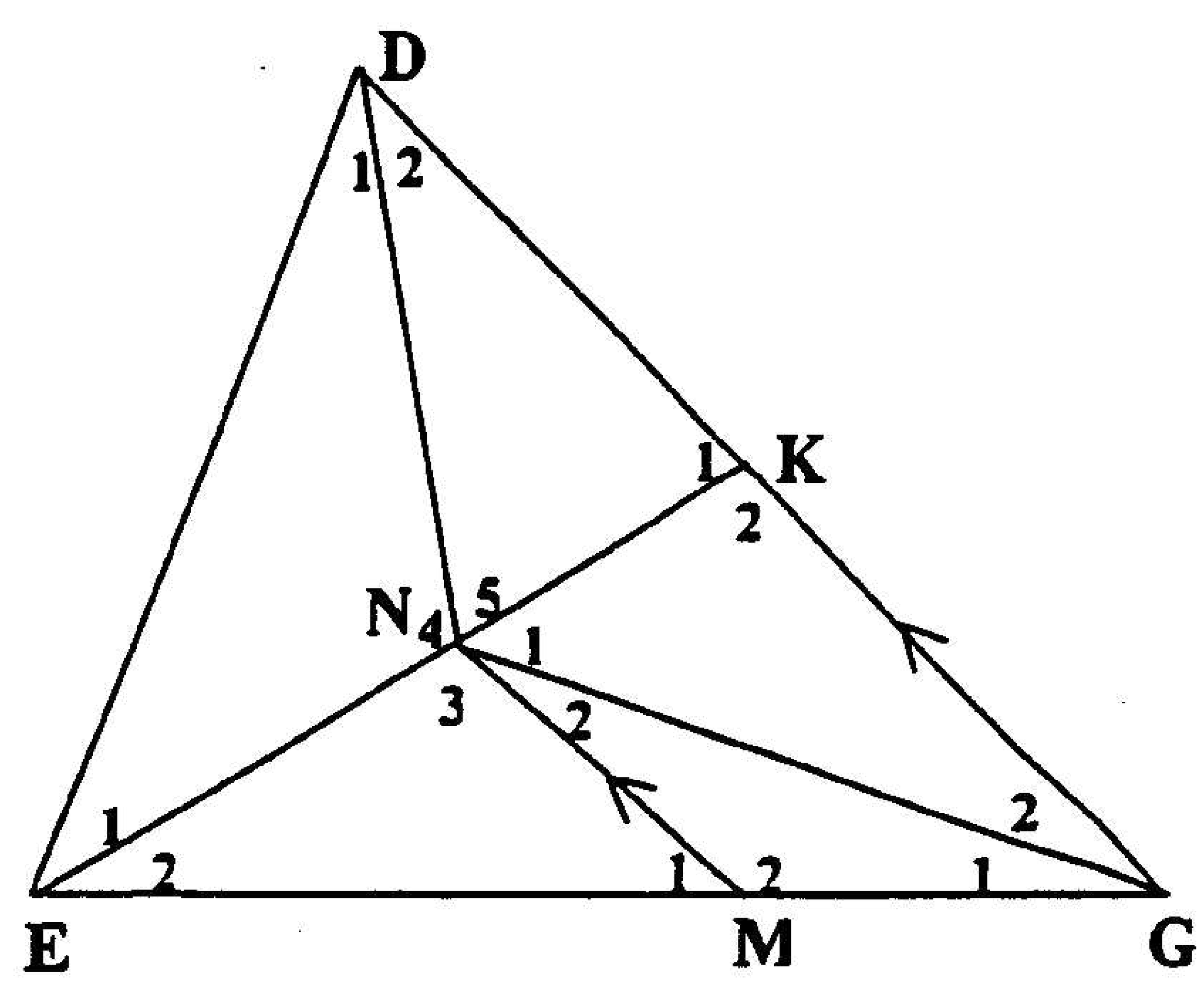
OR

$\hat{G}_1$ is common
GS : GK = 1 : 2 and ✓ S
GT : GM = 1 : 2 ✓ S
✓ S
Δ GST ||| Δ GKM (an <u>included angle</u> between 2 sets of sides which are in proportion) ✓ R

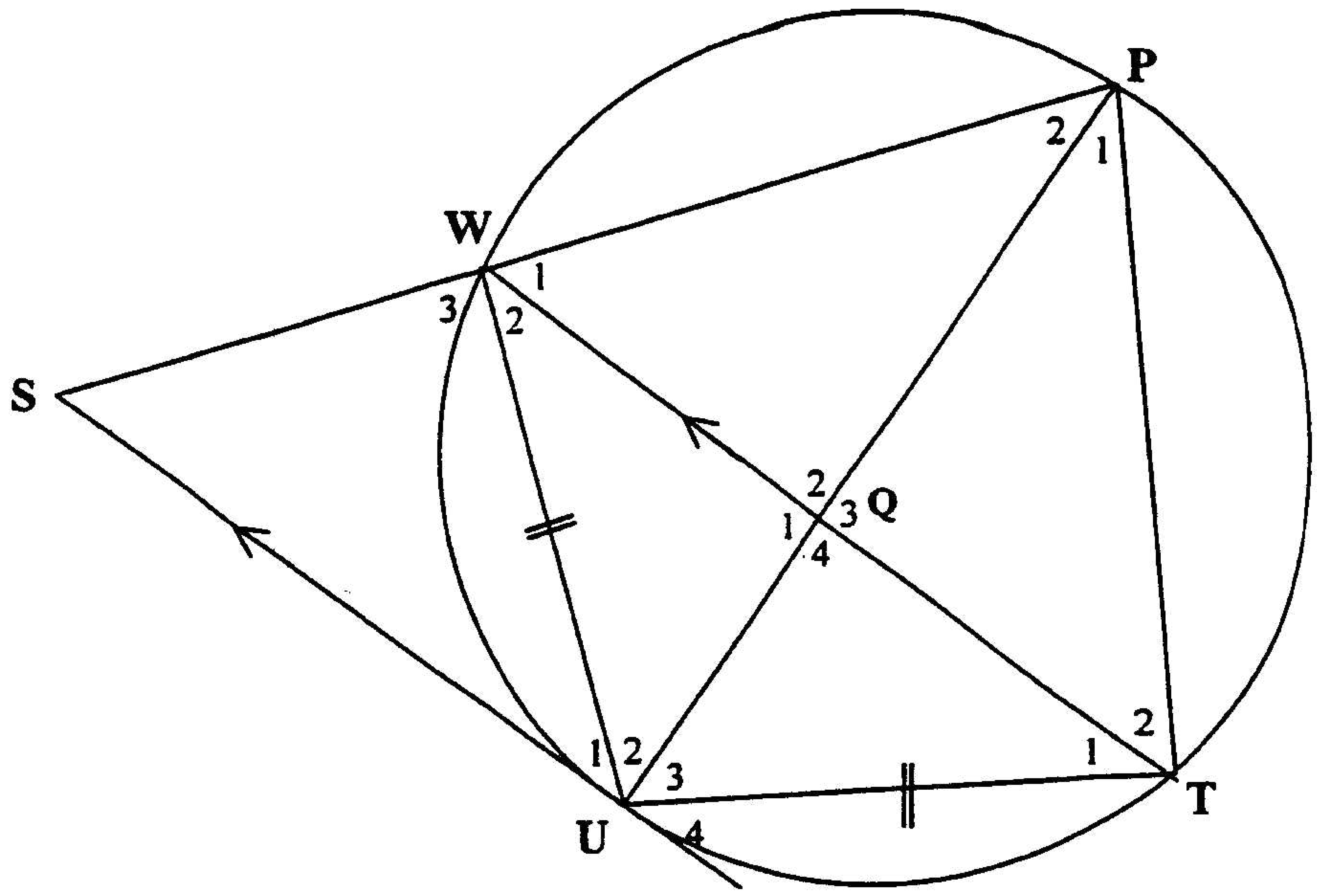
(4)

8.2.2 $\hat{S}_3 = \hat{K}$ ✓S (Δ s ||| from 8.2.1) ✓R OR $\hat{T}_2 = \hat{M}_2$ ✓R ∴ KMST is a cyclic quad. (ext ∠ = int. opp. ∠) (3) [15]	If after 8.2.2 candidate says "**therefore**" ∴ $\hat{S}_3 = \hat{K}$ no penalty for omitting reason

QUESTION 9

9.1	
	
9.1.1 N ✓S (1)	
9.1.2 $\hat{G}_2 = \hat{G}_1$ (NG is an ∠ bisector, N is incentre/ ∠ bisectors are concurrent) ✓S ✓R $\hat{N}_2 = \hat{G}_2$ (‖ lines, alt. ∠s) ✓ S/R $\therefore\ \hat{N}_2 = \hat{G}_1$ $\therefore$ Δ NMG is an isosceles triangle (2 ∠s equal) ✓ R (4)	If omit the last reason, but have $\hat{N}_2 = \hat{G}_1$, award the last mark.
9.1.3 $\frac{EN}{NK} = \frac{EM}{MG}$ ✓S (line parallel one side triangle) ✓R $\therefore \frac{EN}{NK} = \frac{EM}{MN}$ (MN = MG; sides opp equal angles) ✓ S/R $\therefore$ EN . MN = EM . NK (3)	

9.2



9.2.1 $\hat{U}_1 = \hat{W}_2$ ✓ S (|| lines, alt. ∠s) ✓ R

$\hat{W}_2 = \hat{T}_1$ ✓ S (angles opp equal sides) ✓ R

$\therefore \hat{U}_1 = \hat{T}_1$

$\therefore$ US is a tangent (∠ between line & chord = ∠ subt by chord) ✓ R

OR

$\hat{U}_4 = \hat{T}_1$ ✓ S (|| lines, alt. ∠s) ✓ R

$\hat{T}_1 = \hat{W}_2$ ✓ S (angles opp equal sides) ✓ R

$\therefore$ US is a tangent (∠ between line & chord = ∠ subt by chord) ✓ R

can give last reason as "converse of tan/chord theorem"
"tan-chord theorem" is wrong and does **not** get mark.

OR

$\hat{U}_1 = \hat{W}_2$ ✓ S (|| lines, alt. ∠s) ✓ R

$\hat{W}_2 = \hat{P}_2$ ✓ S (subtended by equal chords) ✓ R

$\therefore$ US is a tangent (∠ between line & chord = ∠ subt by chord) ✓ R

OR

$\hat{U}_4 = \hat{T}_1$ ✓ S (|| lines, alt. ∠s) ✓ R

$= \hat{P}_2$ (same segment) ✓ S/R

$= \hat{P}_1$ (subtended by equal chords) ✓ S/R

$\therefore$ US is a tangent (∠ between line & chord = ∠ subt by chord) ✓ R

(5)

9.2.2 In Δ PUS and Δ UWS

1. $\hat{S}$ is common ✓ S
2. $\hat{P}_2 = \hat{U}_1$ ✓ S (∠ between tangent and chord) ✓ R
3. $S\hat{U}P = \hat{W}_3$ (3rd ∠ of Δ).

$\therefore$ Δ SPU ||| Δ SUW (equiangular) ✓ R (4)

Last mark allocated for either the third ∠ **or** the reason – equiangular or (∠∠∠) or (∠∠)

9.2.3 $\frac{SU}{SW} = \frac{SP}{SU}$ ✓ S (Δ SPU ||| Δ SUW) $\therefore SU^2 = SP.SW$ ✓ S In Δ SPU $\frac{PS}{WS} = \frac{PU}{QU}$ ✓ S (line parallel one side triangle) ✓ R $\therefore PS = \frac{PU.WS}{QU}$ ✓ S $\therefore SU^2 = \frac{PU.SW^2}{QU}$ ✓ S $\therefore SU^2.QU = PU.SW^2$ (6) [23] **TOTAL : 200**	or equivalent proportion