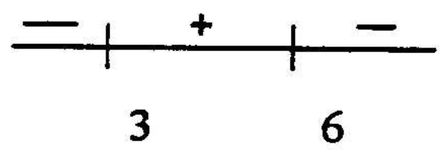
POSSIBLE ANSWERS FOR :

MATHEMATICS HG P1 FINAL COPY

SENIOR CERTIFICATE EXAMINATION 13 NOV 2003

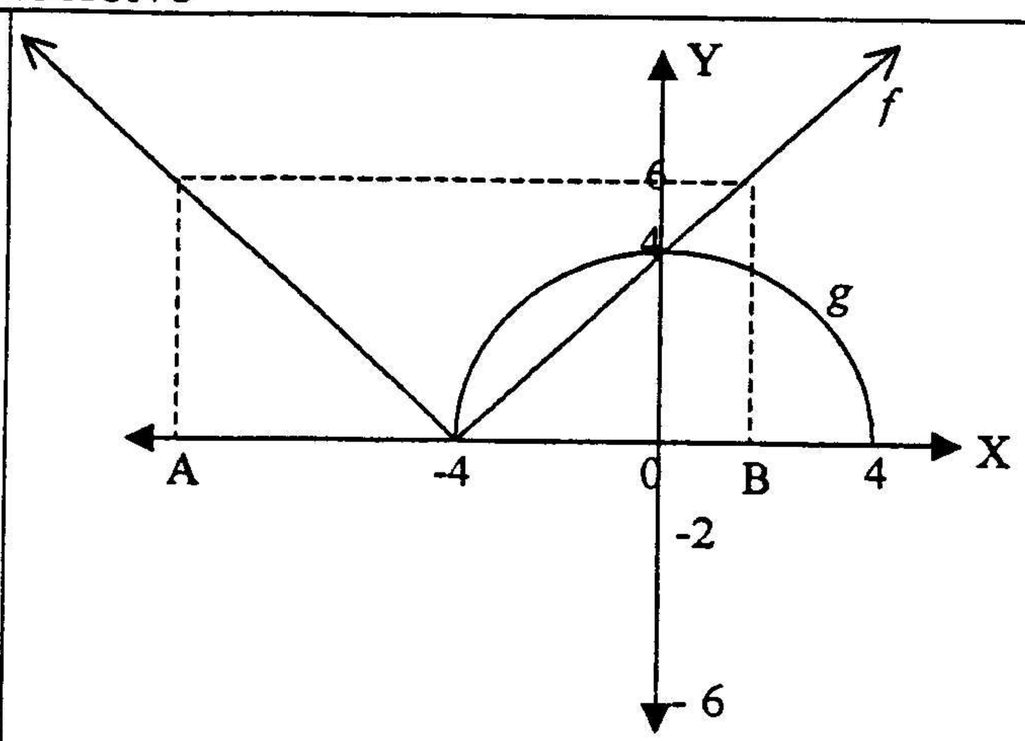
QUESTION 1				
1.1	1.1.1	$(x-2)(x+3)=6$ $x^2+x-6=6$ $x^2+x-12=0$ $(x+4)(x-3)=0$ $x=-4$ *or* $x=3$	(4)	✓ removing the brackets ✓ std form ✓ factors ✓ both answers **[once-off -1 for omitting =0 for rest of paper]**
	1.1.2	$\lvert x-3\rvert>3$ $x^2-6x+9>9$ $x^2-6x>0$ $x(x-6)>0$ $x<0$ or $x>6$ **OR** $x-3<-3$ or $x-3>3$ $x<0$ or $x>6$ **NOTE:** If $-3<x-3<3$ $0<x<6$ max $\frac{1}{4}$ If $-3>x-3>3$ $0>x>6$ max $\frac{2}{4}$	(4)	✓ square both sides ✓ factors ✓✓ each inequality ✓✓ definition ✓✓ each inequality **[No "or" :deduct 1 mark once for rest of paper]** **[";" not acceptable: -1]**
	1.1.3	$x=\sqrt{2x-1}+2$ $x-2=\sqrt{2x-1}$ $x^2-4x+4=2x-1$ $x^2-6x+5=0$ $(x-1)(x-5)=0$ $x=1$ *or* $x=5$ $\therefore$ $x=5$ only answer If $x^2=2x-1+4$ BD : Solve and check $\frac{1}{6}$; Solve not checking : $\frac{0}{6}$	(6)	✓ Sq. root sign alone ✓ sq. both sides ✓ std form ✓ factors ✓ both answers ✓ choice

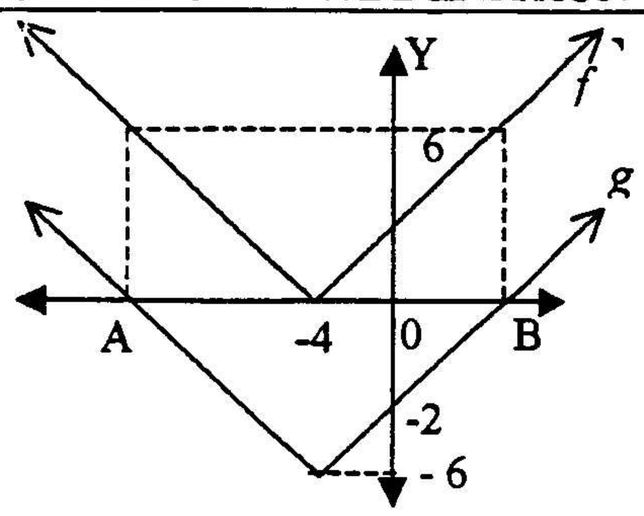
1.2	$\frac{x}{x-3}<2$ $\frac{x}{x-3}-2<0$ $\frac{x-2(x-3)}{x-3}<0$ $\frac{-x+6}{x-3}<0$  $x<3 \text{ or } x>6$ **Max marks: 2 only**	(5)	**[BD if multiply both sides by (x – 3): max $\frac{1}{5}$ x – 2x + 6 < 0 etc.]** ✓ 0 or RHS ✓ one denominator ✓ diagram (graph, table, line, etc ✓✓ for correct interpretation of diagram **[marks 2 or 0]**
1.3	$x^2+y^2=100$ ……(1) $4y+3x=50$ $\therefore\ y=\frac{50-3x}{4}$ ……(2) (2) in (1): $x^2+\left(\frac{50-3x}{4}\right)^2=100$ $\frac{16x^2+2500-300x+9x^2}{16}=100$ $25x^2-300x+2500=1600$ $25x^2-300x+900=0$ $x^2-12x+36=0$ $(x-6)^2=0$ $x=6$ $\therefore\ y=8$	(7)	✓ eq. (2) ✓ substitution ✓ bracket squared ✓ standard form ✓ factors ✓ x- value ✓ y-value
OR	$x^2+y^2=100$ …… ……(1) $4y+3x=50$ $\therefore\ x=\frac{50-4y}{3}$ …… ….(2) (2) in (1): $\left(\frac{50-4y}{3}\right)^2+y^2=100$ $\frac{2500-400y+16y^2+9y^2}{9}=100$ $25y^2-400y+2500=900$ $25y^2-400y+1600=0$ $y^2-16x+64=0$ $(y-8)^2=0$ y = 8 and x = 6		✓ eq. (2) ✓ substitution ✓ bracket squared ✓ standard form ✓ factors ✓ x- value ✓ y-value

OR	$x^2 + y^2 = 100$ $4y = 50 - 3x$ $16x^2 + 16y^2 = 1600$ $16x^2 + (50 - 3x)^2 = 1600$ $16x^2 + 2500 - 300x + 9x^2 = 1600$ $25x^2 - 300x + 900 = 0$ $x^2 - 12x + 36 = 0$ *etc.*		$x^2 + y^2 = 6^2 + 8^2$ $x = 6, y = 8$ *Marks* $\frac{4}{7}$ *if* $y = 6, x = 8$ *Marks* $\frac{0}{7}$
1.4.1	$x^2 - 2x + 3x - kx + k - 3 = 0$ $x^2 + x(1-k) + k - 3 = 0$ $\Delta = b^2 - 4ac$ $= (1-k)^2 - 4(1)(k-3)$ $= 1 - 2k + k^2 - 4k + 12$ $= k^2 - 6k + 13$	(3)	✓ standard form ✓ substitution into Δ ✓ remove brackets
1.4.2	$\Delta = k^2 - 6k + 9 - 9 + 13$ $= (k-3)^2 + 4$ $\therefore \Delta \geq 4$ for all $k \in$ R $\Delta \geq 0$ $\therefore$ roots are real	(4)	✓ completion of square ✓ form ✓ $\Delta \geq 4$ ✓ $\Delta > 0$ **[If $\Delta \geq 0$ indicated at least 1 mark]**
OR	Using the turning point: Minimum value = 4 $\therefore \Delta \geq 4$ for all $k \in$ R $\therefore$ roots are real	(4) **[33]**	✓✓ "Minimum value" stated anywhere ✓✓ 4

QUESTION 2			
2.1	(0; 1) satisfies the equation $y = k^x$ since $k^0 = 1$	(1)	✓ explanation
2.2	substitute $\left(\frac{1}{2};2\right)$: $2 = k^{\frac{1}{2}}$ $\therefore k = 4$	(2)	✓ substitution ✓ answer
2.3	$y = a(x-b)^2 + q$ $y = a\left(x - \frac{1}{2}\right)^2$ (0; 1): $1 = a\left(0 - \frac{1}{2}\right)^2$ $= a.\frac{1}{4}$ $\therefore a = 4$	(4)	✓ form ✓ sub TP ✓ sub (0; 1) ✓ a value

OR	$y = ax^2 + bx + 1$ $0 = \frac{1}{4}a + \frac{1}{2}b + 1$ $0 = a + 2b + 4$.....(1) $1 = a + b + 1$ $0 = a + b$ $a = -b$ $0 = a + (-2a) + 4$ $a = 4$		✓ form and c = 1 ✓substituting x = $\frac{1}{2}$ ✓ substitute (0;1) ✓ answer
2.4	$f(x) = 4^x$ for f^{-1}: $x = 4^y$ $\therefore\ y = \log_4 x$	(2)	[$\log_k x$ **accepted: 2 marks**] ✓ definition ✓ answer
2.5	$h(x) = -4^x$ or $h(x) = -k^x$	(1) **[10]**	✓ answer $(-k^x)$

QUESTION 3			
3.1	 $f(x) = \lvert x+4 \rvert$ and $g(x) = \sqrt{16 - x^2}$	(6)	For f: ✓ x-intercept (-4) ✓y-intercept (4) ✓ correct shape: V to top ✓ symmetry **[If salient point is at $x = 4$: $\frac{3}{4}$, and CA marks apply thereafter]** for g: ✓ semi circle ✓ intercept/showing radius **[Graphs on separate axes: marks $\frac{5}{6}$]**
3.2	$y \geq 0$ **OR** [$y \in [0;\infty)$ *or* $Range = \{y \mid 0 \leq y < \infty\}$	(2)	✓✓ answer **[y>0: subtract 1]**
3.3	$-4 < x < 0$	(2)	✓ critical values ✓correct inequalities
3.4	$x + 4 = 6$ or $x + 4 = -6$ $x = 2$ or $x = -10$	(2)	✓ set-up equations ✓ both answers **[Only 1 answer: 1 mark]**

3.5		(2) [14]	✓✓ A & B on x-axis **[Only 1 mark if indicate on the line y = 6]** **[If only −10 and 2 on x-axis: zero but if A=-10 and B =2 on x-axis: full marks]** **Alternatively** the graph can be moved 6 units down and the x- intercepts of the new graph can be labeled A & B

QUESTION 4

4.1	$h(x) = x^3 + px^2 + 2x + q$ $x^2 - x - 2 = (x-2)(x+1)$ $h(2) = 0$ $h(2) = 8 + 4p + 4 + q = 0$ ..... (1) $4p + q = -12$ $h(-1) = 0$ $h(-1) = -1 + p - 2 + q = 0$ .....(2) $p + q = 3$ (1) – (2): $3p = -15$ $p = -5$ and $q=8$	(7)	✓ Attempt to factorise (M) ✓ factorise correct (A) ✓ substitute in (1) ✓Either $h(2) = 0$ or $h(-1) = 0$ ✓ substitute in (2) ✓p-value ✓ q- value
OR	$x^3 + px^2 + 2x + q = (x^2 - x - 2)(x-4)$ $\therefore q = 8$ *and* $p = -5$		✓✓✓✓ ✓ q ✓✓ p
4.2	$f(x) = 2x^3 + kx^2 + 4x - 7$ $= (x+2).g(x) - 3$ $f(-2) = 0 - 3$ $2(-2)^3 + k(-2)^2 + 4(-2) - 7 = -3$ $-16 + 4k - 8 - 7 = -3$ $4k = 28$ $k = 7$	(4) [11]	✓ f(−2) = −3 **[If f(-2) = 0 BD: Max $\frac{1}{4}$]** ✓substituting -2 ✓ simplification ✓ answer

QUESTION 5

5.1	$\frac{5^{a-2}.2^{a+2}}{10^a - 2.10^{a-1}} = \frac{10^a\left(\frac{1}{25}.4\right)}{10^a\left(1-\frac{2}{10}\right)}$ $= \frac{4}{25} \div \frac{8}{10}$ $= \frac{4}{25}.\frac{10}{8}$ $= \frac{1}{5}$	(5)	✓ splitting up ✓same bases ✓common factor ✓ simplification ✓ answer

OR	$\frac{5^a.5^{-2}.2^a.2^2}{10^a - 2.10^a.10^{-1}}$ $= \frac{10^a.5^{-2}.2^2}{10^a(1-2.10^{-1})}$ $= \frac{\frac{4}{25}}{1-\frac{2}{10}} = \frac{4}{25} \times \frac{10}{8} = \frac{1}{5}$		✓ splitting up ✓same bases ✓common factor ✓ simplification ✓ answer
OR	$\frac{5^a.5^{-2}.2^a.2^2}{5^a 2^a - 2^{a-1}.5^{a-1}.2}$ $= \frac{5^a.5^{-2}.2^a.2^2}{5^a.2^a(1-2^{-1}.5^{-1}.2)}$ $= \frac{\frac{4}{25}}{\frac{4}{5}}$ $= \frac{1}{5}$		✓ splitting up ✓same bases ✓common factor ✓ simplification ✓ answer

5.2	log1 - log2 + log2 – log 3 + ...+ log99 – log100 = 0 - log100 = - 2	(5)	✓Apply log law – write without fractions ✓ log 99 – log 100 ✓ 0 ✓ -log100 ✓ answer
OR	$log(\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times ..... \times \frac{99}{100})$ $= log\frac{1}{100}$ $= -2$		✓Write as one log ✓ $\frac{99}{100}$ ✓✓ $log\frac{1}{100}$ ✓ answer **[Any attempt in 5.2 to use AP or GP formulae: 0 marks]**

5.3	5.3.1	$3x^{\frac{2}{3}} - 12 = 0$ $3x^{\frac{2}{3}} = 12$ $x^{\frac{2}{3}} = 4$ $x^2 = 4^3 = 64$ $x = 8$ or -8	(3)	✓ dividing by 3 ✓ exp law ✓ any one answer

OR		$3x^{\frac{2}{3}} - 12 = 0$ $3x^{\frac{2}{3}} = 12$ $x^{\frac{2}{3}} = 4$ $x = \left(2^2\right)^{\frac{3}{2}}$ $= 8$		Full marks (3)
OR		$3x^{\frac{2}{3}} - 12 = 0$ $x^{\frac{2}{3}} - 4 = 0$ $(x^{\frac{1}{3}} - 2)(x^{\frac{1}{3}} + 2) = 0$ $x^{\frac{1}{3}} = \pm 2 \quad and \quad x = \pm 8$		Full marks (3)
	5.3.2	$3^{x^2-1} = \frac{27^{-x}}{3} = \frac{3^{-3x}}{3}$ $= 3^{-3x-1}$ $x^2 - 1 = -3x - 1$ $x^2 + 3x = 0$ $x(x+3) = 0$ $x = 0 \quad \text{or} \quad x = -3$	(4)	✓ same base ✓ using second law ✓ standard form ✓ both answers
	5.3.3	$3\log_8 x - 5 = 2\log_x 8$ $3\log_8 x - 5 = \frac{2}{\log_8 x}$ $3(\log_8 x)^2 - 5\log_8 x = 2$ $3(\log_8 x)^2 - 5\log_8 x - 2 = 0$ $(3\log_8 x + 1)(\log_8 x - 2) = 0$ $\log_8 x = -\frac{1}{3} \quad \text{or} \quad \log_8 x = 2$ $x = 8^{-\frac{1}{3}} \quad \text{or} \quad x = 8^2$ $x = \frac{1}{2} \quad \text{or} \quad x = 64$	(6)	✓ log law same base ✓ standard form ✓ factors ✓ log forms ✓ exponential forms ✓ both answers

OR		$\frac{3\log x}{\log 8}-5=\frac{2\log 8}{\log x}$ $3(\log x)^2-5\log 8\log x-2(\log 8)^2=0$ $(3\log x+1\log 8)(\log x-2\log 8)=0$ $x^3=\frac{1}{8}$ *or* $x=8^2$ $x=\frac{1}{2}$ *or* $x=64$		
	5.3.4	$\log_2(5-x)+\log_2 x\le 2$ $\log_2(5x-x^2)\le 2$ $5x-x^2\le 4$ $5x-x^2-4\le 0$ $x^2-5x+4\ge 0$ $(x-1)(x-4)\ge 0$ $x\le 1$ or $x\ge 4$ ..... (1) by definition: $5-x>0$ and $x>0$ $x<5$ and $x>0$ $\therefore\ 0<x<5$ .....(2) combining (1) and (2): $4\le x<5$ or $0<x\le 1$	(10) **[33]**	✓ single log ✓ remove logs ✓ factors ✓✓ each answer ✓ definition ✓✓ $4\le x<5$ ✓✓. $0<x\le 1$ **[Critical values: one mark inequality signs: one mark]** $\frac{3}{4}$ **marks for** $4<x<5$ **or** $0<x<1$
QUESTION 6				
6.1		$T_5=\frac{1+15}{2}=8$	(3)	✓✓ 1+15 and divide by 2 ✓ answer 8
OR		$T_1=a=1$ $T_9=1+8d=15$ $d=\frac{14}{8}$ $T_5=1+4d=1+\frac{4}{1}.\frac{14}{8}=8$		✓ 9^{th} term ✓ d ✓ answer
OR		$T_n=\frac{7n-3}{4}$ $Middle\ \ term=\frac{1+\frac{7n-3}{4}}{2}=\frac{\frac{4+7n-3}{4}}{2}=\frac{7n+1}{8}$		

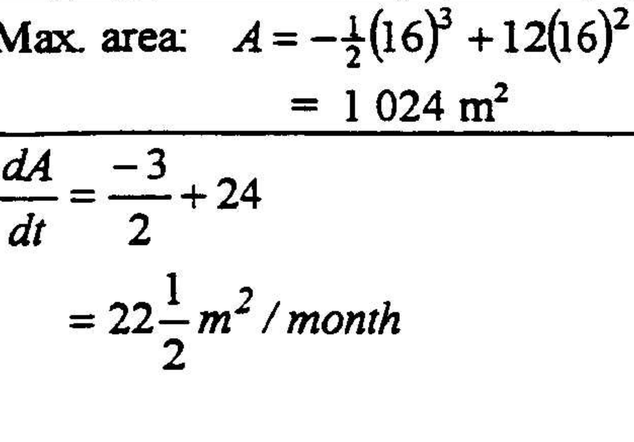
6.2		$S_n = a + ar + ar^2 + \ldots + ar^{n-2} + ar^{n-1}$ $rS_n = \quad ar + ar^2 + ar^3 + \ldots + ar^{n-1} + ar^n$ $S_n - rS_n = a - ar^n$ $S_n(1-r) = a\left(1-r^n\right)$ $S_n = \frac{a\left(1-r^n\right)}{1-r}$	(4)	✓ S_n ✓ rS_n ✓ subtract ✓ factorise
6.3	6.3.1	$6+3+\frac{3}{2}+\frac{3}{4}+ \ldots$ GP: $a=6;\quad r=\frac{1}{2}$ $S_{11} = \frac{a\left(1-r^n\right)}{1-r} = \frac{6\left(1-\left[\frac{1}{2}\right]^{11}\right)}{1-\frac{1}{2}}$ $= 12(1-0{,}000488)$ $= 11{,}99 \text{ or } (\frac{6141}{512} \text{ or } 11\frac{509}{512})$	(4)	✓ formula ✓ substitute r = $\frac{1}{2}$ ✓ substituting n = 11 ✓ answer
	6.3.2	$S_\infty = \frac{a}{1-r}$ $= \frac{6}{1-\frac{1}{2}}$ $= 12$	(3)	✓ formula ✓ substitution ✓ answer **[answer only: full marks]**
	6.3.3	$\sum_{k=1}^{11} 6\left(\frac{1}{2}\right)^{k-1} \quad or \quad \sum_{0}^{10} 6(\frac{1}{2})^k$	(2)	✓✓ [Zero or 2]
6.4		$324 ; 324r ; 324r^2 ; 324r^3 ; 324r^4$ $\therefore 324r^4 = 2500$ $r^4 = \frac{2500}{324} = \frac{625}{81} = \frac{5^4}{3^4} = \left(\frac{5}{3}\right)^4$ $r = \frac{5}{3}$ **OR** $r = 1{,}67$ The second lowest gear is:: $324(1{,}67) = 541{,}08$ **OR** $324(\frac{5}{3}) = 540$	(4)	✓ term 5 = 2500 ✓ dividing by 324 ✓ $\sqrt[4]{\ }$ ✓ term 1 × r

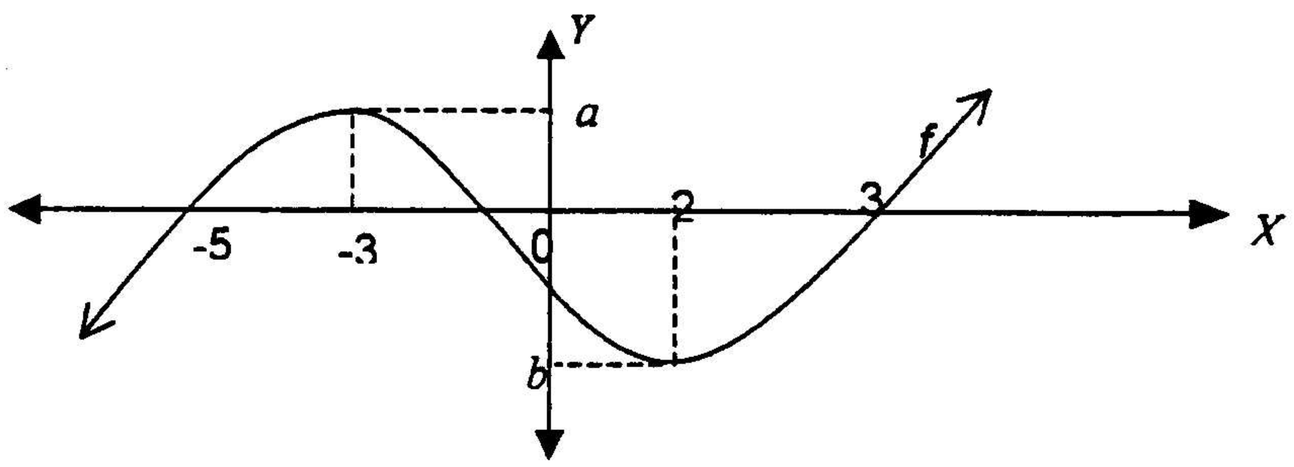
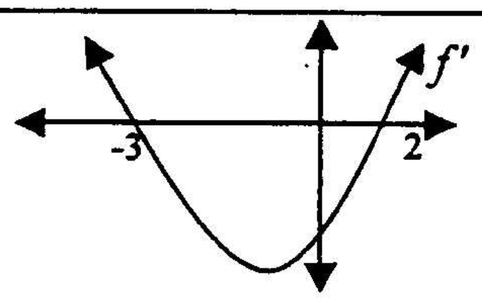
6.5	$S_{2n} = \frac{2n}{2}[2a + (2n-1)(4)]$ $= 2an + 8n^2 - 4n$ $2S_n = n[2a + (n-1)4] = 2an + 4n^2 - 4n$ $S_{2n} - 2S_n = 4n^2$ $\sqrt{S_{2n} - 2S_n} = 2n$	(7) **[27]**	✓Formula ✓substitution of 2n ✓ substitution of d = 4 ✓ simplification ✓ $2S_n$ (substitution of n and 4) ✓ simplification ✓ $S_{2n} - 2S_n$

QUESTION 7

7.1	$f(x) = 6x^2 - 3x - 1$ $f(x+h) = 6(x+h)^2 - 3(x+h) - 1$ $= 6(x^2 + 2xh + h^2) - 3x - 3h - 1$ $= 6x^2 + 12xh + 6h^2 - 3x - 3h - 1$ $f(x+h) - f(x) = 12xh + 6h^2 - 3h$ $\frac{f(x+h) - f(x)}{h} = 12x + 6h - 3$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 12x - 3$	(6)	✓ $f(x+h)$ ✓ simplification ✓ subtract $f(x)$ ✓ $\div h$ ✓ definition ✓ answer **[Penalty –1 for incorrect notation]** **[Answer only: 0 marks]**
7.2	$\frac{d}{dx}\left(4\sqrt{x} - \frac{8}{\sqrt{x}} + \pi x^3\right) = \frac{d}{dx}\left(4x^{\frac{1}{2}} - 8x^{-\frac{1}{2}} + \pi x^3\right)$ $= 2x^{-\frac{1}{2}} + 4x^{-\frac{3}{2}} + 3\pi x^2$	(4)	✓ both exponential forms ✓✓✓ each derivative
7.3	$y = (x^2 - \sqrt{x})^2$ $= x^4 - 2x^2.x^{\frac{1}{2}} + x$ $= x^4 - 2x^{\frac{5}{2}} + x$ $\frac{dy}{dx} = 4x^3 - 5x^{\frac{3}{2}} + 1$	(5)	✓ remove bracket ✓ exponential law ✓✓✓ each derivative **[Maximum –1 for both 7.2 and 7.3 for abuse of equal sign]**

7.4	$f(x)=ax^3+bx$ $(-1;3)$: $3=a(-1)^3+b(-1)$ $3=-a-b$ ..... (1) tangent: $y=x+4 \Rightarrow m=1$ $f'(-1)=1$ $f'(x) = 3ax^2 + b$ $\therefore 3a(-1)^2 + b = 1$ $\therefore 3a + b = 1$ .....(2) (1) + (2): $2a = 4$ $a = 2$ From (1): $b = -5$	(7) [22]	✓ sub (-1 ; 3) ✓ $m = 1$ ✓ derivative $f'(-1)$ ✓ derivative = 1 ✓ equation (2) ✓ value of a ✓ value of b

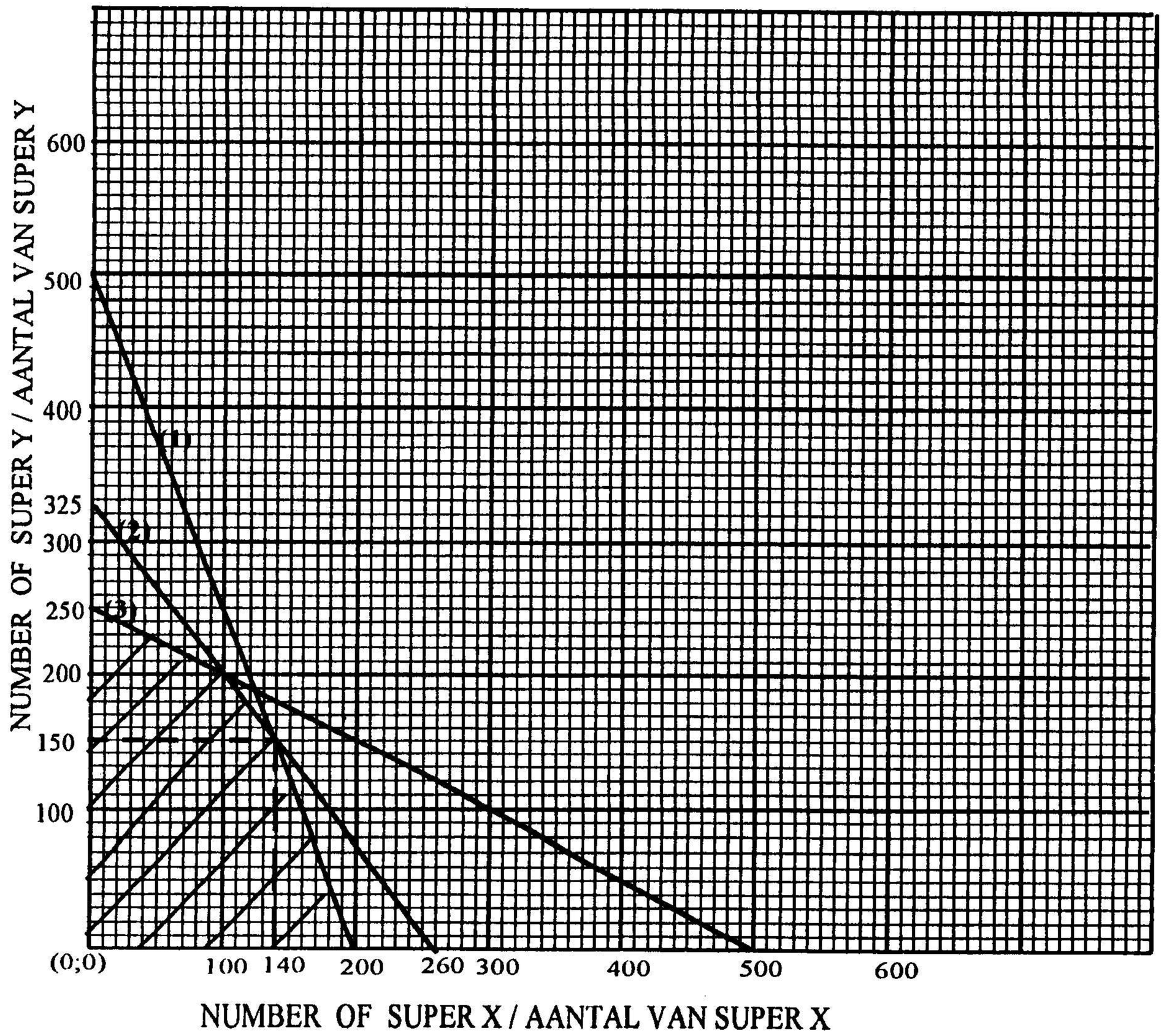
QUESTION 8				
8.1	8.1.1	A maximum is obtained when $\frac{dA}{dt}=0$ $\frac{dA}{dt}=-\frac{3}{2}t^2+24t = 0$ $\times 2: \quad -3t^2+48t=0$ $\div -3: \quad t^2-16t=0$ $t(t-16)=0$ $t=16$	(4)	✓ derivative ✓ = 0 ✓ factors ✓ correct value of t **[t = 0 or 16: -1]**
	8.1.2	Max. area: $A=-\frac{1}{2}(16)^3+12(16)^2$ $= 1\,024\ m^2$	(2)	✓ substitution ✓ answer
	8.1.3	$\frac{dA}{dt}=\frac{-3}{2}+24$ $=22\frac{1}{2}\,m^2/month$	(3)	**[If substitute into original equation: 0/3]** ✓ $\frac{dA}{dt}$ ✓ sub $t = 1$ into derivative ✓ answer **[Ignore the unit]**
	8.1.4	$-\frac{1}{2}t^3+12t^2=0$ $-\frac{1}{2}t^2(t-24)=0$ $t=0$ *or* $t=24$  	(5)	✓ value t ✓ TP(16; 1024) ✓ (0; 0) ✓ shape ✓ indicating t on graph **[No marks deducted if full graph drawn]**

	8.2			
	8.2.1	$f'(x) = 0$ for $x = -3$ or $x = 2$	(2)	✓✓ x values
	8.2.2	$f'(x) > 0$ for $x < -3$ or $x > 2$	(4)	✓✓ for each inequalities **[-1 if = signs are included]**
	8.2.3		(3)	✓ parabola (shape) ✓ arms upward ✓ roots - 3 and 2
	8.2.4	$k > a$ or $k < b$	(3)	✓✓✓ $k > a$;/or $k < b$ **[Partial marks:** ✓✓ **k > a or** ✓✓ **k < b** ✓ **k ≤ b** ✓ **k ≥ a** ✓✓ **k ≥ a or/; k ≤ b]**
8.3	8.3.1	Athlete B is running in front at the start of the race or (B)	(1)	✓ answer
	8.3.2	Athlete B trips and falls at the hurdle	(1)	✓ answer
	OR	Athlete slow down or decelerate or he stayed at the same place for a small period of time		
	8.3.3	Athlete A is leading the race at the halfway mark or (A)	(1)	✓ answer
	8.3.4	Athlete C finishes the race faster and perhaps breaks the record or (C)	(1)	✓ answer
			[30]	

QUESTION 9			
9.1	Constraint (1): $150x + 60y \leq 30\,000$ Constraint (2): $50x + 40y \leq 13\,000$ Constraint (3): $10x + 20y \leq 5\,000$	(6)	✓✓ correct values & ineq. ✓✓ correct values & ineq. ✓✓ correct values & ineq. **[-1 if <]** **[$\frac{3}{6}$ for all signs "="]**
9.2	On graph paper	(6)	1 mark per intercept

9.3	On graph paper	(2)	✓✓ correct shading of feasible region
9.4	Profit : $P = 20\,000x + 10\,000y$ **OR** $y = -2x + \frac{P}{10\,000}$	(2)	✓✓ correct equation
9.5	140 Super X and 150 Super Y	(2)	**[140 ± 10 or 150 ± 10]**
9.6	P = 20 000(140) + 10 000(150) = R 4 300 000	(2)	✓ substitution ✓ answer **or some number between R4 000 000 and R4 600 00**
		[20]	
	TOTAL : 200		

QUESTION 9


NUMBER OF SUPER Y / AANTAL VAN SUPER Y
600
500
400
325
300
250
200
150
100
(0;0)
100 140 200 260 300 400 500 600
NUMBER OF SUPER X / AANTAL VAN SUPER X
(2)
(3)