POSSIBLE ANSWERS FOR:

WISKUNDE SG / MATHEMATICS SG
VRAESTEL II / PAPER II
SET B

3 UUR
150 PUNTE

3 HOURS
150 MARKS

✓M = 1 mark for a certain method used

✓A = 1 mark for accuracy

✓CA = 1 mark for consistent accuracy

✓CAO = 1 mark for the correct answer only

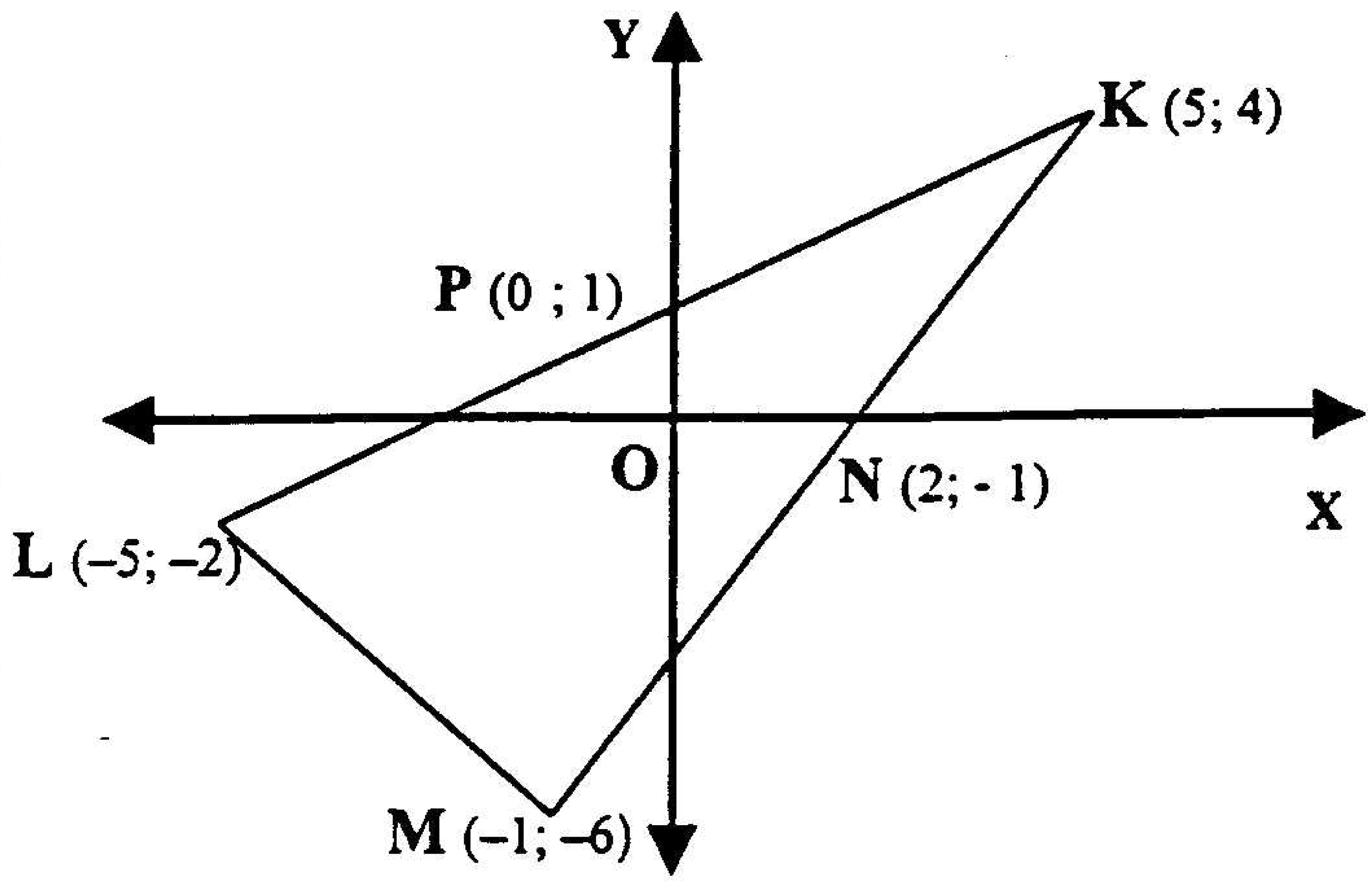
✓S = 1 mark for the correct geometric statement

✓R = 1 mark for the correct reason given

✓S/R = 1 mark for the correct statement with the correct reason

QUESTION 1

1.1



Wrong formula anywhere – 0 marks

✓M

$$\left(\frac{-1+5}{2};\frac{-6+4}{2}\right)=\left(\frac{4}{2};\frac{-2}{2}\right)$$

✓A ✓A

N(2: –1) **or** $x = 2$; $y = -1$ (3)

1M mark for stating or using midpoint formula

1A mark per coordinate

1.2 ✓M

$$m_{LM}=\frac{-2+6}{-5+1}=\frac{4}{-4}=-1 \quad ✓A \quad (2)$$

1 M mark for stating or using gradient formula
1A mark for answer
Answer only ⇒ full marks

1.3 ✓M ✓CA

$-1=(-1)2+c$

c =1

$y=-x+1$ **or** $x+y-1=0$ ✓CA

1M mark for use of any variant of straight line equation
1CA mark for substitution
1CA mark for answer – any form straight line eqaution acceptable

OR

✓M

$y-(-1)=-1\,(x-2)$ ✓A

$y = -x + 2 - 1$
$y = -x + 1$ ✓CA (3)

1M mark for use of any variant of straight line equation
1CA mark for substitution
1CA mark for answer

1.4 at P(0 ; 1)

$\therefore$ RHS = – 0 + 1 ✓M
= 1 ✓A
= LHS

OR

$m_{PN}=\frac{1+1}{0-2}=-1$ ✓A

gradient of the line is – 1 ✓M
P lies on line

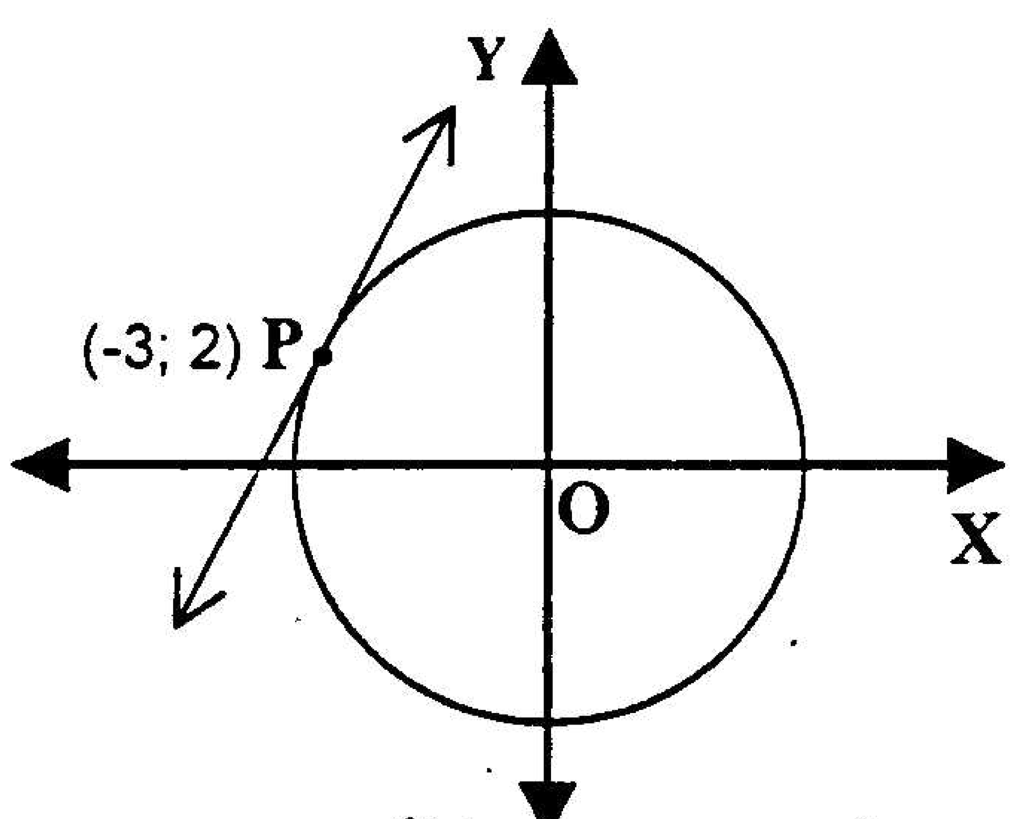
OR

P is y-intercept of the line in 1.3 (2)

IM mark for substitution
1A mark for answer
No penalty if LHS and RHS is kept simultaneously
Substitution into wrong equation resulting in LHS ≠ RHS - max 1 mark

1.5 $LM = \sqrt{(-5+1)^2 + (-2+6)^2}$ ✓M ✓A $= \sqrt{(-4)^2 + (4)^2} = \sqrt{32}$ or $4\sqrt{2}$ ✓CA $\approx 5{,}65$ (3)	1 M mark for stating or using distance formula 1 A mark for correct substitution 1 CA mark for answer – accept rounded off to 6
1.6 $PN = \sqrt{(0-2)^2 + (1+1)^2}$ ✓M $= \sqrt{(-2)^2 + (2)^2}$ ✓CA $= \sqrt{8}$ or $2\sqrt{2}$ ✓CA $\approx 2{,}83$	1 M mark for using distance formula 1 CA mark for substitution of N from 1.1 1 CA mark for answer – accept rounded off to 3
$LM = 2(2\sqrt{2}) = 4\sqrt{2} = 2(2{,}83)$ ✓M ✓A $LM = 2PN$ (5) [18]	1 M mark for manipulion in order to provide justification 1 A mark for conclusion. Midpoint theorem is quoted ⇒ 0 marks 2PN = 2(3) = 6 = LM ⇒ max 4 marks

QUESTION 2



2.1. 2.1.1 $x^2 + y^2 = (-3)^2 + (2)^2$ ✓M ✓A $r^2 = 9 + 4 = 13$ ✓CA $\therefore x^2 + y^2 = 13$ (3)	"If $r^2=13$ only give 2 out of 3." 1 M mark for stating or using circle formula 1 A mark for correct subst into circle formula 1 C A mark for answer
2.1.2 $m_{OP} = \dfrac{0-2}{0+3} = \dfrac{-2}{3}$ ✓M ✓A (2)	1 M mark for stating or using gradient formula 1 A mark for answer Answer only → 2 marks
2.1.3 $m_{\perp op} = \dfrac{3}{2}$ ✓CA (1)	1 CA mark ; follows on from 2.1.2

2.1.4 $2 = \frac{3}{2}.(-3) + c$ ✓M ✓CA $c = \frac{9}{2} + 2$ $= \frac{13}{2}$ ✓CA $y = \frac{3}{2}x + \frac{13}{2}$ ✓CA **OR** $y - y_P = m_{\perp op}(x - x_P)$ $y - 2 = \frac{3}{2}(x+3)$ ✓M ✓A $2y - 4 = 3x + 9$ ✓CA $2y = 3x + 13$ or $2y - 3x - 13 = 0$ ✓CA **OR** $x.x_1 + y.y_1 = r^2$ ✓M ✓A $-3x + 2y = 13$ ✓CA $2y = 3x + 13$ ✓CA $y = \frac{3}{2}x + \frac{13}{2}$	(4)	1 M mark for stating or using a straight line formula 1 CA mark for correct substitution 1 CA mark for manupulation 1 CA mark for final answer 1 M mark for stating or using a straight line formula 1 A mark for correct substitution 1 CA mark for manipulation 1 CA mark for final answer – any form of line
2.1.5 ✓M $m_{tan} = \tan\theta = \frac{3}{2}$ ✓CA $\therefore \theta = 56{,}3^0$ ✓CA	(3)	1 M mark for using tan ratio 1 CA for gradient from 2.1.4 1 CA mark for answer - θ must be in [0°; 180°] (penalise for rounding off error) Use any trig ratio in a triangle correctly is acceptable
2.2 $x^2 + (x+5)^2 = 25$ ✓M $x^2 + x^2 + 10x + 25 - 25 = 0$ ✓A $2x^2 + 10x = 0$ ✓CA $x(x+5) = 0$ ✓M $x = 0$ *or* $x = -5$ ✓CA $y = 5$ or $y = 0$ ✓CA (0; 5) or (–5; 0) **OR** $(y-5)^2 + y^2 = 25$ $y^2 - 10y + 25 + y^2 - 25 = 0$ $2y^2 - 10y = 0$ $y(y-5) = 0$ $y = 0$ or $y = 5$ $x = -5$ or $x = 0$ (–5; 0) ; (0; 5)	(6)	1 M mark for subst. the straight line into the circle 1 A mark for correct multiplication 1 CA mark for standard form of quadratic 1M mark for factorising 1 CA mark for x-values 1 CA mark for y-values (the x- and y-values must be correctly paired) Writing down (0; 5) and (–5; 0) only without showing any work or diagram ⇒ max 4 marks Writing down (0; 5) and (–5; 0) only with work or diagram correctly ⇒ 6 marks

2.3 $AP^2 = PC^2$

✓A ✓M ✓A

$$(x+1)^2 + (y-5)^2 = (x-1)^2 + (y+1)^2$$ ✓CA

$$x^2 + 2x + 1 + y^2 - 10y + 25 = x^2 - 2x + 1 + y^2 + 2y + 1$$

$$2x - 10y + 26 = -2x + 2y + 2$$

$$4x - 12y = -26 + 2$$

$$4x - 12y = -24$$

$$4x - 12y + 24 = 0$$

$$x - 3y + 6 = 0$$ ✓CA

OR

Midpoint of AC is (0; 2)

Gradient of AC = –3

Gradient of perp bisector is $\frac{1}{3}$

Equation of locus is:

$y = \frac{1}{3}x + 2$ **or**

$3y = x + 6$ **or**

$3y - x - 6 = 0$

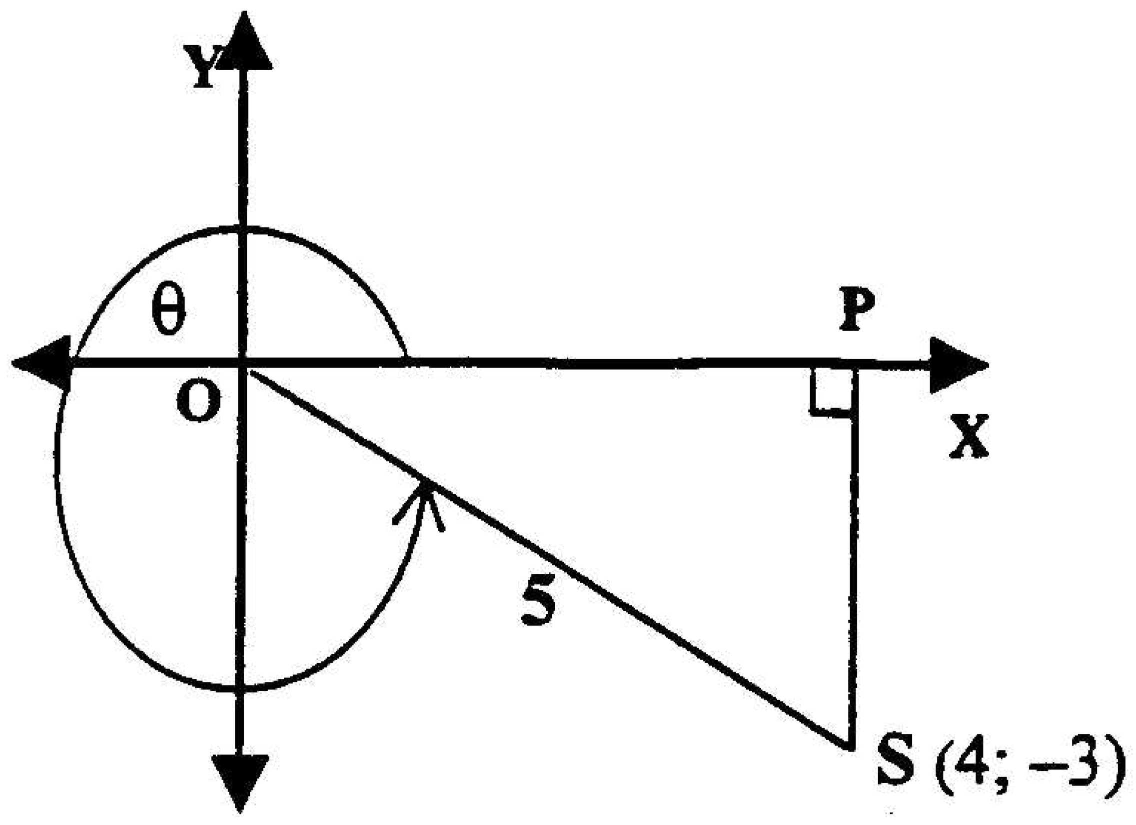
(5)

[24]

1 M mark for equating distances
1 A mark for substituting on LHS
1 A mark for subsituting on RHS
1CA mark for correct multiplication
1 CA mark for final equation

2A mark for midpoint
1A mark for gradient of AC
1CA mark for gradient of perp bisector
1CA mark for final answer – any form of straight line acceptable

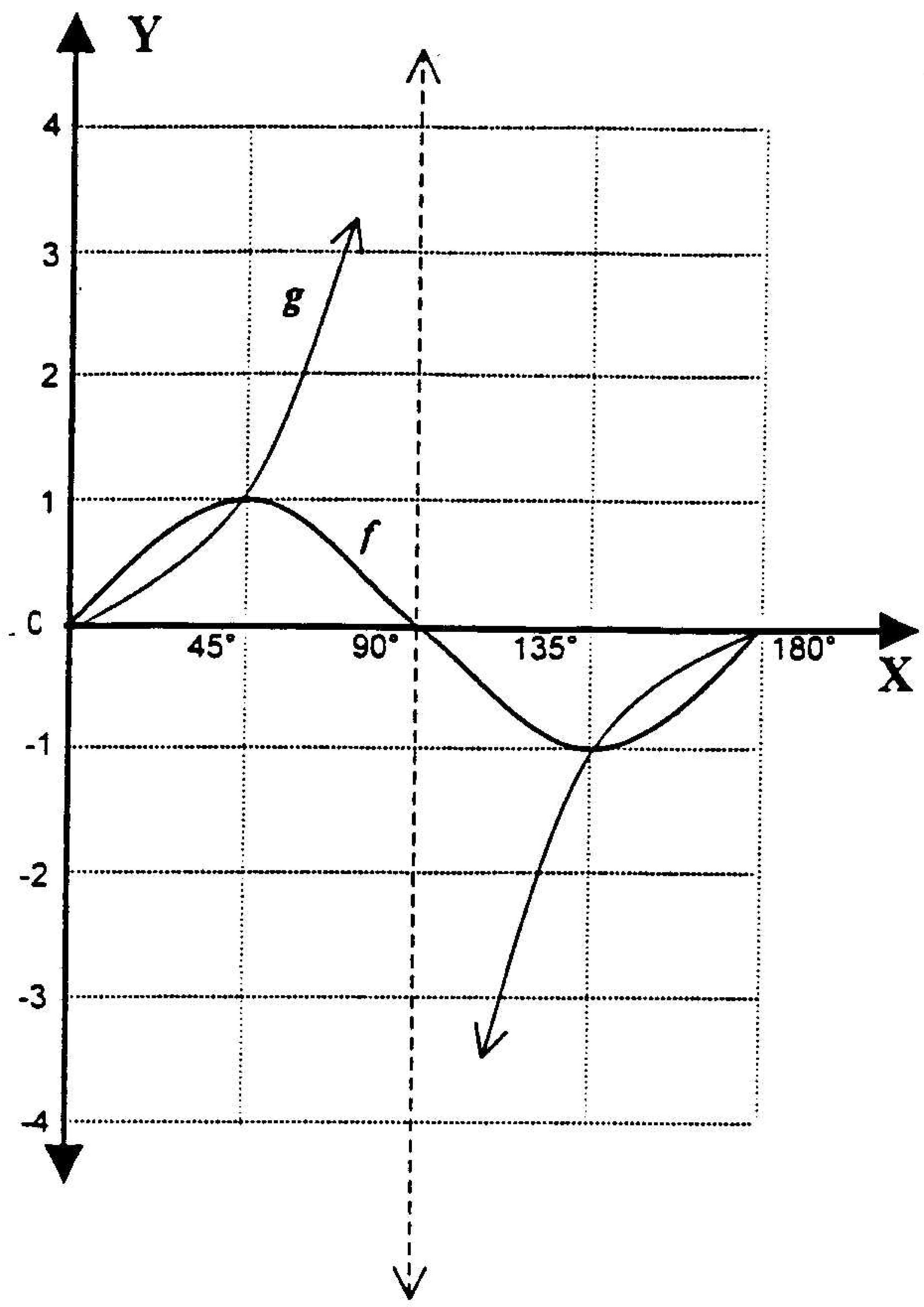
QUESTION 3



3.1.1 $OS = \sqrt{16+9} = \sqrt{25}$ ✓M ✓A $= 5$ ✓CA (3)	1M – stating or using distance formula 1A – correct substitution 1CA – final answer Answer only ⇒ 3 marks OS = –5 ⇒ max 2 marks
3.1.2 $\cos\theta = \frac{4}{5}$ ✓CA (1)	
3.1.3 from 3.1.2 ref. angle = 36,9° ✓CA $\theta = (360° - 36,9°)$ $= 323,1°$ ✓CA (2)	1CA – reference angle 1CA – final answer in correct quadrant If rounding error already penalised in 2.1.5 no penalty here.
3.2 $\frac{-\cos x . \sec x}{\sin x . \cot x}$ ✓A ✓A ✓A ✓A $= \frac{-\cos x . \frac{1}{\cos x}}{\sin x . \frac{\cos x}{\sin x}}$ ✓CA ✓CA $= -\frac{1}{\cos x}$ $= -\sec x$ ✓CA (7)	If most (all) ratios written without angle (x) –1 for notation. Do not penalise if it is left out in one or two cases. Any of last two acceptable as the answers
3.3 LHS $= \frac{-\cot 30°(-\operatorname{cosec} 60°)}{-\tan 45°}$ ✓A ✓A ✓A $= -\frac{\frac{\sqrt{3}}{1} \times \frac{2}{\sqrt{3}}}{1}$ ✓CA ✓CA ✓CA $= -2$ ✓CA (7) [20]	Note: the correct sign must be shown at each reduction. If the ratios are written down without the reduction: 1A for the **sign** and 1A for the **value** in each case. If 1 – sign only ⇒ max 5 marks

QUESTION 4

4.1



f: x-intercepts: 0°, 90°, 180° ✓A turning points: (45°; 1) (135°; -1) ✓A y-intercept ✓A		No need to indicate coordinates on graph
g: shape ✓A (45°;1) (135°;-1) ✓A asymptote ✓A	(6)	Note: for the shape we require two branches Asymptote must be indicated by a line different from the grid lines (preferably a broken line) No penalty if arrowhaeds on curve are omitted
4.2.1 $x = 90°$ ✓CA	(1)	This answer must be in the form of an equation
4.2.2 $y \in [-1; 1]$ ✓CA ✓A or $\{y : -1 \leq y \leq 1\}$ ✓CA ✓A	(2)	1 A mark for notation; 1CA mark for endpoints of interval. No penalty if curly brackets or $y \in$ not shown. If $-1 \leq x \leq 1 \Rightarrow$ 1 mark only
4.2.3 $h(45°) = \tan 45° - \sin(2 \times 45°)$ ✓CA $= (1) - 1$ ✓CA $= 0$ ✓CA	(3)	answer only ⇒ full marks
	[12]	

QUESTION 5

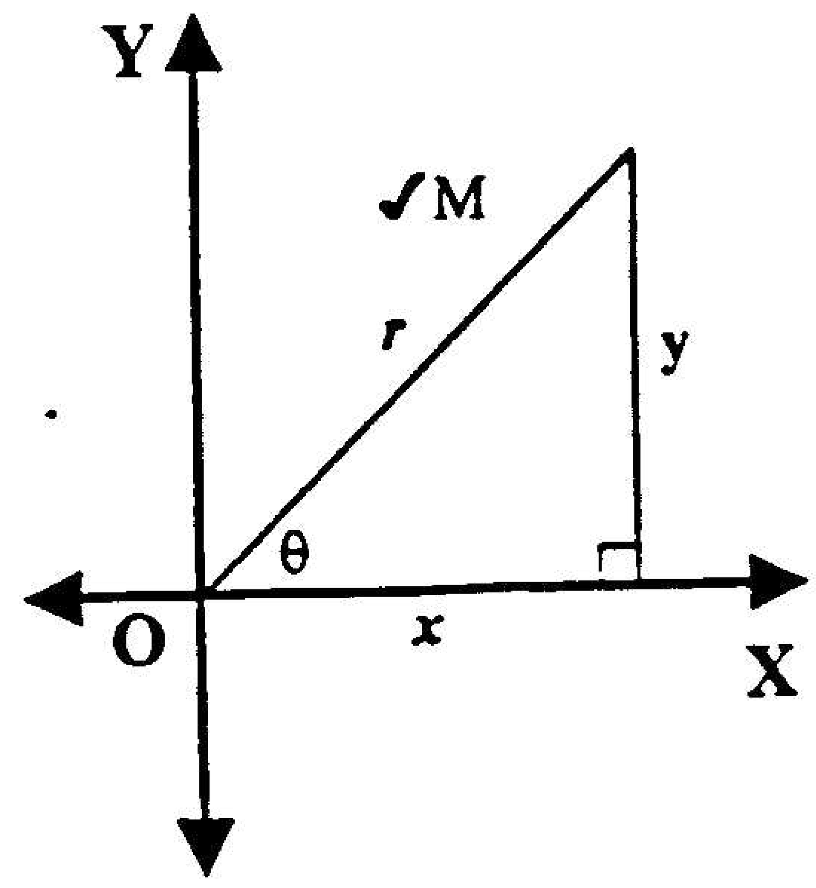
5.1 LHS $= \sin^2\theta + \cos^2\theta$

$= (\frac{y}{r})^2 + (\frac{x}{r})^2$ ✓A ✓A

$= \frac{y^2 + x^2}{r^2}$ ✓A

$= \frac{r^2}{r^2}$ ✓A

$= 1 =$ RHS (5)



No marks if proven for a specific case (i.e. if e.g. sides indicated as 3;4;5 or a special angle e.g. 45° or 60°)

If x, y and r are used without an accompanying diagram : 2 marks only.

5.2 $\cos A \sin^2 A + \cos^3 A$

$= \cos A\,(\sin^2 A + \cos^2 A)$ ✓A ✓M

$= \cos A\,(1)$ ✓A

OR

$\cos A \sin^2 A + \cos^3 A$

$= \cos A(1 - \cos^2 A) + \cos^3 A$ ✓M

$= \cos A - \cos^3 A + \cos^3 A$ ✓A

$= \cos A$ ✓A (3)

Dividing by cos A prior to factorization 0 marks.
1 need not be shown in final answer

5.3.1 $\cos 2x = -0{,}53$

$2x = 180° - 58°$ ✓A

$= 122°$ ✓CA

$x = 61°$ ✓CA (3)

1 A mark for reference angle
1 CA mark for angle in 2^{nd} Q
1 CA mark for dividing by 2
If ref. Angle divided by 2 and then angle in 2^{nd} Q or final answer given only as 29° ⇒2 marks
Divide by 2 at start, ignore further calculations ⇒ 0 marks

5.3.2 $\sqrt{2}\sin x - 1 = 0$

$\sqrt{2}\sin x = 1$ ✓A

$\sin x = \frac{1}{\sqrt{2}}$ ✓CA

$x = 45°; 135°$ ✓CA ✓CA (4)

[15]

If sin x = ½, max 2 out of 4 marks

Only if it is given that $x = 30°, 150°$

(one mark for each answer)

QUESTION 6

6.1 $\sin C = \dfrac{AD}{b}$

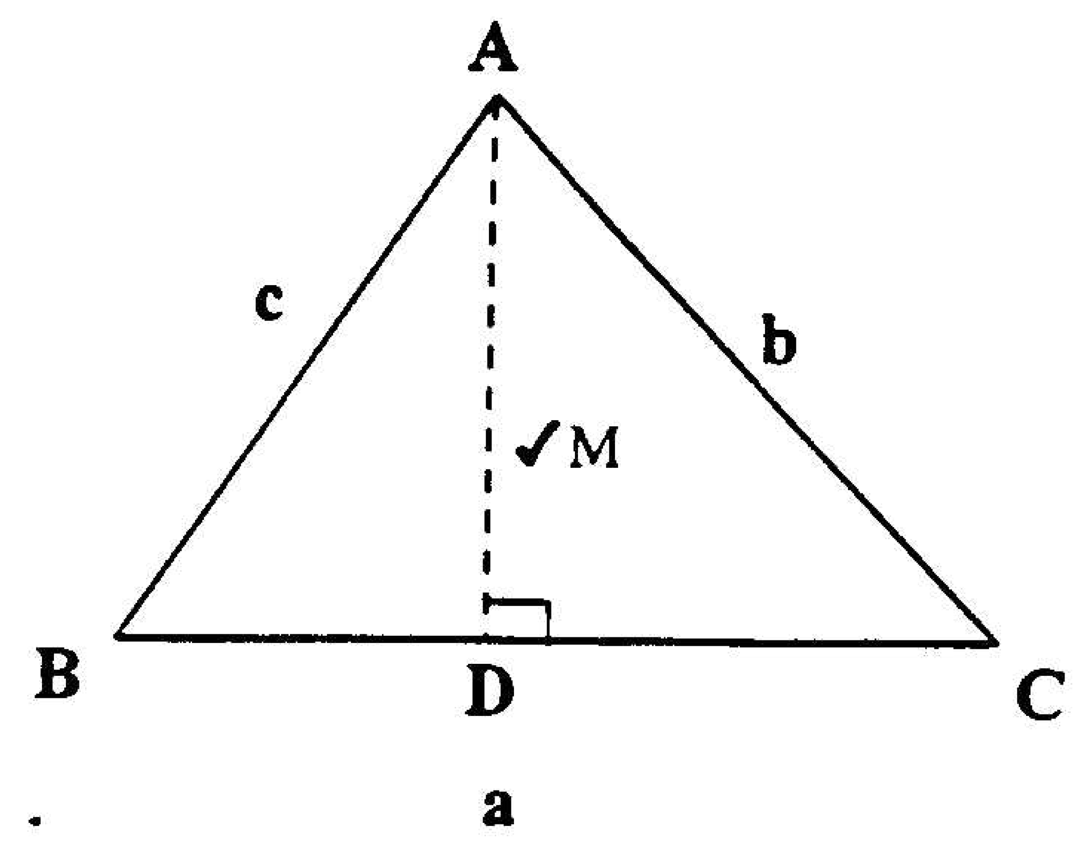
$AD = b\sin C$ ✓A

similarly

$AD = c\sin B$ ✓A

$b\sin C = c\sin B$ ✓A

$\therefore \dfrac{\sin C}{c} = \dfrac{\sin B}{b}$ (4)



OR ✓M ✓A ✓A

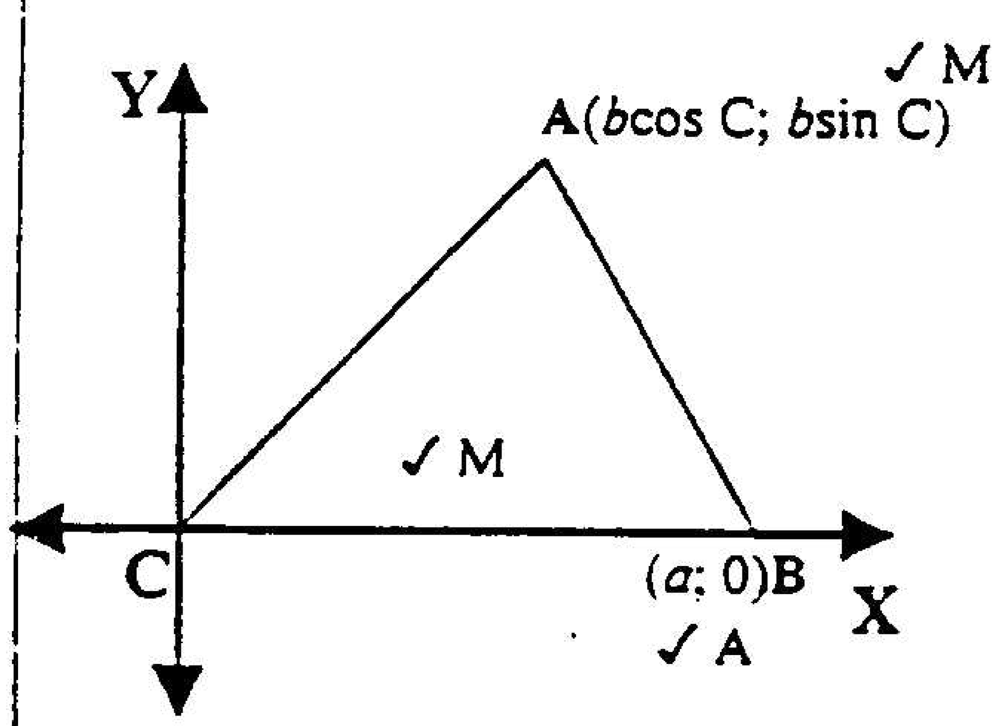
Area $\Delta ABC = \dfrac{1}{2}ab\sin C = \dfrac{1}{2}ac.\sin B$

Divide by $\dfrac{1}{2}abc$ ✓A

Thus $\dfrac{\sin C}{c} = \dfrac{\sin B}{b}$

OR bsin C = c.sin B

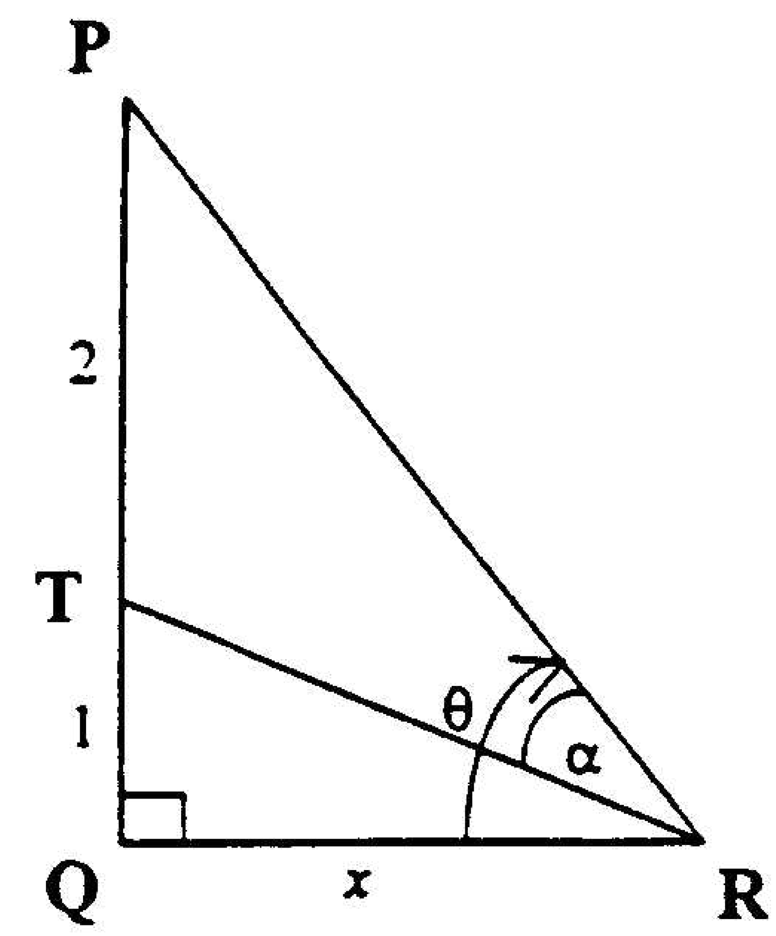
OR



Correct y coordinate of A

Placing the triangle in standard position on the axes

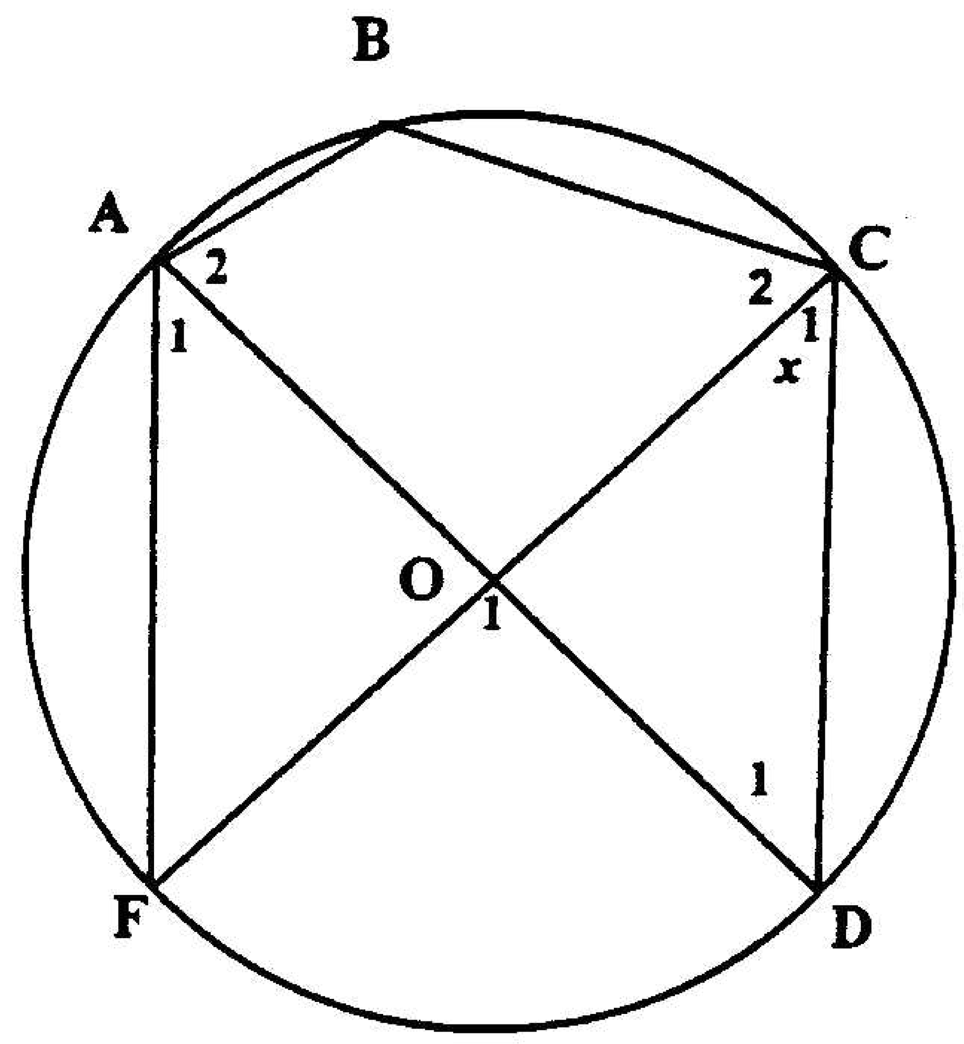
Indicating that CB = a

6.2.1 In ΔTQR, $\hat{Q}=90°$ (a) $\cos(\theta-\alpha)=\frac{x}{TR}$ ✓A ✓A $TR=\frac{x}{\cos(\theta-\alpha)}$ (2) (b) $T\hat{P}R=(90°-\theta)$ ✓A (1)	 Note $(\theta-\alpha)$ as well as $(90°-\theta)$ may be indicated on the sketch.
(c) $\frac{TR}{\sin(90°-\theta)}=\frac{2}{\sin\alpha}$ ✓M $TR=\frac{2\cos\theta}{\sin\alpha}$ ✓CA $\frac{x}{\cos(\theta-\alpha)}=\frac{2\cos\theta}{\sin\alpha}$ ✓CA $\therefore x=\frac{2.\cos\theta\cos(\theta-\alpha)}{\sin\alpha}$ (3)	1 M mark for using the sine-rule 1 CA mark for reduction os sin(90°–θ) 1 CA for substituting for TR from (a)
6.2.2 $x=\frac{2\cos 50°\cos 20°}{\sin 30^0}$ ✓M ✓A $=2{,}4$ m ✓CA (3) [13]	Alternatives : Using the tan ratio or the sine rule in triangleTQR or PQR In Δ PQR tan ratio results in x = 2,517 sin rule results in x = 2,517 In Δ TQR tan ratio results in x = 2,747 sin rule results in x = 2,747

QUESTION 7

7.1 Opposite angles ✓CAO (1)

7.2



7.2.1 $\hat{D}_1 = \hat{C}_1 = x$ ✓S (OC = OD = radius) ✓R

$\hat{A}_1 = \hat{C}_1 = x$ ✓S ($\angle^s$ subt by same chord FD) ✓R

$\hat{F} = \hat{D}_1 = x$ ✓S ($\angle^s$ subt by same chord **AC**) ✓R (6)

May not use parallel lines as reason unless lines proved parallel

7.2.2

(a) $\hat{B} = 180° - x$ ✓S (opp int $\angle^s$ of cyclic quad) ✓R (2)

(b) $\hat{O}_1 = 2x$ ✓S ($\angle$ at centre = 2 $\angle$ at circumf) ✓R
OR
(ext $\angle$ of Δ = sum int opp $\angle^s$) (2)
[11]

May also use central angle theorem with reflex angle AOC

QUESTION 8

8.1 Construction: Draw diameter AE and join EB. ✓M

Proof:

$\hat{A}_1 + \hat{A}_2 = 90°$ ✓S (tang ⊥ diam) ✓R

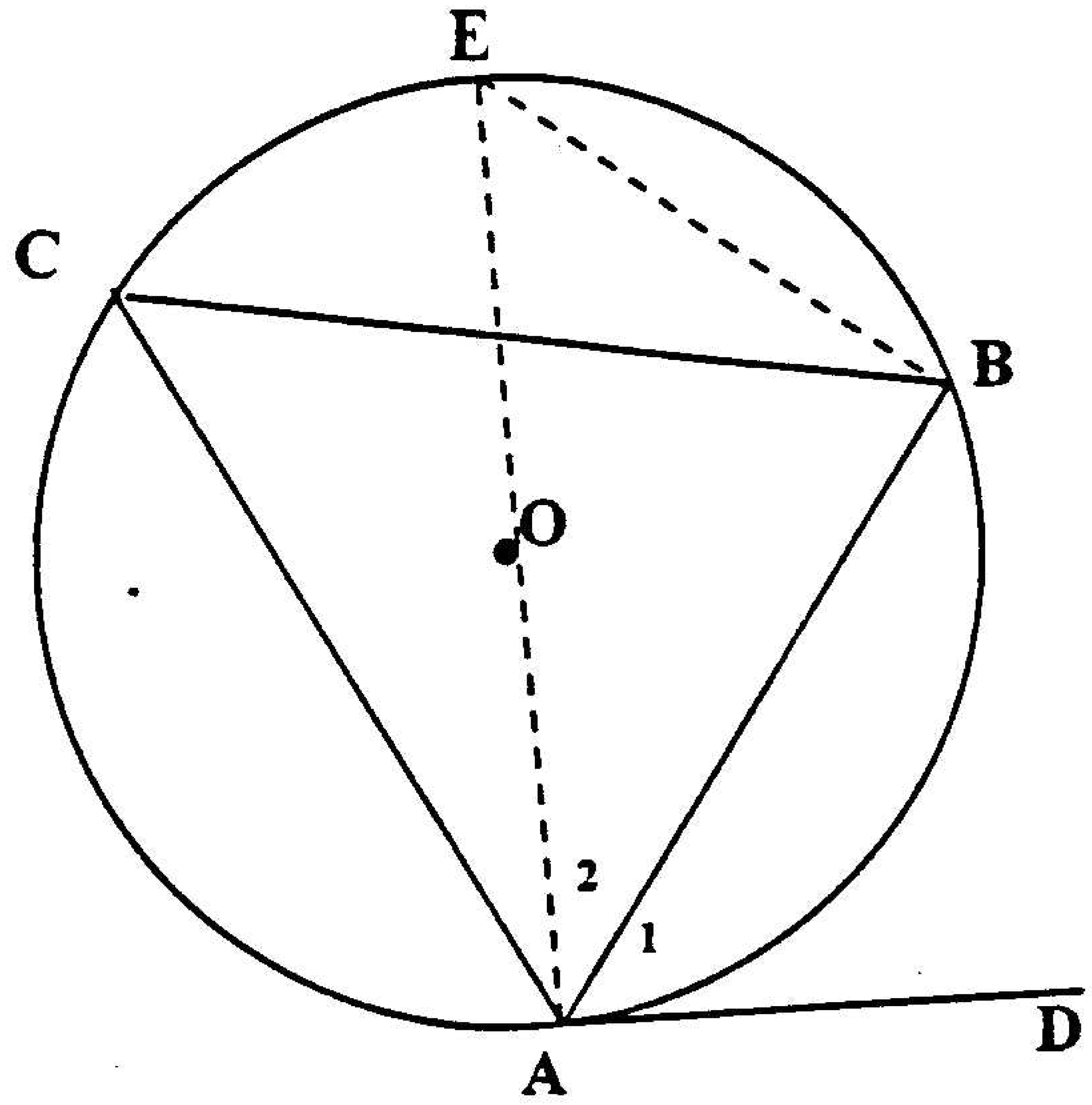
$\therefore \hat{A}_1 = 90° - \hat{A}_2$

$E\hat{B}A = 90°$ ✓S (∠ in semi circle) ✓R

$\therefore \hat{E} = 90° - \hat{A}_2$ (int ∠ˢ of ΔAEB supl) ✓S/R

$\therefore \hat{A}_1 = \hat{E}$

$= \hat{C}$ (subt by same chord AB) ✓S/R



OR ✓M

Construction: Draw diameter AE and chord EC

Proof:

$\hat{A}_1 + \hat{A}_2 = 90°$ ✓S (tang ⊥ diam) ✓R

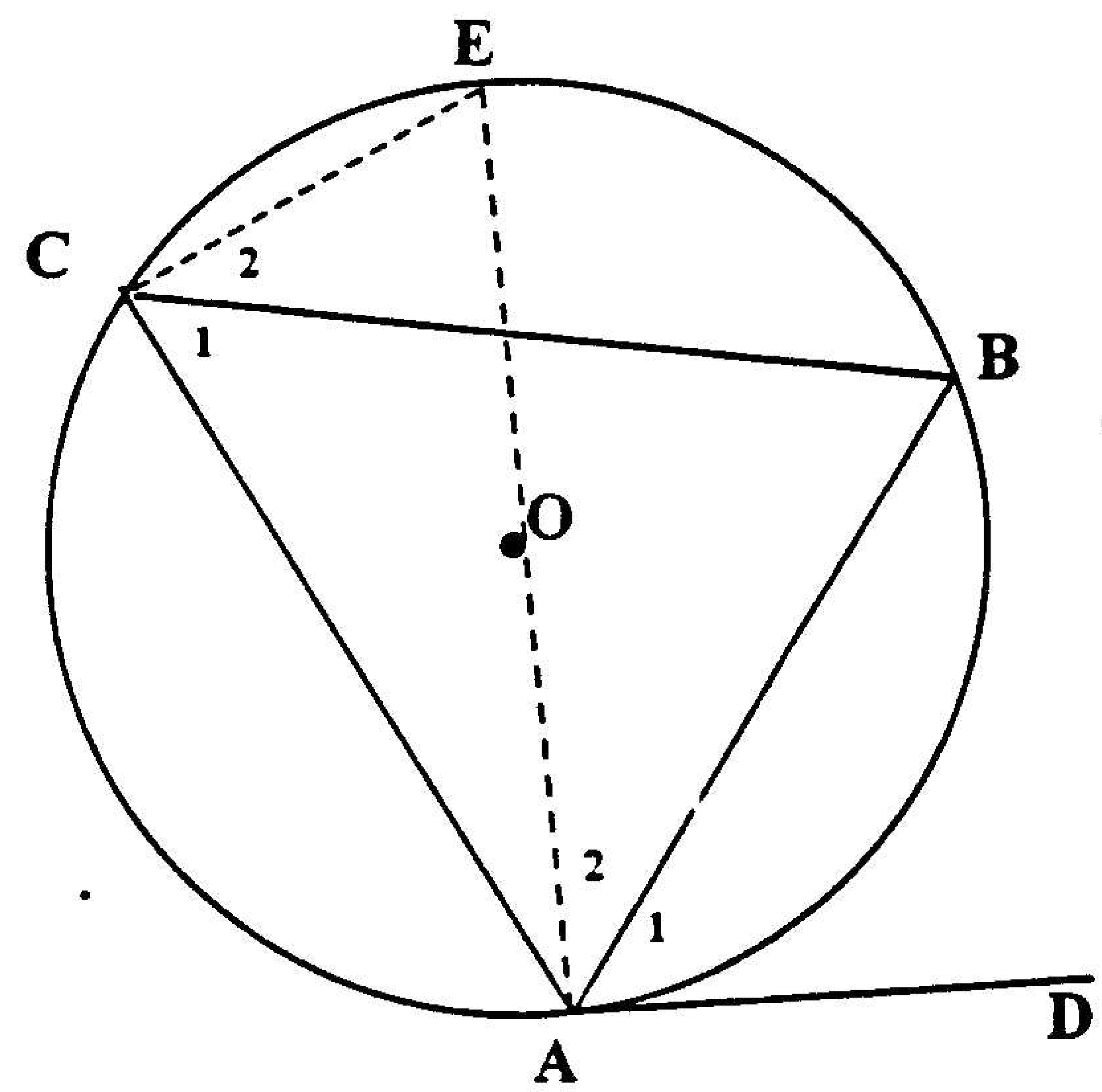
$\therefore \hat{A}_1 = 90° - \hat{A}_2$

$E\hat{C}A = 90°$ ✓S (∠ in semi circle) ✓R

$\therefore \hat{C}_1 = 90° - \hat{C}_2$ (adj comp angles) ✓S/R

$\therefore \hat{C}_2 = \hat{A}_2$ (subt by chord EB) ✓S/R

$\hat{A}_1 = \hat{C}_1$



OR

Construction: Draw radii OB and OA ✓M

Proof:

$\hat{A}_1 + \hat{A}_2 = 90°$ ✓S ✓S (tang ⊥ radius) ✓R

$\therefore \hat{A}_2 = 90° - \hat{A}_1$

$\hat{B}_1 = \hat{A}_2$ ✓S

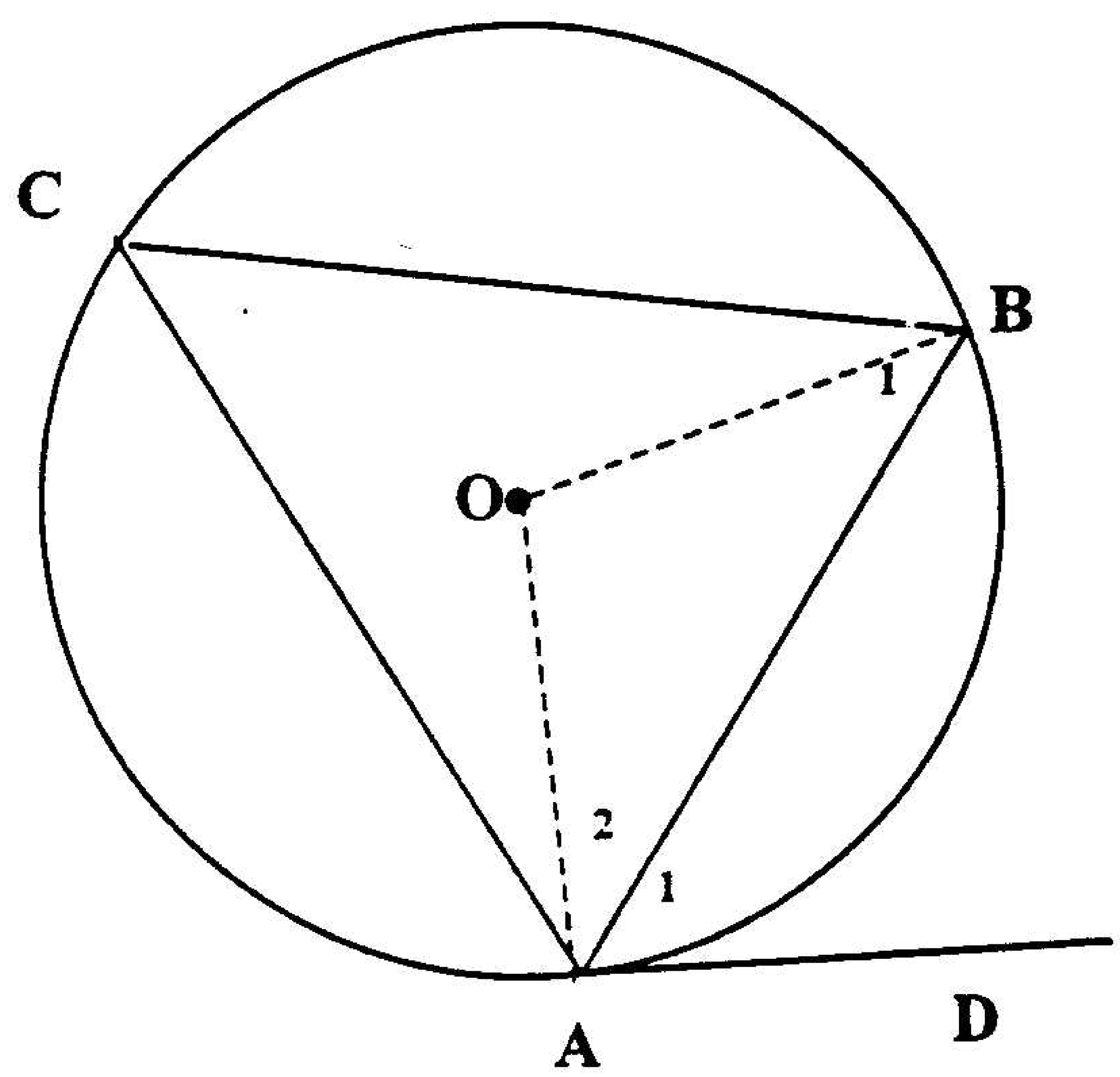
$\therefore A\hat{O}B = 2\hat{A}_1$ (sum of ∠s of Δ) ✓S/R

$\hat{C} = \frac{1}{2} A\hat{O}B$ ✓S (∠at centre) ✓R

$= \hat{A}_1$

(note that the order of statements in this alternative may not be as above. Candidates may start at ∠C)

(7)



If the construction does not pass through the centre of the circle it is a breakdown and a max of 1 out of 7 might be awarded if the angles in the same segment is given as a reason in the appropriate place.

8.2

8.2.1 $\hat{D}_1 = 90°$ ✓S (given)

$\hat{C}_1 = 90°$ ✓S (∠ in semi circle) ✓R

$\therefore$ EO || CA (corresp ∠ˢ =) ✓R (4)
OR
(co-int. ∠ˢ suppl.)

8.2.2 $\hat{C}_2 = \hat{A} = x$ ✓S (∠ betw tang and chord) ✓R

$= \hat{O}_1 = x$ ✓S (OE || AC, corresp ∠ˢ =) ✓R

(4)

8.2.3

$\therefore \hat{B}_1 = 90° - x$ ✓S (int ∠ˢ of Δ suppl) ✓R

$\therefore \hat{P} = \hat{B}_1 - \hat{C}_2$ (ext ∠ of Δ = sum opp int ∠ˢ) ✓S/R

$= 90° - x - x$

$= 90° - 2x$

OR

In ΔAPC: ✓S ✓S

$\hat{P} = 180° - (x + 90° + x)$ (int ∠ˢ of Δ suppl) ✓R

$= 90° - 2x$

OR

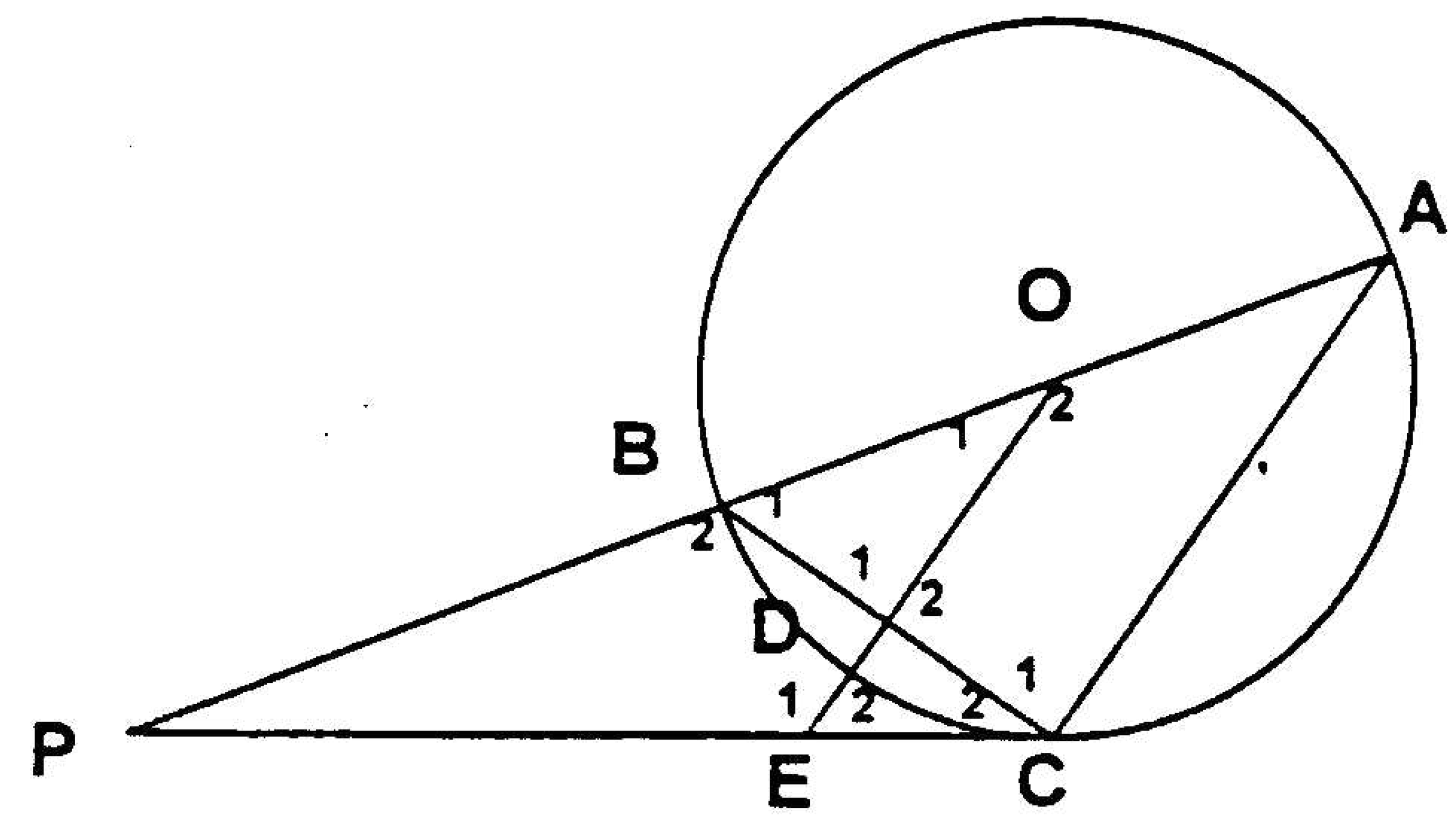
In Δ POE:

$\hat{P} = 180° - (\hat{O}_1 + \hat{E})$ ✓S

$= 180° - (x + 90° + x)$ ✓S (int ∠ˢ of Δ suppl) ✓R

$= 90° - 2x$

(3)
[18]



QUESTION 9

9.1 Construction: Draw heights k and h from F and E respectively and join EC and BF. ✓M

Proof:

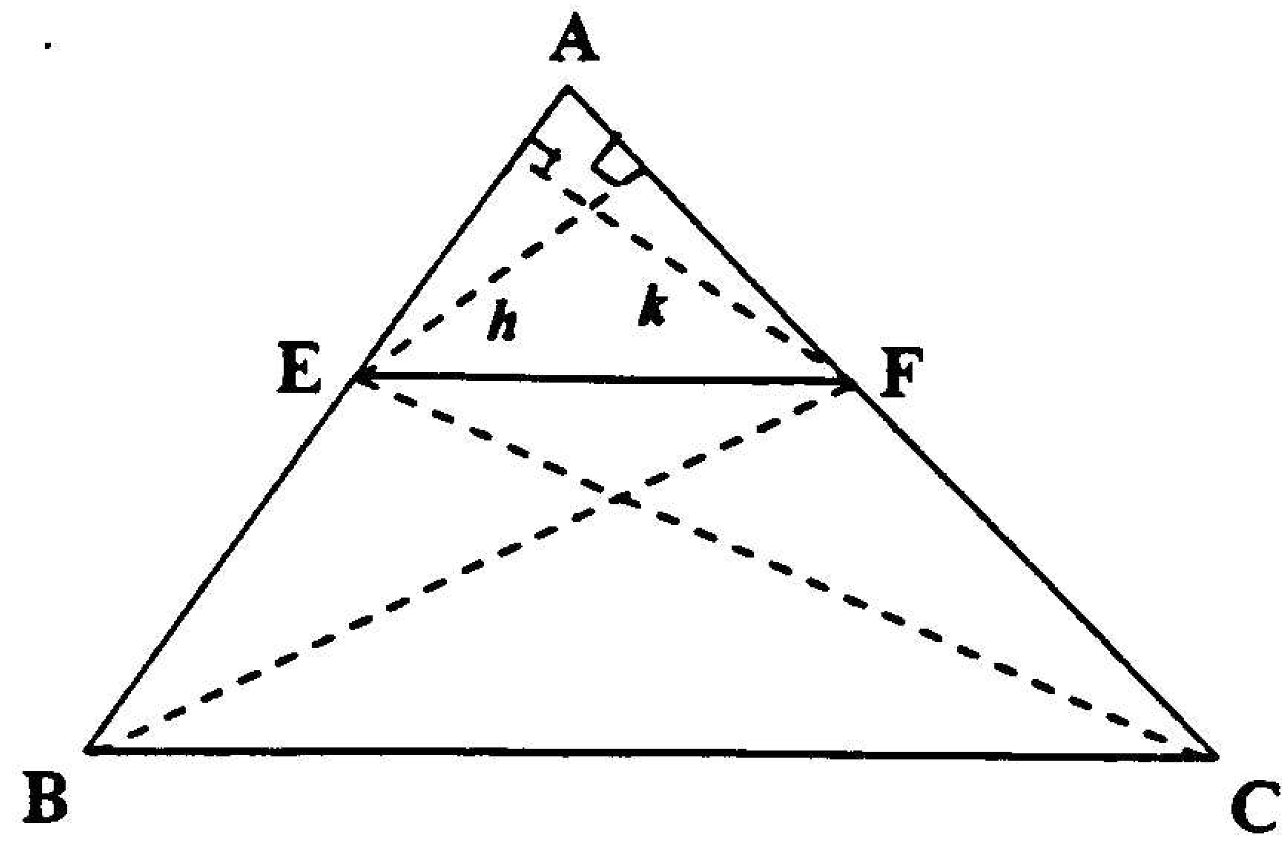
$$\frac{\text{Area of } \Delta AEF}{\text{Area of } \Delta BEF} = \frac{\frac{1}{2} AE.k}{\frac{1}{2} EB.k} = \frac{AE}{EB}$$ ✓M ✓S

$$\frac{\text{Area of } \Delta AEF}{\text{Area of } \Delta EFC} = \frac{\frac{1}{2} AF.h}{\frac{1}{2} FC.h} = \frac{AF}{FC}$$ ✓S

But Area of ΔBEF = Area of ΔEFC (Δ^s on same base and betw same || lines) ✓S/R

$$\therefore \frac{\text{Area of } \Delta AEF}{\text{Area of } \Delta BEF} = \frac{\text{Area of } \Delta AEF}{\text{Area of } \Delta EFC}$$ ✓S

$$\therefore \frac{AE}{EB} = \frac{AF}{FC}$$ (6)



Note: It is essential to differentiate between the different heights in step one and two.
In step one it is not essential to show the calculation based on the area formula, but then a reason is required e.g. same height or same vertex.
If candidates do not use the constructions h and k the ✓ **M** can be used for **Join EC and BF**

9.2.1 $\frac{QT}{TP} = \frac{QW}{WR}$ ✓S/R (line drawn || to one side of Δ) OR (TW // VR) OR (proportionality theorem)

$\frac{15}{x+2} = \frac{x+4}{x}$ ✓A

$15x = (x+2)(x+4)$

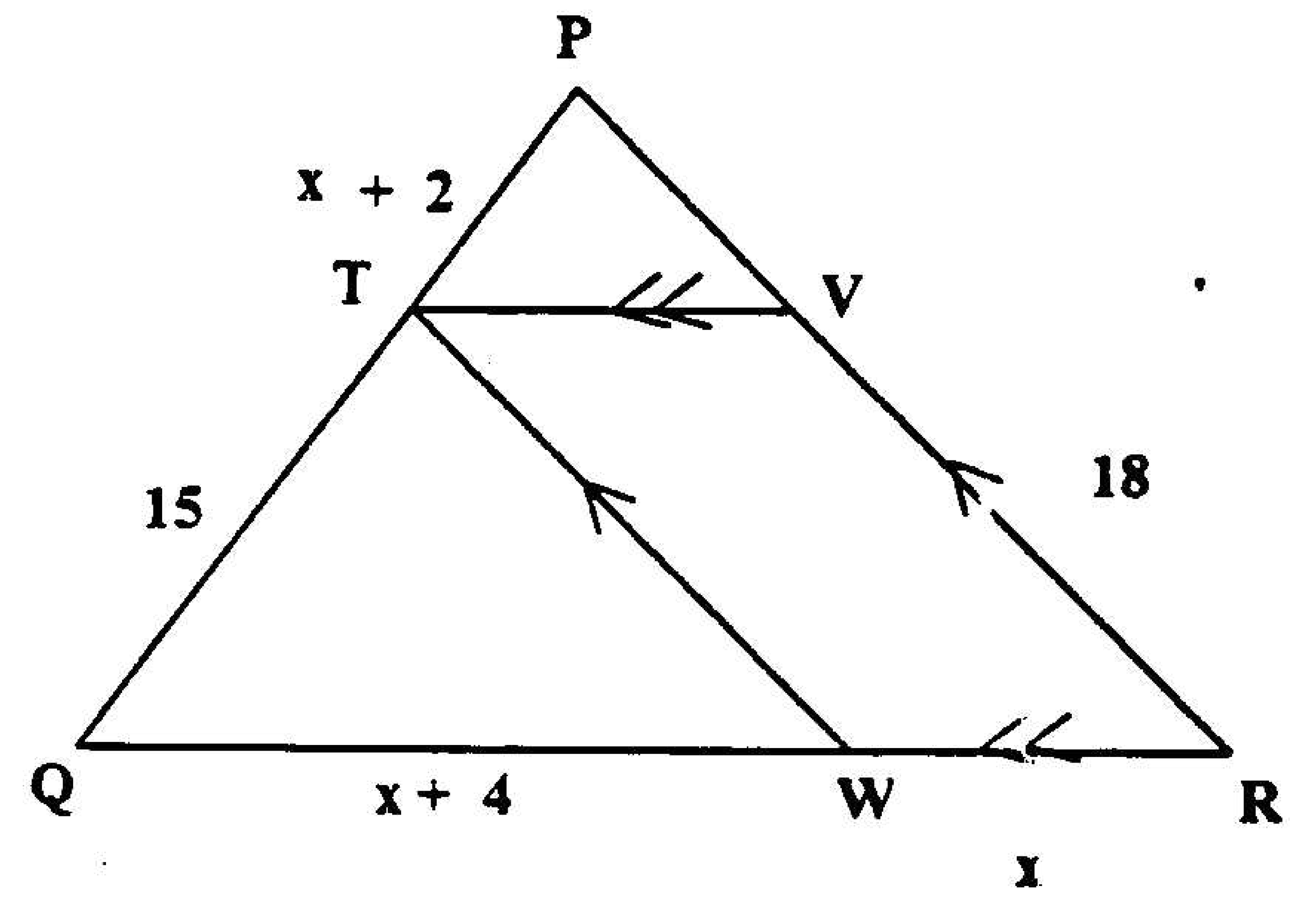
$= x^2 + 6x + 8$

$0 = x^2 + 6x - 15x + 8$

$= x^2 - 9x + 8$ ✓C/A

$= (x-8)(x-1)$ ✓C/A

$x = 8 \text{ or } x = 1$ ✓C/A (5)

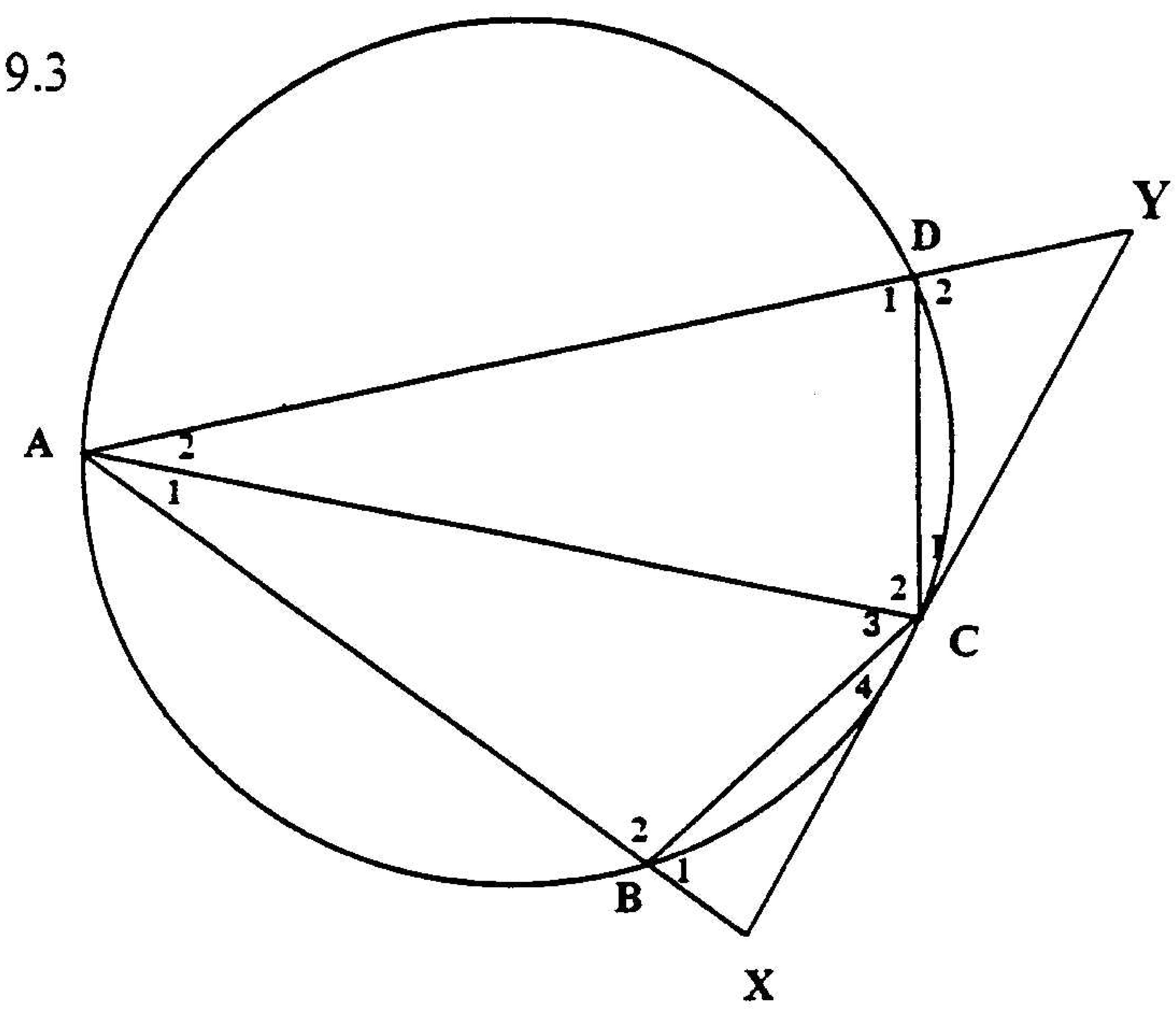


9.2.2 $\frac{PV}{VR} = \frac{PT}{TQ} = \frac{x+2}{15}$ ✓S/R (line drawn || to one side of Δ) OR TV//QR

$\frac{PV}{18} = \frac{8+2}{15} = \frac{10}{15} = \frac{2}{3}$ ✓CA

PV = 12 units ✓CA (3)

9.3



9.3.1 (i) $\hat{A}_1 = \hat{A}_2$ ✓S (given)

$= \hat{C}_1$ ($\angle$ betw tang and chord) ✓S/R

(ii) $\hat{B}_2 = \hat{D}_2$ (ext $\angle$ of cyclic quad = int opp $\angle$) ✓S/R

(iii) $\hat{C}_3 = \hat{Y}$ (int $\angle^s$ of Δ suppl)

$\therefore$ $\Delta ABC \,|||\, \Delta CDY$ (equiangular) ✓R (4)

9.3.2 $\frac{AB}{CD} = \frac{BC}{DY} = \frac{AC}{CY}$ ✓CAO (1)

[19]