## POSSIBLE ANSWERS FOR:

# MATHEMATICS HIGHER GRADE PAPER 2 NOVEMBER 2002

✓M = 1 mark for a certain method used

✓A = 1 mark for accuracy

✓CA = 1 mark for consistant accuracy

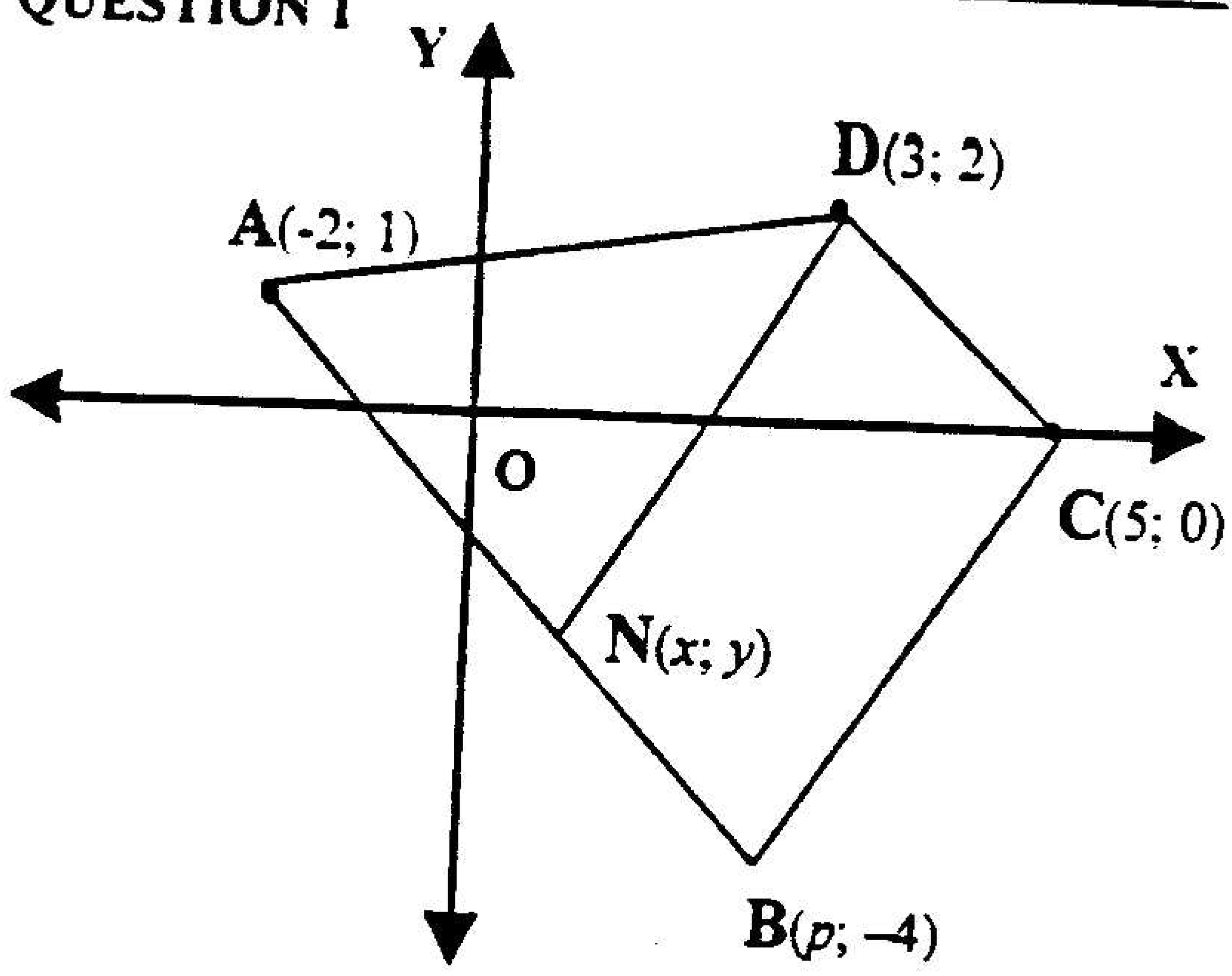
✓CAO = 1 mark for the correct answer only

✓S = 1 mark for the correct geometric statement

✓R = 1 mark for the correct reason given

✓S/R = 1 mark for the correct statement with the correct reason

**QUESTION 1**




| Answer | Marks | Comments |
|---|---|---|
| | | Assumption that $p = 3 \Rightarrow$ 0 marks |
| 1.1 $m_{AB} = m_{CD}$ ✓M | | equating gradients |
| $m_{AB} = \dfrac{-4-1}{p+2}$ | | |
| $= \dfrac{-5}{p+2}$ ✓A | | $m_{AB} = \dfrac{-5}{p+2}$ |
| $m_{CD} = \dfrac{2-0}{3-5}$ | | |
| $= -1$ ✓A | | $m_{CD} = -1$ |
| $\therefore \dfrac{-5}{p+2} = -1$ | | |
| $\therefore p+2 = 5$ ✓CA | | correct manipulation |
| $\therefore p = 3$ | | |
| **OR** | | |
| DC: $\dfrac{y-2}{x-3} = \dfrac{0-2}{5-3} = -1$ ✓A | | $m_{CD} = -1$ |
| ✓M | | equating gradients |
| $\therefore$ AB: $y = -x + c$ $(-2; 1)$ | | |
| $\therefore 1 = 2 + c$ | | |
| $\therefore c = -1$ | | |
| $\therefore y = -x - 1$ ✓A | | equation of AB |
| and B$(p; -4)$ | | |
| $\therefore -4 = -p - 1$ ✓CA | | correct manipulation |
| $\therefore p = 3$ | (4) | |
| 1.2 $AB^2 = (-2-3)^2 + (1+4)^2$ ✓M | | correct substitution into correct formula |
| $= 50$ | | |
| $AB = \sqrt{50}$ ✓CA | | correct manipulation |
| $= 5\sqrt{2} = 7{,}071$ | | decimal form also accepted |
| $CD^2 = (3-5)^2 + (2-0)^2$ ✓M | | correct substitution into correct formula |
| $= 8$ | | |
| $CD = \sqrt{8}$ ✓CA | | manipulation |
| $= 2\sqrt{2} = 2{,}83$ | | decimal form also accepted |
| $\dfrac{AB}{CD} = \dfrac{\sqrt{50}}{\sqrt{8}} = \dfrac{5\sqrt{2}}{2\sqrt{2}} = \dfrac{5}{2} = \dfrac{2{,}5}{1} = 2\tfrac{1}{2}$ ✓CA | | any of the last three forms acceptable; value will depend on cadidate's two calculated lengths |
| AB:CD $= 5:2$ | (5) | |

| | |
|---|---|
| 1.3 $\Delta x_{NB} = \Delta x_{DC}$ | |
| $x_N - 3 = 3 - 5$ ✓M | Correct substitution |
| $x_N = -2+3$ | |
| $x_N = 1$ ✓CA | $x = 1$ only $\Rightarrow$ 2 marks |
| $y_N + 4 = 2 - 0$ ✓M | correct substitution |
| $y_N = 2 - 4$ | |
| $y_N = -2$ ✓CA | $y = -2$ only $\Rightarrow$ 2 marks |
| **OR** | |
| Midpoint of BD $(3; -1)$ ✓A | Finding midpt of BD : follow on if incorrect |
| $\therefore$ midpoint of NC $(3; -1)$ ✓M | Diags of parm bisect e.o. |
| $\therefore \frac{x+5}{2} = 3$ | |
| $\therefore x = 1$ ✓CA | Finding $x$ |
| and $\frac{y+0}{2} = -1$ | |
| $\therefore y = -2$ ✓CA | Finding $y$ |
| $\therefore$ N$(1; -2)$ | |
| **OR** | |
| $m_{BN} = m_{CD}$ | |
| $\therefore \frac{y+4}{x-3} = \frac{2-0}{3-5} = -1$ | |
| $\therefore y + 4 = -x + 3$ | |
| $\therefore y = -x - 1$ ✓A | Correct equation |
| and $m_{DN} = m_{CB}$ | |
| $\frac{y-2}{x-3} = \frac{0+4}{5-3} = 2$ | |
| $\therefore y - 2 = 2x - 6$ | |
| $\therefore y = 2x - 4$ ✓A | Correct equation |
| $\therefore 2x - 4 = -x - 1$ | |
| $\therefore 3x = 3$ | |
| $\therefore x = 1$ ✓CA | Calc. the value of $x$ |
| $\therefore y = -2$ ✓CA | Calc. the value of $y$ |
| $\therefore$ N$(1; -2)$ | |
| **OR** | |
| $BN^2 = CD^2$ | |
| $(x-3)^2 + (y+4)^2 = (5-3)^2 + (0-2)^2$ | |
| $x^2 - 6x + 9 + y^2 + 8y + 16 = 4 + 4$ | |
| $x^2 - 6x + y^2 + 8y + 17 = 0$ ... ① ✓A | Correct equation |
| $DN^2 = CB^2$ | |
| $(x-3)^2 + (y-2)^2 = (5-3)^2 + (0+4)^2$ | |
| $x^2 - 6x + 9 + y^2 - 4y + 4 = 4 + 16$ | |
| $x^2 - 6x + y^2 - 4y - 7 = 0$ ... ② ✓A | Correct equation |
| $12y + 24 = 0$ ... ① − ② | |
| $12y = -24$ | |
| $y = -2$ ✓CA | Correct calculation of the value of $y$ |
| $x = 1$ ✓CA | Correct calculation of the value of $x$ |
| N$(1; -2)$ | |
| **OR** | |
| **OR** | |

| | | |
|---|---|---|
| $m_{BN} = m_{CD}$<br>$\therefore \quad \dfrac{y+4}{x-3} = \dfrac{2-0}{3-5} = -1$<br>$\therefore \quad y + 4 = -x + 3$<br>$\therefore \quad y = -x - 1 \quad \dots \text{①}$ ✓A | | Correct equation |
| and<br>$BN^2 = CD^2$<br>$(x-3)^2 + (y+4)^2 = (5-3)^2 + (0-2)^2$<br>$x^2 - 6x + 9 + y^2 + 8y + 16 = 4 + 4$<br>$x^2 - 6x + y^2 + 8y + 17 = 0 \quad \dots \text{②}$ ✓A | | Correct equation |
| $x^2 - 6x + (-x-1)^2 + 8(-x-1) + 17 = 0$<br>$x^2 - 6x + x^2 + 2x + 1 - 8x - 8 + 17 = 0$<br>$2x^2 - 12x + 10 = 0$<br>$x^2 - 6x + 5 = 0$<br>$(x-1)(x-5) = 0$<br>$x = 1 \text{ or } x = 5$ ✓CA<br>$y = -2 \text{ or } y = -6$ | | Correct calculation of the values of $x$ and $y$ |
| $N(1;-2)$ ✓CA | | Chosing correct set for N |
| **OR**<br>The previous method with<br>$m_{DN} = m_{CB} \Rightarrow y = 2x - 4 \quad \dots \text{①}$<br>and $BN^2 = CD^2 \Rightarrow x^2 - 6x + y^2 + 8y + 17 = 0 \quad \dots \text{②}$<br>**or**<br>$m_{BN} = m_{CD} \Rightarrow y = -x - 1 \quad \dots \text{①}$<br>and $DN^2 = CB^2 \Rightarrow x^2 - 6x + y^2 - 4y - 7 = 0 \quad \dots \text{②}$<br>**or**<br>$m_{DN} = m_{CB} \Rightarrow y = 2x - 4 \quad \dots \text{①}$<br>$DN^2 = CB^2 \Rightarrow x^2 - 6x + y^2 - 4y - 7 = 0 \quad \dots \text{②}$ | | |
| **OR**<br>$AN = AB - CD$<br>$= \sqrt{50} - \sqrt{8}$<br>$= 5\sqrt{2} - 2\sqrt{2}$<br>$= 3\sqrt{2}$ ✓A | | Correct culculation of AN |
| $\therefore \quad AN:NB = 3\sqrt{2} : 2\sqrt{2} = 3:2$ ✓M | | Finding the ration AN:NB correct |
| $\therefore \quad x = \dfrac{mx_B + nx_A}{m+n}$<br>$= \dfrac{3 \times 3 + 2 \times (-2)}{3+2}$<br>$= 1$ ✓CA | | Not in the curriculum, but accepted<br>Correct calculation of the value of $x$ |
| and $\quad y = \dfrac{my_B + ny_A}{m+n}$<br>$= \dfrac{3 \times (-4) + 2 \times 1}{3+2}$<br>$= -2$ ✓CA<br>$\therefore \quad N(1;-2)$ | (4) | Correct calculation of the value of $y$ |
| 1.4 $\quad x = 3$ ✓CAO | (1) | Must not get $x = 3$ from cross multiplication by 0 |

| Solution | Marks | Comments |
|---|---|---|
| 1.5 $90°$ ✓CA | (1) | Can use angle of inclination formula<br>Answer will depend on answer of 4.1<br>Accept angle of inclination only in [0; 180] |
| 1.6 area $\|\|^{m}$ NBCD = 2 area $\Delta$ DBC ✓M<br>✓A ✓A<br>$= 2(\frac{1}{2})(6)(5-3)$<br>= 12 square units ✓A | | Diag of parm bisect area of parm<br>Correct length of base<br>Correct value for perp height<br>Correct manipulation |
| **OR** | | |
| $DC = DN = 2\sqrt{2}$<br>Altitude from D to AB $= 3\sqrt{2}$ ✓A<br>Area NBCD = 2 area $\Delta$DCB ✓M<br>$= 2(\frac{1}{2}DC.h)$<br>$= 2\sqrt{2} \times 3\sqrt{2}$ ✓CA<br>= 12 sq units ✓CA | | Correct value for perpendicular height<br>Diag of parm bisect area of parm<br>Correct substitution<br>Correct manipulation |
| **OR** | | |
| $\tan \alpha = DCm = -1$<br>$\alpha = 135°$<br>$\tan \beta = BCm = 2$<br>$\beta = 63{,}4°$<br>$D\hat{C}B = 108.4°$ ✓A | | Finding the value of $D\hat{C}B$ correctly |
| $BC = \sqrt{(0+4)^2 + (5-3)^2}$<br>$= \sqrt{20}$<br>$= 2\sqrt{5}$ ✓A | | Calculating the length of BC correctly |
| Area NBCD = 2 area $\Delta$DCB<br>$= 2(\frac{1}{2}DC.BC.\sin D\hat{C}B)$<br>$= 2\sqrt{2} \times 2\sqrt{5} \times \sin 108{,}4°$ ✓M<br>$\approx$ 12 sq units ✓CA | (4) | Using area rule incorrect or incorrect formula ⇒ no further marks<br>Correct use of and substitution into area rule<br>Correct calculation(simplification) |
| 1.7 R(-1;q) A(-2;1) C(5;0) | | |
| $m_{AR} = \frac{q-1}{-1+2}$<br>$= q-1$ ✓A | | Correct calculation of gradient of AR |
| $m_{AC} = \frac{1-0}{-2-5}$<br>$= \frac{1}{-7}$ ✓A | | Correct calculation of gradient of AC |
| $m_{AR} = m_{AC}$<br>$q - 1 = \frac{1}{-7}$ ✓M | | Equating gradients |
| $q = \frac{6}{7}$ ✓CA | | Correct calculation(simplification) |
| **OR** | | |
| AC: $\frac{y-0}{x-5} = \frac{0-1}{5+2} = -\frac{1}{7}$ ✓A | | Correct calculation of gradient of AC |
| $y = -\frac{1}{7}x + \frac{5}{7}$ ✓CA R(−1; $q$)<br>$q = \frac{1}{7} + \frac{5}{7}$ ✓M<br>$= \frac{6}{7}$ ✓CA | (4)<br>[23] | Simplified form of equation of AC<br>Substitute R(−1; $q$) correctly into equation of AC<br>Correct calculation(simplification) |

**QUESTION 2**

2.1.1 subst. (1; –3) in the LHS

LHS $= (1)^2 + 4(1) + (-3)^2 + 2(-3) - 8$ ✓ M

$= 0$ ✓ A

$=$ RHS

N(1; –3) lies on the circle.

Correct substitution into correct equation
Correct calculation
Accept if 0 of RHS is kept

**OR**

Let $x = 1$ in $x^2 + 4x + y^2 + 2y - 8 = 0$

$1 + 4 + y^2 + 2y - 8 = 0$ ✓ M

$y^2 + 2y - 3 = 0$

$(y + 3)(y - 1) = 0$

$y = -3$ or $y = 1$ ✓ A

N(1; –3)

Correct substitution into correct equation

Correct calculation of $y$ values

**OR**

Let $y = -3$ in $x^2 + 4x + y^2 + 2y - 8 = 0$

$x^2 + 4x + (-3)^2 + 2(-3) - 8 = 0$ ✓ M

$x^2 + 4x - 5 = 0$

$(x + 5)(x - 1) = 0$

$x = -5$ or $x = 1$ ✓ A

N(1; –3) is on circle (2)

Correct substitution into correct equation

Correct calculation of $x$ values

2.1.2 $x^2 + 4x + 4 + y^2 + 2y + 1 = 8 + 4 + 1$ ✓ M ✓ A

$(x + 2)^2 + (y+1)^2 = 13$ ✓ CA

centre M(–2; –1) ✓ CA

$m_{MN} = \dfrac{-3 + 1}{1 + 2} = \dfrac{-2}{3}$ ✓ CA

$m_{tan} = \dfrac{3}{2}$, N(1; –3) ✓ M

$y + 3 = \frac{3}{2}(x - 1)$ **or** $y = \frac{3}{2}x + c$ ✓ M

$2y + 6 = 3x - 3$     $-3 = \frac{3}{2} + c$

$c = -\frac{9}{2}$

$3x - 2y - 9 = 0$ **or** $y = \frac{3}{2}x - \frac{9}{2}$ **or** $3x - 2y = 9$ ✓ CA (8)

Completion of the squares on LHS / Adding same on RHS
Writing into centre/radius form ⇒ even if RHS ≠ 13

Finding the centre of the circle

Correct calculation of gradient of MN

Determining gradient of tangent correctly

Substituting N(1;–3) into equation of tangent

Correct simplification into any form of straight line equation

2.1.3 $\tan\theta = \dfrac{3}{2}$ ✓ M

$\theta = 56{,}3°$ ✓ CA (2)

Correct use of inclination rule depending on grad of 2.1.2
Correct calc of angle of incl ; must be $0° \le \theta \le 180°$
Penalise for incorrect rounding off and never again in paper

2.1.4 $3x - 2y = 9$

$y = 0$ ✓ M

$3x = 9$

$x = 3$ ✓ CA

(3; 0) (2)

Substituting $y = 0$
Calculating $x$
Only $(x = 3) \Rightarrow$ full marks; Any other form $(a; 0) \Rightarrow$ 1 mark

2.1.5 $x = 0$ ✓ M

$y^2 + 2y - 8 = 0$ ✓ A

$(y + 4)(y - 2) = 0$ ✓ CA

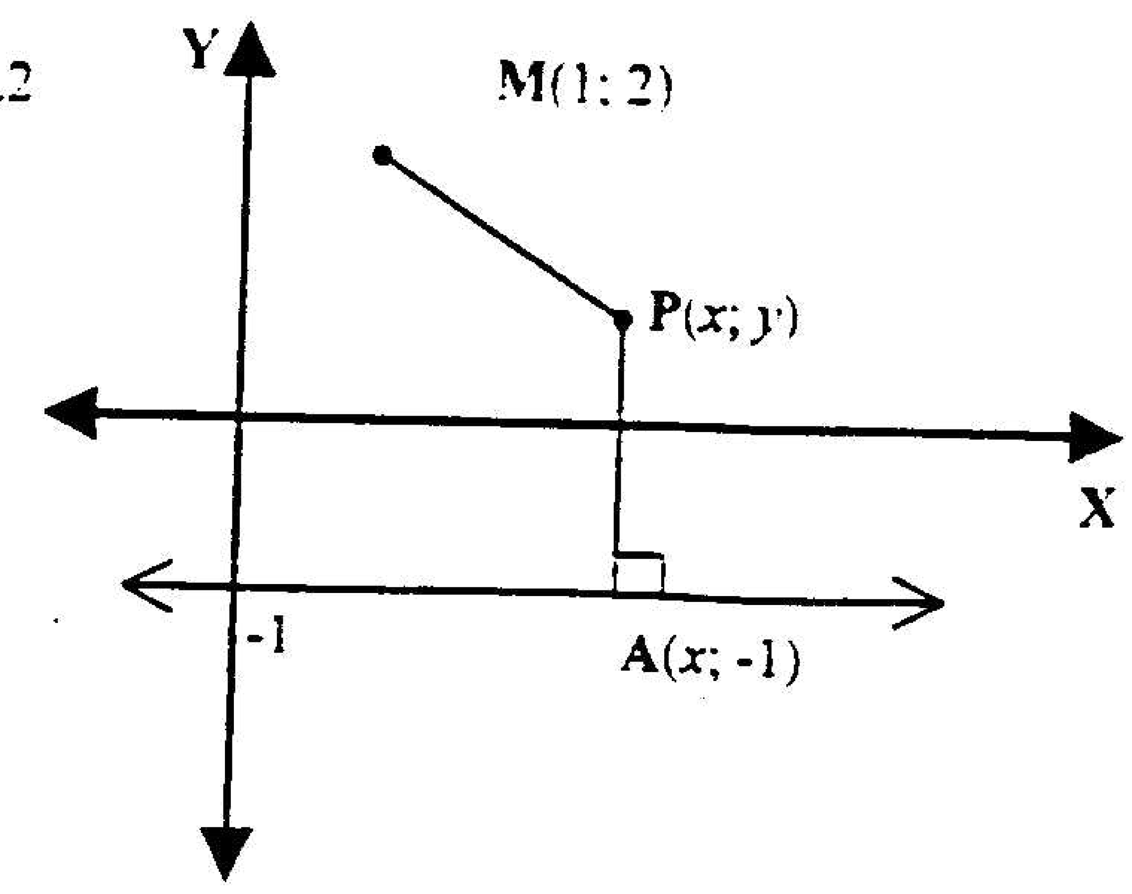
✓ CA ✓ CA

$y = -4$ or $y = 2$

(0; –4); (0; 2) (5)

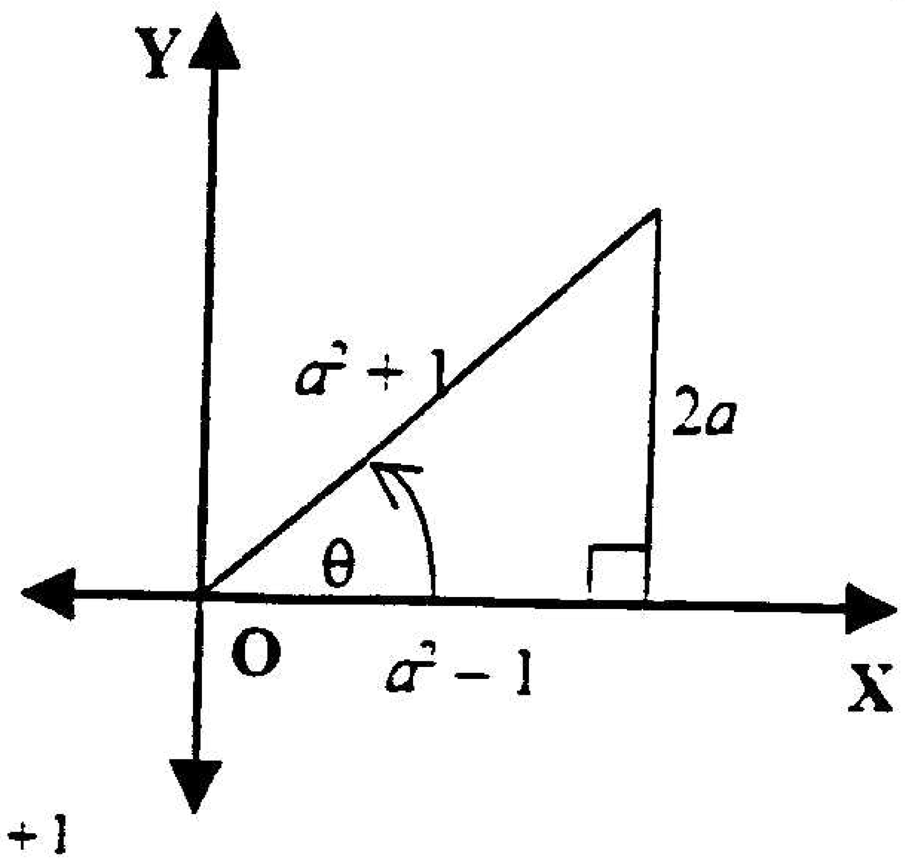
Substituting $x = 0$
Correct simplification
Correct factorising or using of formula from quadr equa only
Determining the values of $y$
Any answer of form $(0; a) \Rightarrow$ 1 mark only

2.2




| | |
|---|---|
| 2.2 $PA^2 = PM^2$ ✓M | Equating PA and PM |
| $(x-x)^2 + (y+1)^2 = (x-1)^2 + (y-2)^2$ ✓A ✓A | Correct subst into correct formulas : $(x-x)^2$ not necessary. If $(x-0)^2$ on LHS ⇒ loses 2 marks if all else is correct ⇒ max 4 marks |
| $y^2 + 2y + 1 = x^2 - 2x + 1 + y^2 - 4y + 4$ ✓CA ✓CA | Correct squaring on both sides |
| $6y = x^2 - 2x + 4$ ✓CA | Correct simplification |
| **OR** | |
| Turning point $(1; \frac{1}{2})$ ✓A ✓A | Correct coordinates of turning point of parabola |
| $y = a(x-1)^2 + \frac{1}{2}$ ✓M ✓CA | Turning point form of quadratic function. Correct substitution of coordinates of turning point |
| $= a(x^2 - 2x + 1) + \frac{1}{2}$ ✓CA | Correct squaring |
| $y = ax^2 - 2ax + a + \frac{1}{2}$ ✓CA (6) | Correct simplification |
| [25] | |

**QUESTION 3**




3.1 $\operatorname{cosec}\theta = \dfrac{a^2+1}{2a}$

$y = 2a;\ r = a^2 + 1$ ✓A

Correctly determining $y$ and $r$

$x^2 = (a^2+1)^2 - 4a^2$ ✓M Pyth

Correct use of theorem of Pyth

$= a^4 + 2a^2 + 1 - 4a^2$

$= a^4 - 2a^2 + 1$ ✓CA

Correct simplification

$= (a^2 - 1)^2$

$x = a^2 - 1$ ✓CA

Determining $x$ or $y$ correctly depending on first mark
If all this is shown on diagram without calc ⇒ 4 marks

✓CA ✓CA

$\sec\theta + \tan\theta = \dfrac{a^2+1}{a^2-1} + \dfrac{2a}{a^2-1}$ **or** $\sec\theta + \tan\theta = \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

Correct ratios for $\sec\theta$ and $\tan\theta$ **or** correct ratios for $\sin\theta$ and $\cos\theta$

$= \dfrac{a^2+2a+1}{a^2-1}$

$= \dfrac{(a+1)^2}{(a+1)(a-1)}$ ✓CA

$= \dfrac{a+1}{a-1}$

**or**

$= \dfrac{1+\sin\theta}{\cos\theta}$

$= \dfrac{1+\dfrac{2a}{a^2+1}}{\dfrac{a^2-1}{a^2+1}}$ ✓CA ✓CA

Correct manipulation

$= \dfrac{a^2+2a+1}{a^2+1} \times \dfrac{a^2+1}{a^2-1}$

$= \dfrac{(a+1)^2}{(a+1)(a-1)}$ ✓CA

$= \dfrac{a+1}{a-1}$ (7)

3.2 $\text{LHS} = \dfrac{1}{\sqrt{2}}\sec(-45°) + \cos 210° - \cot 840°$

✓A ✓A ✓A

$= \dfrac{1}{\sqrt{2}}\sec 45° - \cos 30° + \cot 60°$

Reduction formula applied correctly, including the signs

✓CA ✓CA ✓CA

$= \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{2}}{1} - \dfrac{\sqrt{3}}{2} + \dfrac{1}{\sqrt{3}}$

Correct ratios

$= 1 - \dfrac{\sqrt{3}}{2} + \dfrac{1}{\sqrt{3}}$

$= \dfrac{2\sqrt{3} - 3 + 2}{2\sqrt{3}}$ ✓M

Absolute correct manipulation

$= \dfrac{2\sqrt{3}-1}{2\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$ ✓A

Rationalising denominater

$= \dfrac{6-\sqrt{3}}{6}$

**OR**

| Solution | Marking guidelines |
|---|---|
| **OR** | |
| $\frac{1}{\sqrt{2}}\sec(-45°)+\cos 210° - \cot 840° = 0.7113248$ ✓A ✓A ✓A ✓A | Correct calculation of value of LHS |
| $\frac{6-\sqrt{3}}{6} = 0.7113248$ ✓A ✓A ✓A ✓A | Correct calculation of value of RHS<br>Ignore rounding off, but both values must be exactly the same, otherwise max 4 marks |
| $\frac{1}{\sqrt{2}}\sec(-45°)+\cos 210° - \cot 840° = \frac{6-\sqrt{3}}{6}$ (8) | |
| 3.3 $\sin 163° \sec 73° - \frac{\cot(x-90°)\tan(90°-x)}{\cot^2(x-360°)}$ | |
| $= \cos 73° \sec 73° + \frac{\tan x}{\cot^2 x} \times \frac{\cot x}{1}$ ✓A ✓A ✓A ✓A | Correct reduction formula : sin 163° = sin 17° and sec 73° = cosec 17° also correct<br>Using the calculator to calculate sin 163°sec 73° = 1 also correct for 1 mark |
| $= 1 + \frac{1}{\cot^2 x}$ ✓A ✓CA | Product of inverses = 1 |
| $= 1 + \tan^2 x$ ✓CA | Correct identity |
| $= \sec^2 x = \frac{1}{\cos^2 x}$ ✓A (8) | Correct identity; any of the last two acceptable |
| [23] | |

## QUESTION 4

**4.1**

$\sin x = \cos 2x - 1$

$\sin x = 1 - 2\sin^2 x - 1$ ✓ M

$2\sin^2 x + \sin x = 0$

$\sin x(2\sin x + 1) = 0$ ✓ M

$\sin x = 0$ ✓ CA or $\sin x = -\frac{1}{2}$ ✓ CA

$x = 0°; 180°$ ✓ CA $\quad x = -30°; 210°$ ✓ CA (6)

Correct formula ⇒ $\cos 2x = 1 - 2\sin^2 x$ or any other correct expansion

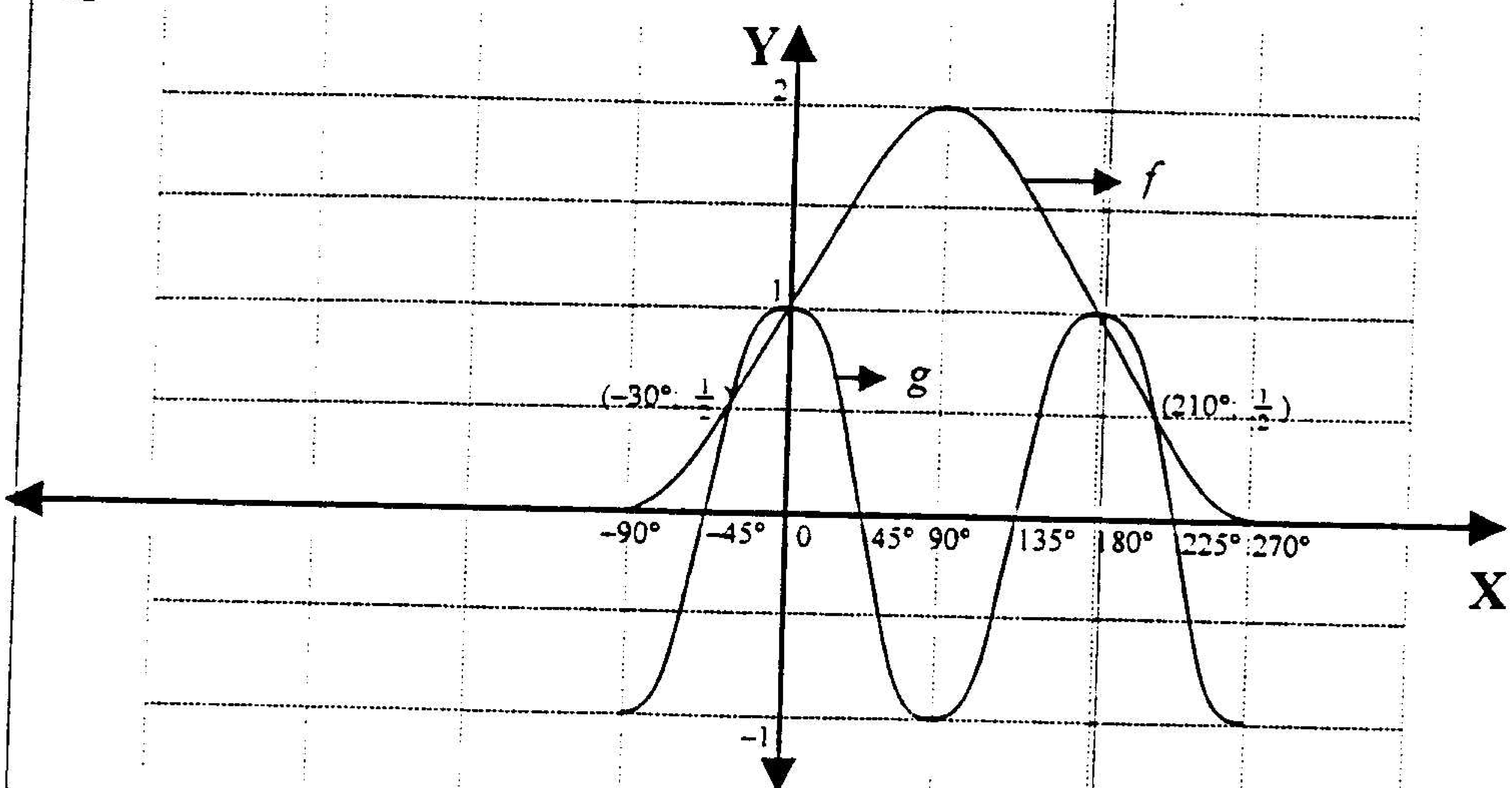
Correct factorisation or use of formula

Solving $\sin x$

Solving $x$ ⇒ 1 mark for any 2 correct angles

Dividing by $\sin x$ somewhere along the line ⇒ max of 3 marks

**4.2**




$f$: $x$-interc ✓ A

$y$-interc ✓ A

turnpts ✓ A

$g$: $x$-inters ✓ A

$y$-inters ✓ A

turnpts ✓ A

$(-30°; \frac{1}{2})$ ✓ CA; $(210°; \frac{1}{2})$ ✓ CA (8)

Loses 1 mark if graph is drawn in incorrect interval

**4.3.1** $y \in [0;2]$ ✓ A or $\{y : 0 \le y \le 2\}$ ✓ A (2)

Correct interval

Correct format

$0 \le x \le 2$ ⇒ 1 mark

**4.3.2.a** $x \in [-30°; 0°]$ ✓ CA or $x \in [180°; 210°]$ ✓ CA ✓ A (3)

Correct intervals : $x = 0°$ or $x = 180°$ ⇒ max of 1 mark

Correct format

**4.3.2.b** $x \in [-90°; -45°]$ ✓ CA or $x \in [45°; 135°]$ ✓ CA or $x \in [225°; 270°]$ ✓ CA (3)

Correct intervals : only mark first first three intervals given and give 1 mark for each correct interval according to candidates graphs

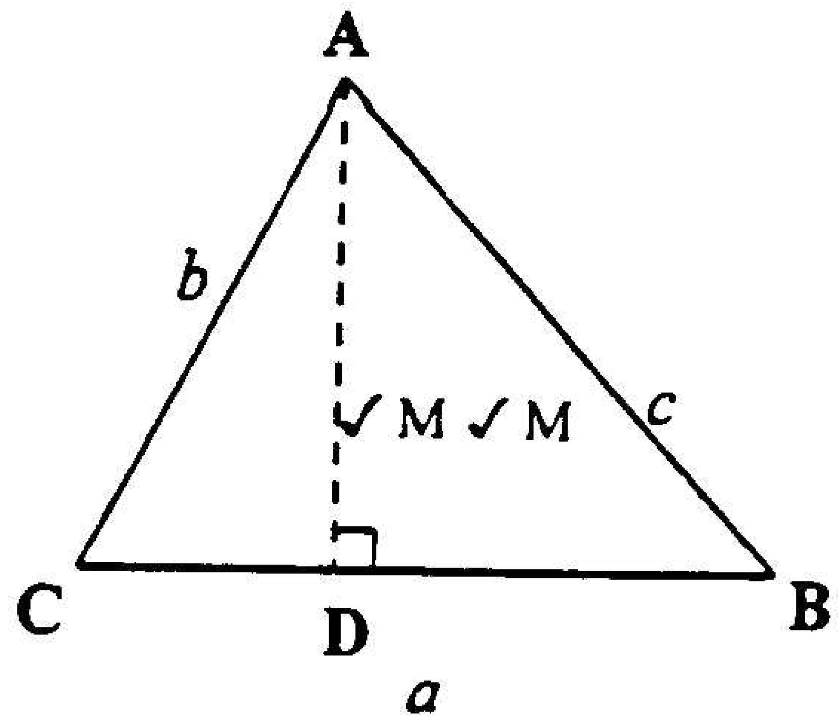
[22]

**QUESTION 5**

| | | | |
|---|---|---|---|
| 5.1.1 | $\cos(A+B) = \cos A\cos B - \sin A\sin B$ ✓ CAO | (1) | |
| 5.1.2 | $\cos(A+A) = \cos A\cos A - \sin A\sin A$ ✓ M<br>$= \cos^2 A - \sin^2 A$<br>$= (1-\sin^2 A) - \sin^2 A$ ✓ M<br>$= 1 - 2\sin^2 A$ ✓ A | (3) | Correct use of cos rule for compound angles<br>$\cos^2 A = 1 - \sin^2 A$: no penalty if not written down<br>Simplification<br>Answ only $(\cos 2A = 1 - 2\sin^2 A) \Rightarrow$ max of 1 mark |
| 5.2.1 | LHS $= \dfrac{\sec x \sin 2x}{(1-\cos 2x)(1+\cot^2 x)}$<br>$= \dfrac{\sec x \times 2\sin x\cos x}{(1-1+2\sin^2 x)(\text{cosec}^2 x)}$ ✓ A ✓ A<br>$= \dfrac{2\sin x}{2\sin^2 x \times \text{cosec}^2 x}$ ✓ A ✓ A<br>✓ CA<br>$= \sin x =$ RHS | (5) | Correct formula for $\sin 2x$<br>Correct formula for $1 + \cot^2 x = \text{cosec}^2 x$<br>$\sec x \times \cos x = 1$; need not write the 1 down<br>Manipulating $1 - \cos 2x$ to $2\sin^2 x$<br>$\sin^2 x \times \text{cosec}^2 x = 1$<br>In case of a break down, max 3 marks |
| 5.2.2 | Undefined where $\cos x = 0$<br>$x = 90°$<br>or where $\cos 2x = 1$<br>$2x = 0°$ or $360°$<br>$x = 0°$ or $180°$<br>or where $\tan x = 0$<br>$x = 0°$ or $180°$<br>✓ CAO ✓ CAO<br>$\therefore x \in \{0°; 90°; 180°\}$<br>✓ CAO | (3) | Mark the first three angles in the final answer |
| 5.3 | $\cos(x - 60°) = 3\cos x$<br>$\cos x\cos 60° + \sin x \sin 60° = 3\cos x$ ✓ M<br>✓ A ✓ A<br>$\dfrac{\cos x}{2} + \dfrac{\sqrt{3}\sin x}{2} - \dfrac{3\cos x}{1} = 0$<br>$\cos x + \sqrt{3}\sin x - 6\cos x = 0$<br>$\sqrt{3}\sin x - 5\cos x = 0$ ✓ CA<br>$\tan x = \dfrac{5}{\sqrt{3}}$ ✓ CA<br>✓ CA<br>$x = 70{,}9° + k.180°;\ k \in Z$ ✓ A<br>**OR**<br>$x = 70.9° + k.360°$<br>or $x = 250{,}9° + k.360°;\ k \in Z$ ✓ A<br>✓ CA | (7)<br><br>**[19]** | Correct formula for $\cos(x - 60°) = \cos x\cos 60° + \sin x \sin 60°$<br>Value of $\cos 60°$<br>Value of $\sin 60°$<br>Correct simplification<br>Correct manipulation<br>Correct angle<br>$k.180°$ and $k \in Z$<br>Both $70{,}9°$ and $250{,}9°$ for 1 mark<br>Both $k.360°$ and $k \in Z$ for 1 mark |

**QUESTION 6**

6.1.1




**OR**

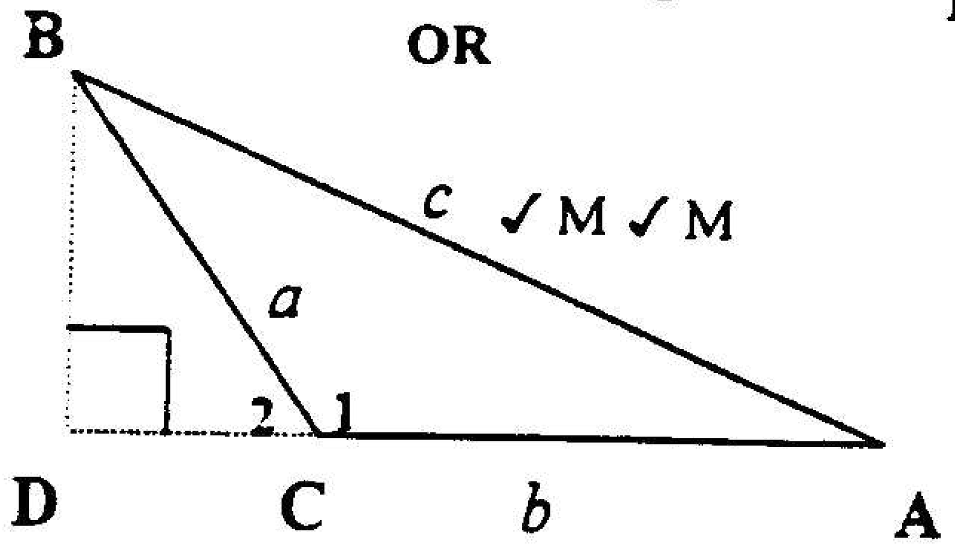



Diagram and construction

Const: Draw BD⊥AC or AC produced.

Proof: Area $\Delta ABC = \frac{1}{2} AC.BD$ ✓A

Correct Area formula for triangle

$\frac{BD}{BC} = \sin \hat{C}_2$ ✓A

Correct ratio for sin C

$\hat{C}_2 = 180° - \hat{C}_1$

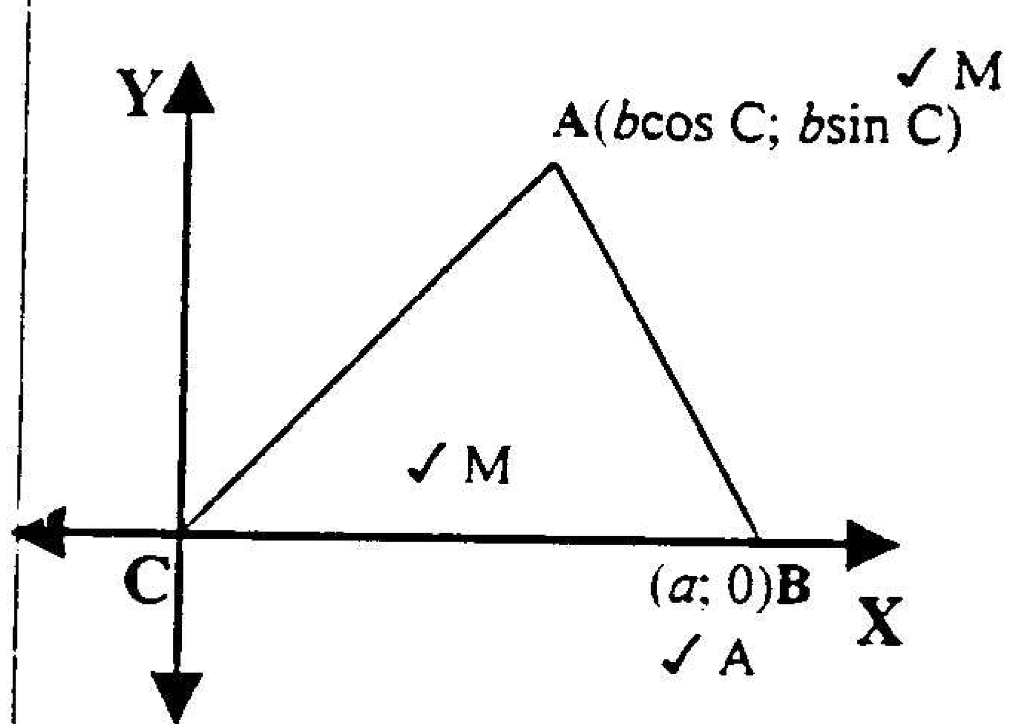
$\therefore \sin \hat{C}_2 = \sin(180° - \hat{C}_1) = \sin \hat{C}_1$

$BD = a \sin C$ ✓A

Changing the subject correctly

$\therefore \Delta ABC = \frac{1}{2} ab \sin C$

**OR**

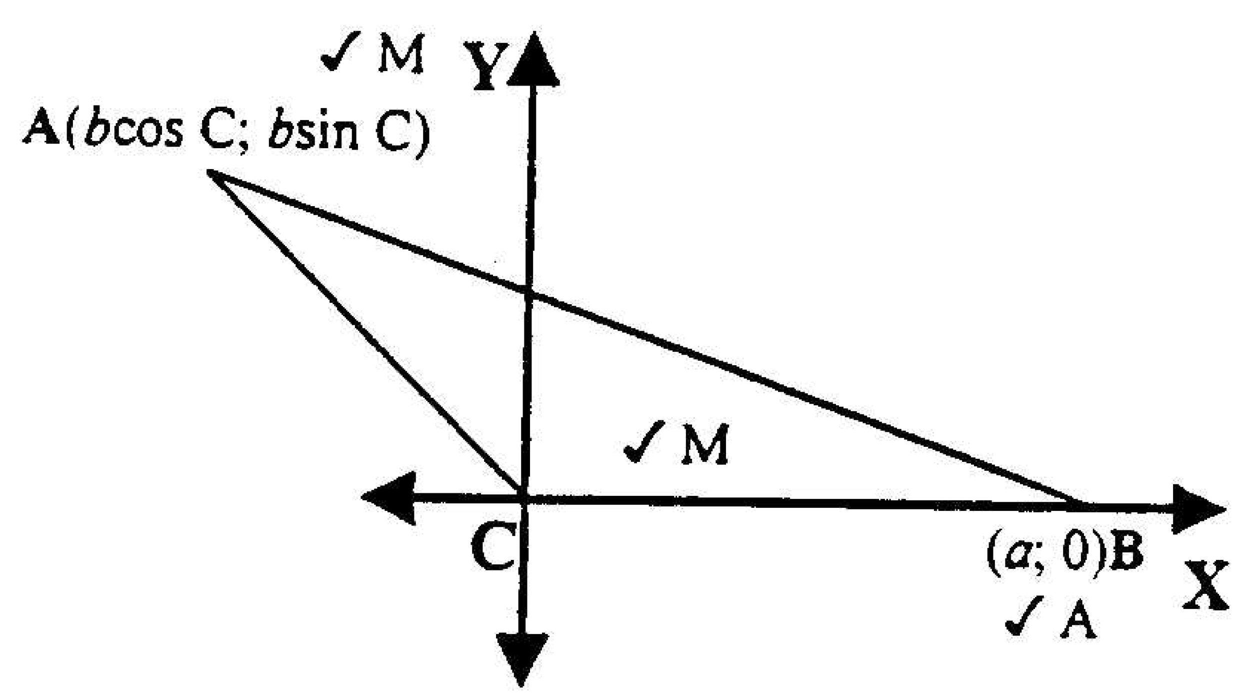



Correct $y$ coordinate of A

Placing the triangle in standard position on the axes

Indicating that CB = $a$

**OR**



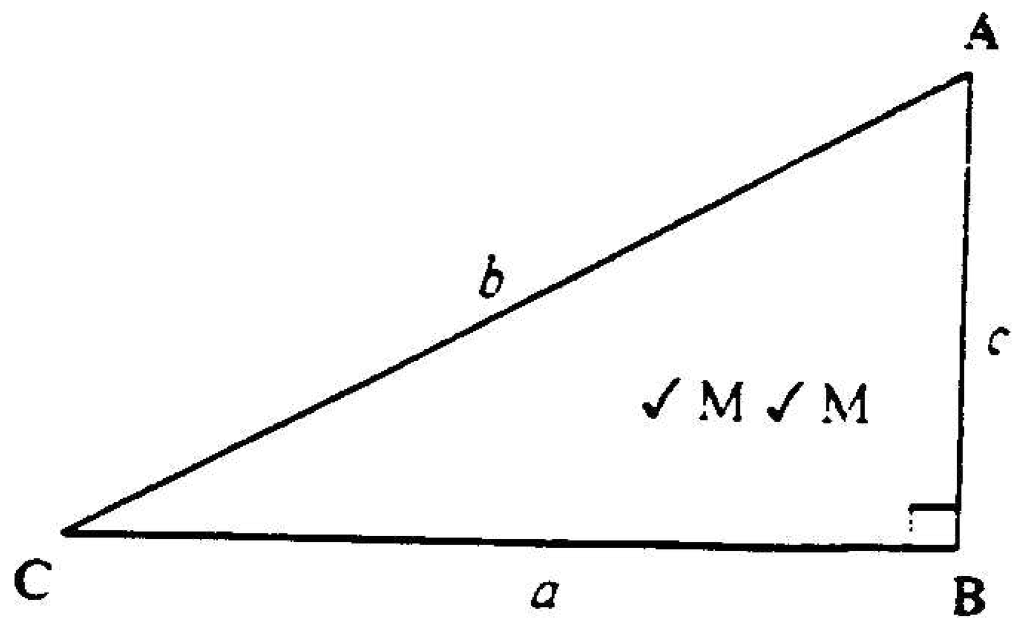

Area $\Delta ABC = \frac{1}{2} b.h$ ✓A

$= \frac{1}{2} ab \sin C$ ✓A

Correct Area formula for triangle
Correct substitution
Proving the theorem correctly for any other angle and then say "Similarly …" is also acceptable
Proving the theorem correctly for any other angle without saying "Similarly …" ⇒ penalty of 1 mark

**OR**

**OR**




✓ M ✓ M

Diagram correct

Area $\Delta ABC = \frac{1}{2}bh$ ✓ A

$= \frac{1}{2}ac$

Correct formula for area of triangle

$\frac{c}{b} = \sin C$ ✓ A

$c = b\sin C$ ✓ A

Area $\Delta ABC = \frac{1}{2}ab\sin C$ (5)

Correct ratio for sin C
Correct simplification

6.1.2 Area $\Delta ABC = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B$ ✓ M

$b\sin A = a\sin B$ ✓ A

$\frac{\sin A}{a} = \frac{\sin B}{b}$ (2)

Equating the correct two versions of area rule
Indicating the correct simplification
If the sine rule is proven correctly in full $\Rightarrow$ full marks

6.2.1 In $\Delta SRQ$

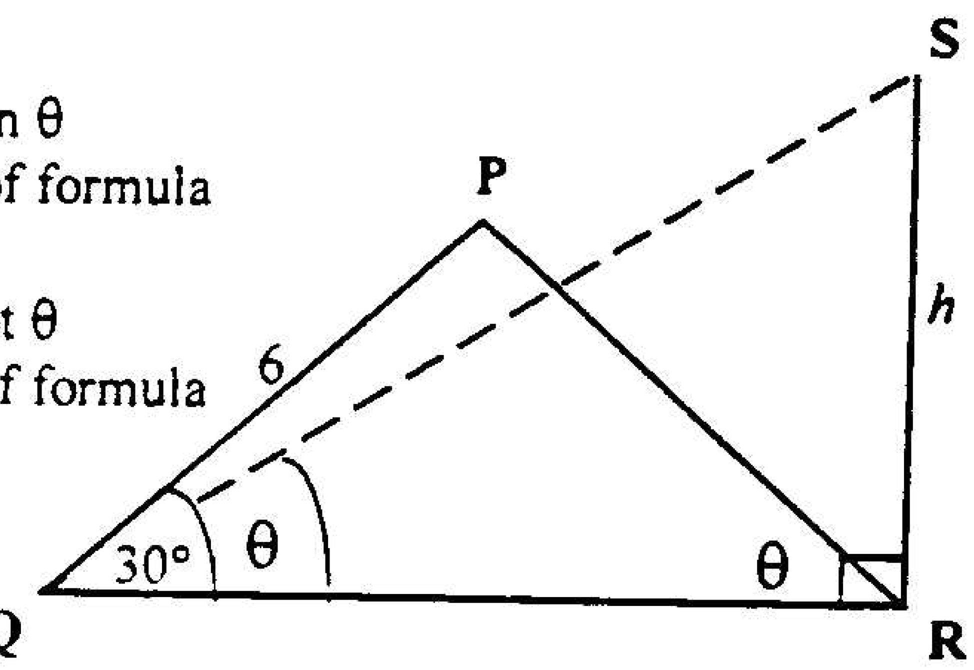
$\frac{h}{QR} = \tan\theta$ ✓ A $\Rightarrow$ $QR = \frac{h}{\tan\theta}$ ✓ A

Correct ratio for tan θ
Changing subject of formula

**OR**

$\frac{QR}{h} = \cot\theta$ ✓ A $\Rightarrow$ $QR = h\cot\theta$ ✓ A (2)

Correct ratio for cot θ
Changing subject of formula




6.2.2 $Q\hat{P}R = 180° - (\theta + 30°)$ or $Q\hat{P}R = 150° - \theta$ ✓ A ✓ A (2)

Correct use of fact that the interior angles of a triangle are supl.

6.2.3 In $\Delta PQR$ $\frac{QR}{\sin P} = \frac{PQ}{\sin R}$ ✓ M

Correct use of sine rule

$\therefore QR = \frac{6\sin[180° - (\theta + 30°)]}{\sin\theta}$ ✓ CA

Correct substitution

$= \frac{6\sin(\theta + 30°)}{\sin\theta}$ ✓ CA

Correct reduction formula

$\therefore \frac{h}{\tan\theta} = \frac{6(\sin 30°\cos\theta + \cos 30°\sin\theta)}{\sin\theta}$ ✓ CA ✓ CA

Substituting QR
Correct application of sin of compound angles

$\therefore h = \frac{6(\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta)}{\sin\theta} \times \tan\theta$ ✓ CA ✓ CA

Correct value of sin 30°
Correct value of cos 30°

$= \frac{3(\cos\theta + \sqrt{3}\sin\theta)}{\sin\theta} \times \frac{\sin\theta}{\cos\theta}$ ✓ CA ✓ CA

Correct simplification
Correct identity for tan θ

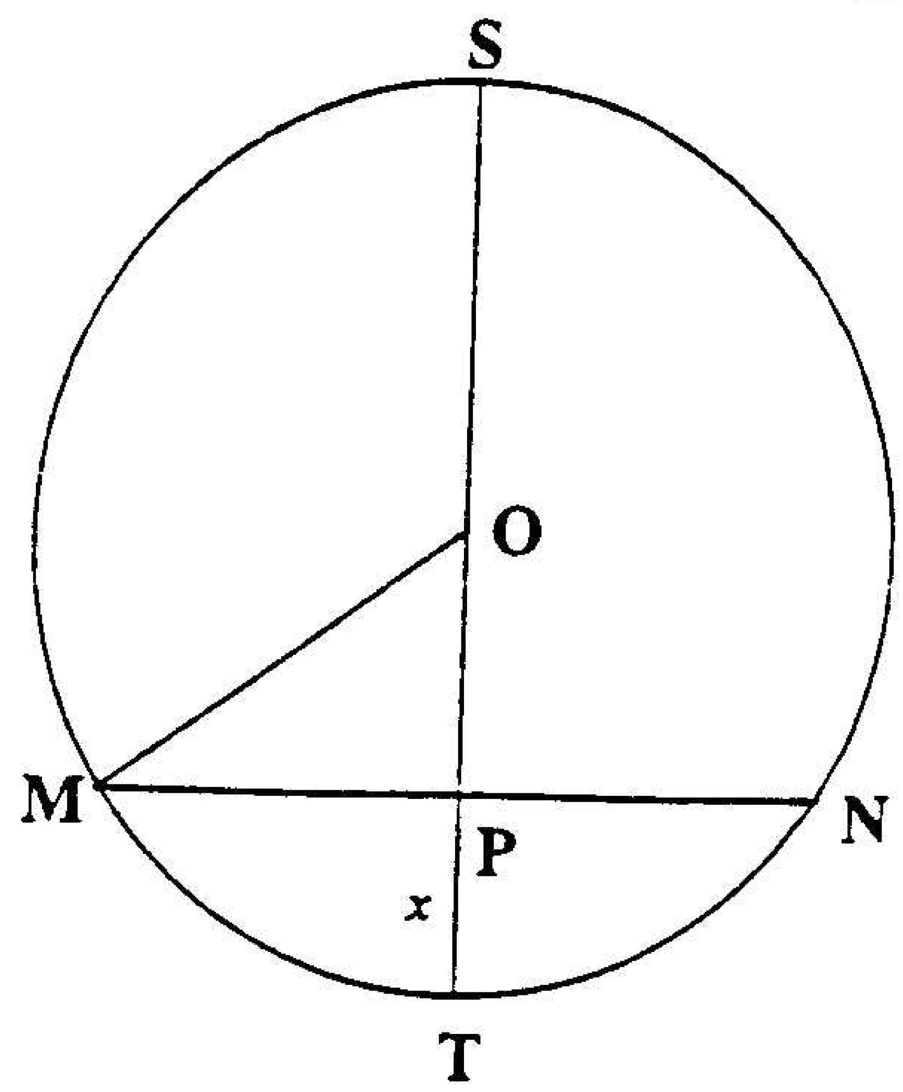
$= 3\left(\frac{\cos\theta}{\cos\theta} + \frac{\sqrt{3}\sin\theta}{\cos\theta}\right)$ ✓ CA

Correct multiplication

$= 3(1 + \sqrt{3}\tan\theta)$ (10)

| | | |
|---|---|---|
| 6.2.4 | $12 = 3\left(1+\sqrt{3}\tan\theta\right)$ ✓ A<br>$\therefore \left(1+\sqrt{3}\tan\theta\right) = 4$<br>$\therefore \sqrt{3}\tan\theta = 3$<br>$\therefore \tan\theta = \frac{3}{\sqrt{3}}$ ✓ A<br>$= \sqrt{3}$<br>$\therefore \theta = 60°$ ✓ CA<br>(3)<br>**[24]** | Correct substitution<br>Correct manipulation<br>Correct value of θ |

**QUESTION 7**




$SP = 4PT = 4x$

$\therefore\ ST = 5x$ ✓ A

$SO = OT = \frac{5}{2}x$ ✓ A (radius = ½ diameter)

$\therefore OM = \frac{5}{2}x$

$OP = \frac{5}{2}x - x$

$= \frac{3}{2}x$ ✓ CA

$MP^2 = MO^2 - OP^2$ (Pythagoras) ✓ S/R

$= \left(\frac{5}{2}x\right)^2 - \left(\frac{3}{2}x\right)^2$

$= \frac{25x^2}{4} - \frac{9x^2}{4}$

$= \frac{16}{4}x^2$

$= 4x^2$ ✓ CA

$\therefore MP = 2x$ ✓ CA

$\therefore MN = 2MP$ (⊥ from center to chord) ✓ R

$= 4x$ ✓ CA

Accept the first three marks if it is indicated on the diagram and not written down in answer book

**OR**

$SP = 4\ PT = 4x$

$\therefore\ ST = 5x$ ✓ A

$SO = OT = \frac{5}{2}x$ ✓ A (radius = ½ diameter)

$\therefore OM = \frac{5}{2}x$

$OP = r - x$ ✓ CA

$MP^2 = r^2 - (r - x)^2$ (Pyth) ✓ S/R

$= 2rx - x^2$

$= 2\left(\frac{5}{2}x\right)x - x^2$

$= 5x^2 - x^2$

$= 4x^2$ ✓ A

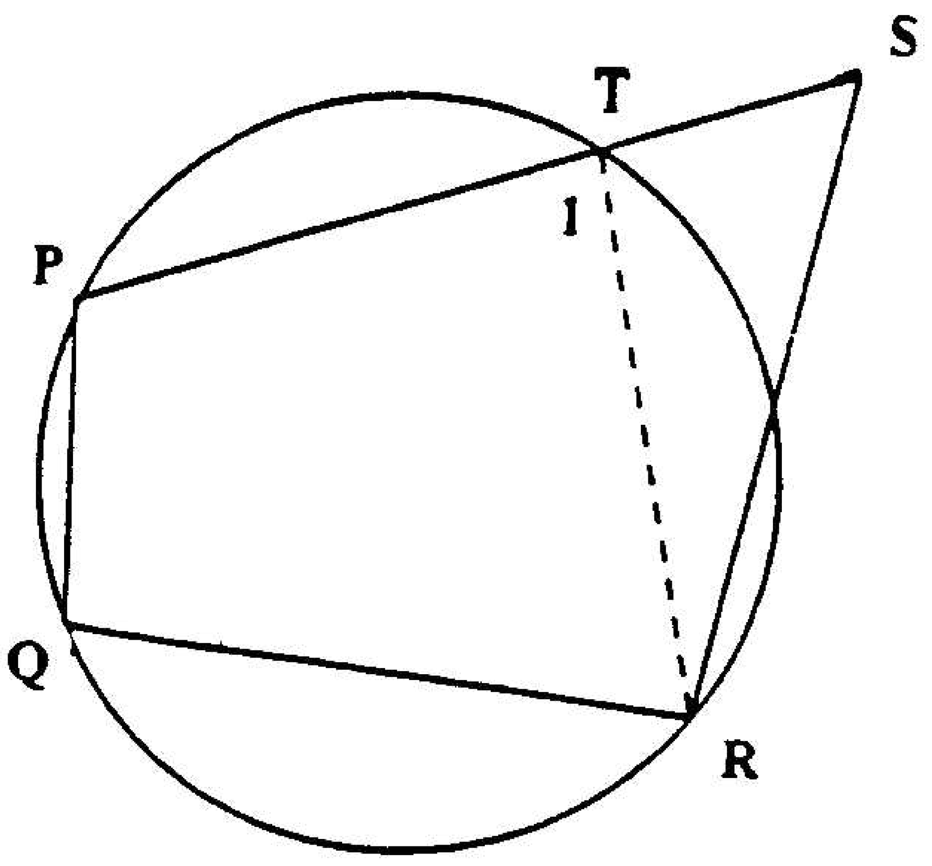
$\therefore MP = 2x$ ✓ CA

$\therefore MN = 2MP$ (⊥ from centre to chord) ✓ R

$= 4x$ ✓ CA

**[8]**

## QUESTION 8




8.1 Proof ✓ M

Assume that PQRS is not a cyclic quad. but that the circle through P, Q and R cuts PS (produced) at T.

$\hat{Q} + \hat{T}_1 = 180°$ ✓ S (opp $\angle^s$ of cycl quad suppl) ✓ R

$\hat{Q} + \hat{S} = 180°$ (given)

$\therefore\ \hat{T}_1 = \hat{S}$ ✓ S

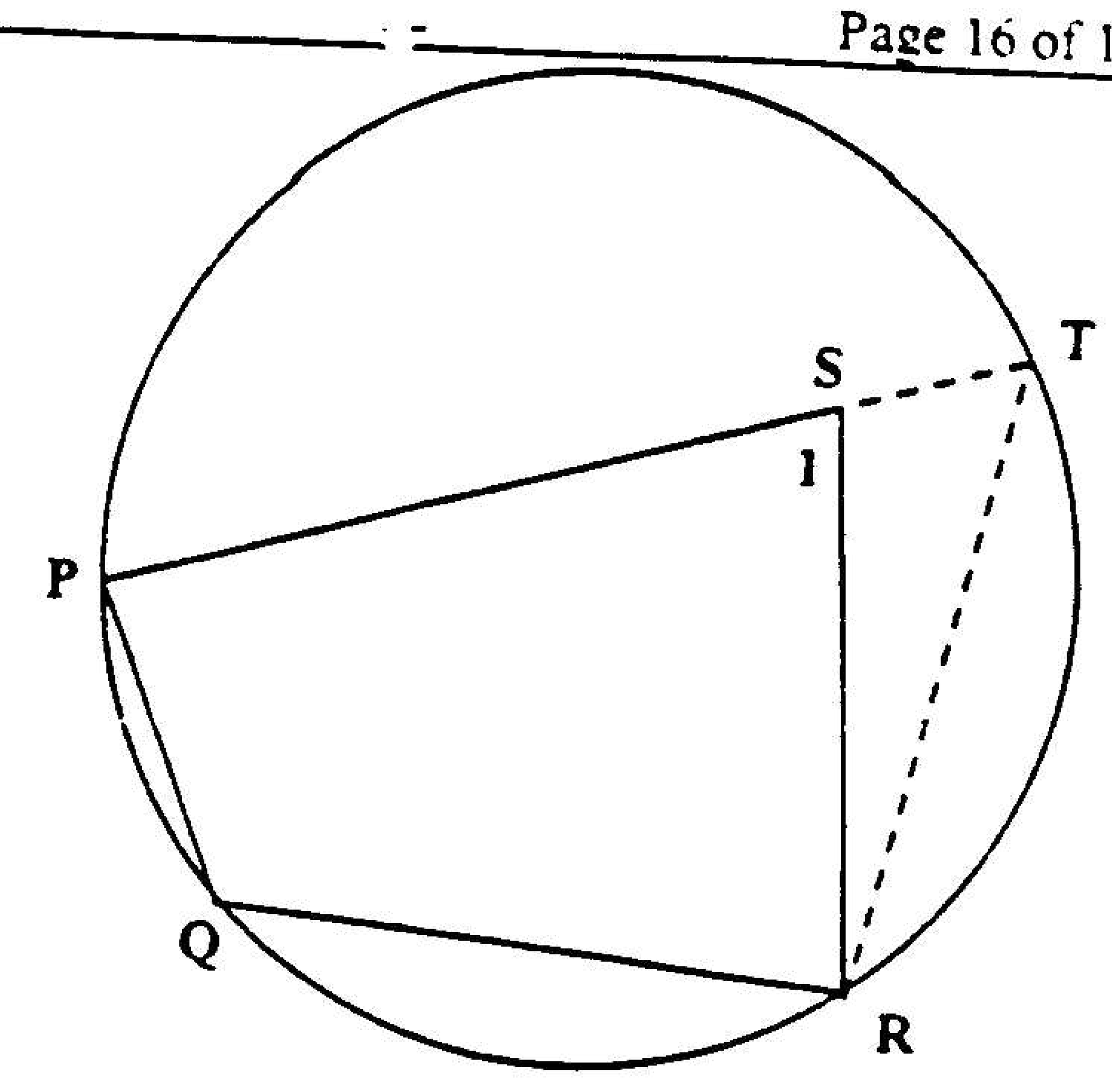
but $\hat{T}_1 = \hat{S} + \hat{R}_1$ (ext $\angle$ of $\Delta$)

but this is impossible

$\therefore$ The assumption is false.

$\therefore$ The circle through P, Q and R does pass through S ✓ S/R

$\therefore$ PQRS is a cyclic quadrilateral. (5)




Construction on diagram only, also acceptable

8.2

8.2.1 $K\hat{L}M = K\hat{N}M$ (opp $\angle^s$ of $||^m =$ ) ✓ S/R

$= E\hat{N}F$ (vert opp $\angle^s$ =) ✓ S/R

$K\hat{L}M + K\hat{D}M = 180°$ (opp $\angle^s$ of cyclic quad supl) ✓ S/R

$\therefore\ E\hat{N}F + K\hat{D}M = 180°$ ✓ S

$\therefore$ DENF a cyclic quad (opp int $\angle^s$ suppl) ✓ R

**OR**

$L\hat{K}E = \hat{E}_2$ (LK | | MN) ✓ S/R

$= 90°$ (ME $\perp$ KD)

$L\hat{M}F = 90°$ (opp int $\angle^s$ of cycl quad supl) ✓ S/R

$= \hat{F}_1$ (KF | | LM) ✓ S/R

$\hat{E}_2 + \hat{F}_1 = 180°$ ✓ S

DENF a cyclic quad (opp int $\angle^s$ supl) ✓ R

**OR**

$\hat{N}_4 = N\hat{M}L$ (KF | | LM) ✓ S/R

$N\hat{M}L + M\hat{L}K = 180°$ (NM | | KL) ✓ S/R

$\hat{N}_4 + M\hat{L}K = 180°$

$K\hat{D}F + M\hat{L}K = 180°$ (opp int $\angle^s$ of cycl quad supl) ✓ S/R

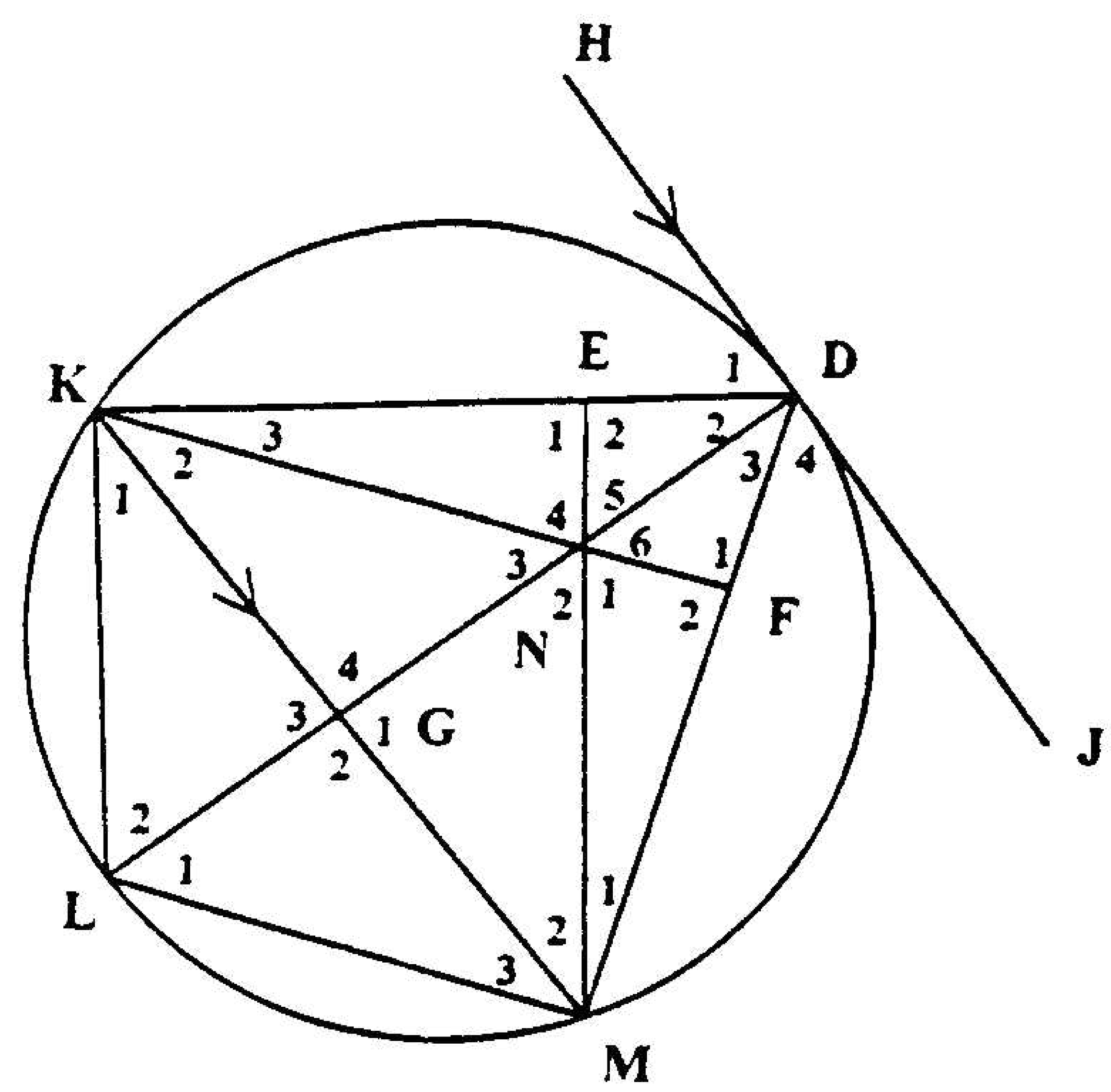
$\hat{N}_4 = K\hat{D}F$ ✓ S

DENF a cyclic quad (ext $\angle$ = opp int $\angle$) ✓ R

**OR**

Same proof as previous one for $\hat{N}_1 = K\hat{D}F$

(5)




In case of a break down, max of 3 marks

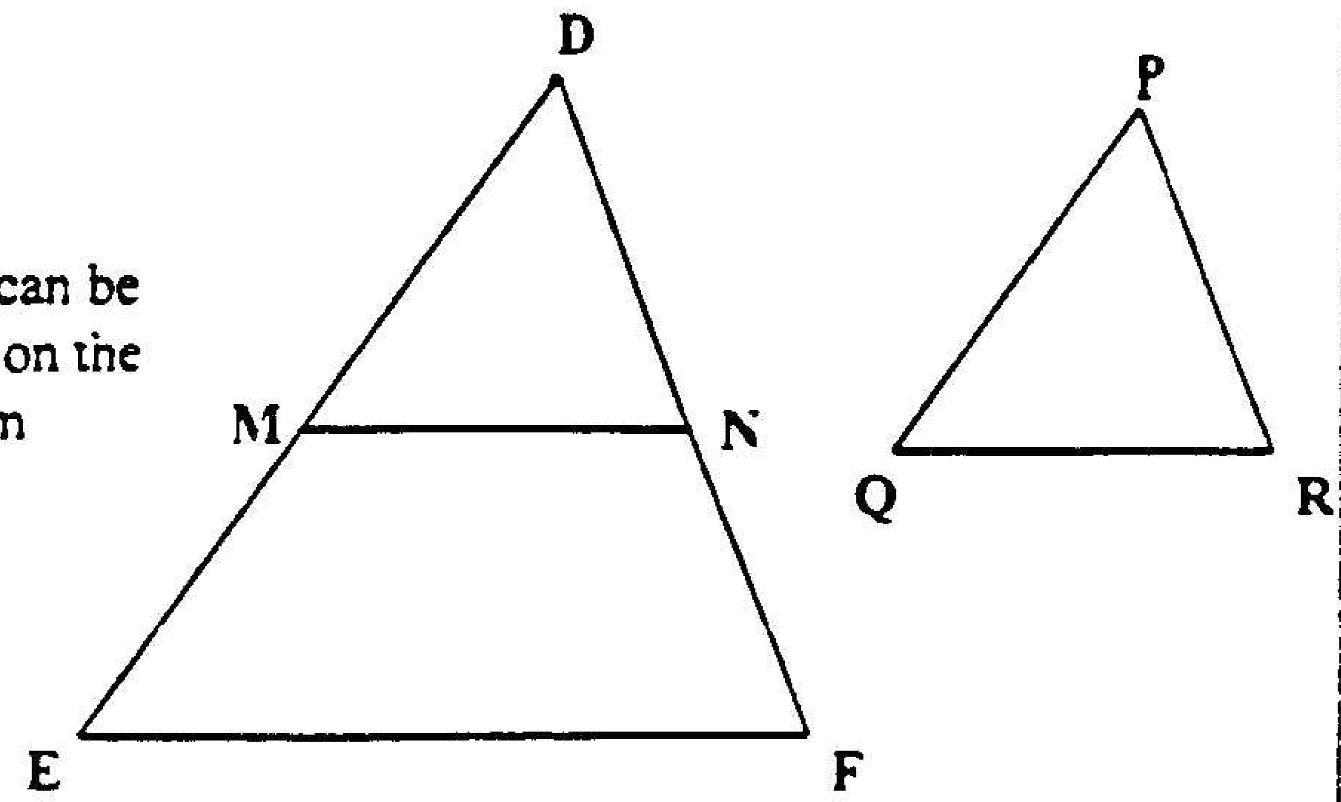
If proven correctly, but reason or two missing $\Rightarrow$ min 4 marks

| | | | |
|---|---|---|---|
| 8.2.2 | $\hat{E}_1 = 90°$ ✓ S | (ME ⊥ KD) | |
| | $\hat{F}_2 = \hat{E}_2 = 90°$ | (ext ∠ of cycl quad ENFD) ✓ S/R | |
| | ∴ ME and KF are altitudes of ΔKMD | | |
| | ∴ DNG is an altitude | (altitudes concurrent at N) ✓ S/R | |
| | ∴ $\hat{G}_1 = 90°$ | | |
| | $= \hat{E}_2$ ✓ S | | |
| | ∴ MGED is a cyclic quad | (DM subt = ∠$^s$) ✓ R | |
| **OR** | | | |
| | $\hat{E}_2 = D\hat{K}L$ | (EM \|\| KL) ✓ S/R | |
| | $= J\hat{D}G$ ✓ S | (∠ betw tang and chord) ✓ R | |
| | $= \hat{G}_1$ | (HJ \|\| KM) ✓ S/R | |
| | MGED is a cyclic quad | (DM subtends = ∠$^s$) ✓ R | |
| **OR** | | | |
| | $\hat{E}_1 = 90°$ ✓ S | (ME ⊥ KD) | |
| | $= D\hat{K}L$ | (EM \|\| KL) ✓ S/R | |
| | ∴ LD a diameter | | |
| | $H\hat{D}G = 90°$ | (tang ⊥ diam/rad) ✓ S/R | |
| | $= D\hat{G}M$ | (HJ \|\| KM) ✓ S/R | |
| | MGED is a cyclic quad | (DM sutends = ∠$^s$) ✓ R (5) | |
| 8.2.3 | $K\hat{M}D = \hat{D}_1$ ✓ S | (∠ betw tang and chord) ✓ R | |
| | $= M\hat{K}D$ | (HD \|\| KM. alt ∠$^s$ =) ✓ S/R | |
| | DK = DM | (sides opposite = ∠$^s$) ✓ R | |
| **OR** | | | |
| | KLMN is a rhombus | (diags bisect perp) ✓ S/R | |
| | ∴ KL = LM | (adj sides of rhombus =) ✓ S/R | |
| | KLMD is a kite | (DL bisects KM perp) ✓ S/R | |
| | ∴ DK = DM | (adj sides of kite =) ✓ S/R | Chief marker must be consulted for other possibilities |
| | | (4) | |
| | | [19] | |

**QUESTION 9**

9.1 Const: On DE cut DM = PQ and on DF cut DN = PR ✓ M

These can be shown on the diagram




Proof:

In $\Delta DMN$ and $\Delta PQR$

(i) $\hat{D} = \hat{P}$ (given)

(ii) DM = PQ (const)

(iii) DN = PR (const)

$\therefore \Delta DMN \equiv \Delta PQR$ ✓ S (S, ∠, S) ✓ R

$\therefore D\hat{M}N = \hat{Q} = \hat{E}$ ✓ S

$\therefore$ MN || EF ✓ S (corresp ∠s =) ✓ R

$\therefore \frac{DE}{DM} = \frac{DF}{DN}$ (line || to one side of Δ) ✓ S/R

$\therefore \frac{DE}{PQ} = \frac{DF}{PR}$ (7)

9.2.1 In $\Delta XYC$ and $\Delta DYX$

(i) $\hat{X}_1 = \hat{D}_1$ (∠ betw tang and chord)

(ii) $\hat{C}_2 = \hat{X}_2$ ✓ S (∠ betw tang and chord) ✓ R

(iii) $X\hat{Y}C = D\hat{Y}X$ (int ∠s of Δ suppl)

$\therefore \Delta XYC \,|||\, \Delta DYX$ ✓ S (equiangular) or (AAA) ✓ R

$\therefore \frac{XY}{DY} = \frac{YC}{YX}$ ✓ S

$\therefore XY^2 = DY.YC$ (5)

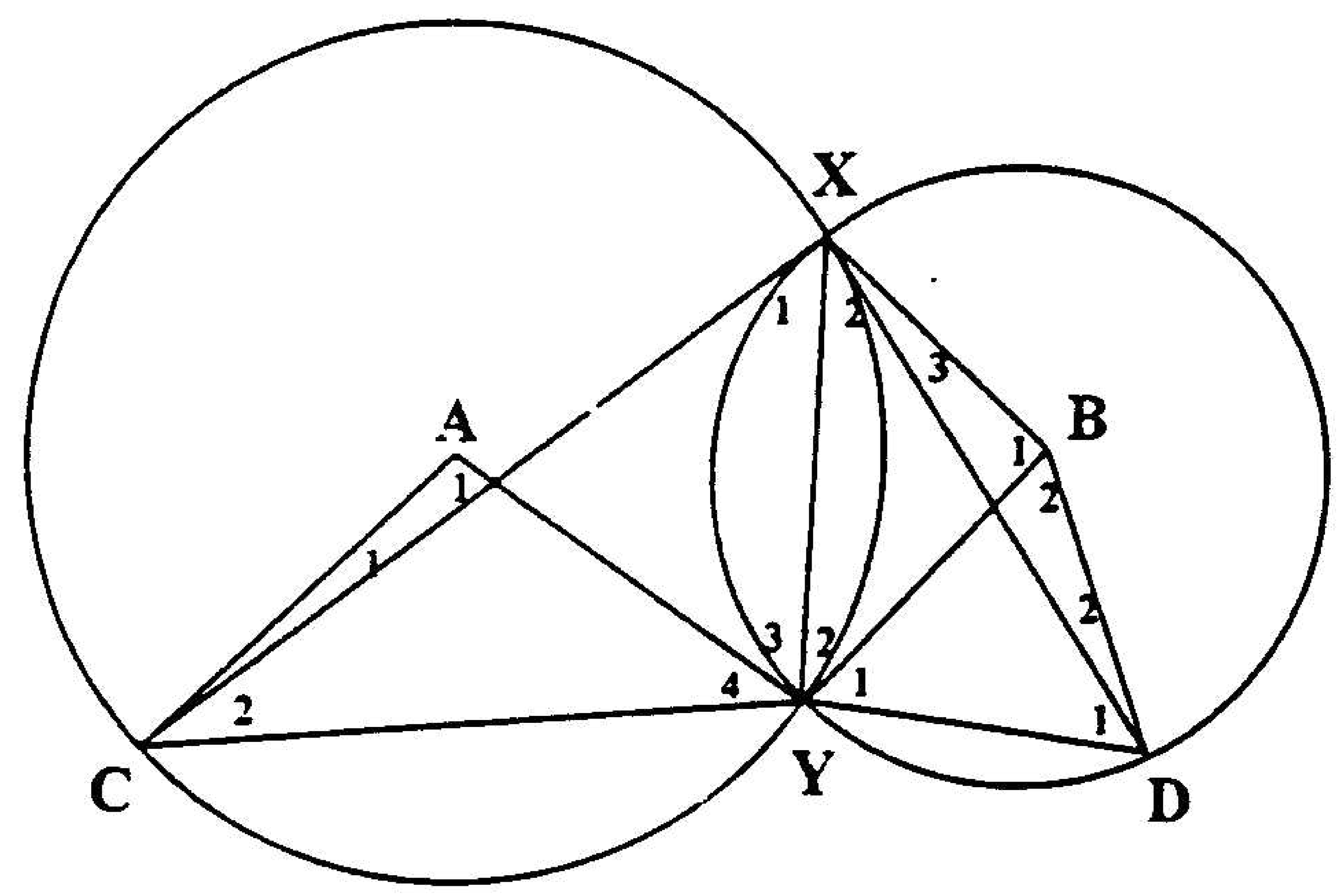
1 mark for first two statements
1 mark for first two reasons
min 4 marks if similarity is proven
max 3 marks if similarity is not proven
If candidates use similarity without reasons ⇒ max 2 marks

9.2.2 $\hat{B}_1 = 2\hat{D}_1$ ✓ S (∠ at centre = 2 ∠ at circumf) ✓ R

$= 2\hat{X}_1$ (∠ betw tang and chord/proven) ✓ S/R

$= \hat{A}_1$ ✓ S (∠ at centre = 2 ∠ at circumf) ✓ R

(5)




9.2.3 In $\Delta CAY$ and $\Delta YBX$

(i) $\hat{A}_1 = \hat{B}_1$ ✓ S (proven) ✓ S/R

(ii) $\hat{Y}_4 = \frac{1}{2}(180° - \hat{A}_1)$ (int ∠s of isos ΔACY suppl)

$= \frac{1}{2}(180° - \hat{B}_1)$ ✓ S/R

$= \hat{Y}_2$ (int ∠s of isos ΔBXY suppl)

(iii) $A\hat{C}Y = B\hat{X}Y$ (int ∠s of Δ suppl)

$\therefore \Delta CAY \,|||\, \Delta YBX$ (equiangular) or (AAA) ✓ R (4)

9.2.4 $\frac{AC}{BX} = \frac{CY}{XY}$ (ΔCAY ||| ΔYBX) ✓ S/R

$\therefore \frac{R}{r} = \frac{CY}{XY}$ ✓ S

$\therefore \frac{R^2}{r^2} = \frac{CY^2}{XY^2}$ ✓ S

$= \frac{CY^2}{DY.YC}$ ✓ S ($XY^2 = DY.YC$ from 9.2.1)

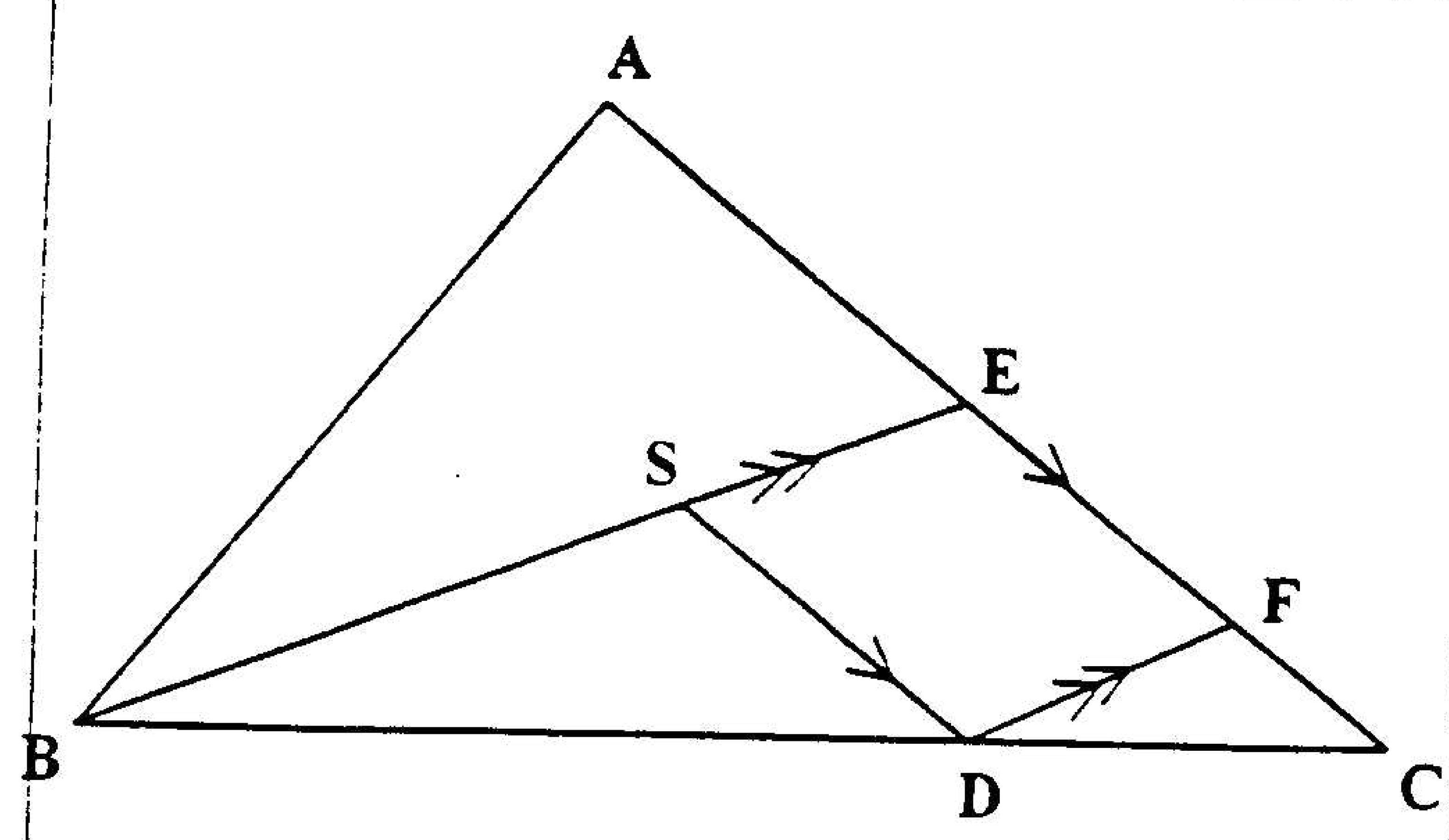
$= \frac{CY}{DY}$ ✓ S

$\frac{r^2}{R^2} = \frac{DY}{CY}$

(4)

[25]

**QUESTION 10**




10.1 In $\Delta$ BEC

$\frac{BS}{SE} = \frac{BD}{DC}$ ✓ S (line parallel to one side of $\Delta$: SD||EC) ✓ R

$\frac{BD}{DC} = \frac{EF}{FC}$ ✓ S (line parallel to one side of $\Delta$: DF||BE) ✓ R

$\frac{BS}{SE} = \frac{EF}{FC}$ (4)

10.2 $\frac{EF}{EC} = \frac{BD}{BC}$ (line parallel to one side of $\Delta$: DF||BE) ✓ S/R

$\frac{BD}{BC} = \frac{BS}{BE}$ (line parallel to one side of $\Delta$: SD||EC) ✓ S/R

$\frac{EF}{EC} = \frac{BS}{BE}$

$\frac{EF}{EC} = \frac{2}{3}$ ✓ S (S the centroid of $\Delta$ABC) ✓ R

$\therefore$ EF = $\frac{2}{3}$EC

**OR**

$\frac{BS}{SE} = \frac{2}{1}$ ✓ S (S the centroid of $\Delta$ABC) ✓ R

$\frac{EF}{FC} = \frac{2}{1}$ ✓ S (from 10.1)

$\frac{EF}{EF + FC} = \frac{2}{2+1}$

$\frac{EF}{EC} = \frac{2}{3}$ ✓ S

EF = $\frac{2}{3}$EC (4)

10.3 SD = EF ✓ S (opp sides of parm =) ✓ R

= $\frac{2}{3}$EC

= $\frac{2}{3}\left(\frac{1}{2}AC\right)$ ✓ S (BE a median of $\Delta$ABC) ✓ R

= $\frac{1}{3}$AC (4)

**[12]**