GAUTENG DEPARTMENT OF EDUCATION

SENIOR CERTIFICATE EXAMINATION

ADDITIONAL MATHEMATICS HG

TIME: 3 hours

MARKS: 400

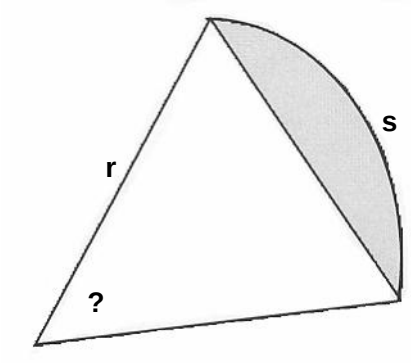
INSTRUCTIONS:

- This examination paper consists of FIVE sections.
- Section A is COMPULSORY.
- A further TWO sections must be answered from Sections B, C, D and E.
- Each section must **be answered in a separate answer book and the relevant section must be clearly indicated on the cover**. Place all answer books inside the answer book for Section A before handing them in.
- Unless otherwise indicated, non-programmable calculators may be used.
- The examination paper consists of 17 pages. Statistical tables and formula sheets can be found on pages, 15, 16 and 17 respectively.
- All essential calculations must be clearly shown.
- All angles are in radians and answers must also be given in radians.
- Writing must be legible.

SECTION A
COMPULSORY
CALCULUS

QUESTION 1

1.1 The sector of the circle in the sketch has an arc length $s = \frac{3\pi}{2}$ and radius r = 3.



Determine

1.1.1 the angle **?**. (2)

1.1.2 the area of the sector. (2)

1.1.3 the area of the shaded portion of the sector. (6)

[10]

QUESTION 2

2.1 $f : x \rightarrow \arcsin x - \frac{p}{4}$

2.1.1 Determine the range of $f(x)$ (4)

2.1.2 Sketch the graph of $f(x)$ (6)

2.2 Answer this question without the use of a calculator. Determine

2.2.1 $\arcsin\left(\cos\frac{7\pi}{6}\right)$ (6)

2.2.2 $\arccos\left(\sin\frac{7\pi}{6}\right)$ (8)

2.2.3 $\cos\left(2\arctan\frac{5}{12}\right)$ (10)

[34]

QUESTION 3

$$f(x) = \begin{cases} |x-2| & \text{if} \quad x > 0 \\ 3 & \text{if} \quad x = 0 \\ 2x+2 & \text{if} \quad x < 0 \end{cases}$$

3.1 Determine if f is continuous at $x = 0$, and substantiate fully. If not continuous name the type of discontinuity. (8)

3.2 Determine if f is differentiable at $x = 0$ (2)

3.3 Assume the function is continuous at $x = 2$. Determine, with substantiation, if f is differentiable at $x = 2$, using algebraic methods. Substantiate fully, but it is not necessary to work from first principles. (8)

[18]

QUESTION 4

Determine the following limits, if possible:

4.1 $\lim\limits_{x\to 0} \frac{1-\cos x}{x^2}$ (8)

4.2 $\lim\limits_{x\to -\infty} \frac{\sqrt{9x^2-3x+2}}{9x-5}$ (6)

[14]

QUESTION 5

Given the function $f(x) = 2x^2 + 1$, determine the area under the graph of $f(x)$ and above the **X** axis between $x = 0$ and $x = 2$. Use the technique of approximating the area by finding n strips of equal width and determining the Riemann sum for the area of these rectangles. Then let $n \to \infty$. **[18]**

QUESTION 6

6.1 $f(x) = \dfrac{\sin x}{\cos x}$. Do not simplify $f(x)$ and determine $f'(x)$. Simplify the answer to only one trigonometric ratio. (8)

6.2 Find the following derivatives:

6.2.1 $f'\left(\dfrac{\pi}{6}\right)$ if $f(x) = \dfrac{\tan x}{\sec^2 x - 1}$ (12)

6.2.2 $\dfrac{dy}{dx}$ if $y = (\arccos(2x+1))^3$ (6)

6.3 If $f(x) = (1+x^2)^3$ show that $f''(x) = 6(x^2+1)(1+5x^2)$ (10)

[36]

QUESTION 7

Find the following integrals:

7.1 $\displaystyle\int_2^6 \frac{3dx}{\sqrt{2x-3}}$ (10)

7.2 $\displaystyle\int \sin(x+1)\sin(3x-2)dx$ (10)

[20]

QUESTION 8

The function $f(x) = \arctan x$ and its derivative $f'(x) = \frac{1}{1+x^2}$ intersect at point A.

8.1 To find point **A** using Newton's Method, a function $g(x) = \frac{1}{1+x^2} - \arctan x = 0$ is used. Show that $g'(x) = \frac{-1(x+1)^2}{(1+x^2)^2}$ (10)

8.2 Newton's formula for finding point **A** is:

$$a_{n+1} = b + \frac{(1+b^2) - (1+b^2)^2 \arctan b}{(b+1)^2} \quad \text{where } b = a_n$$

Determine the x co-ordinate of point **A** correct to 3 decimal places. Use $a_0 = 1$ (8)

[18]

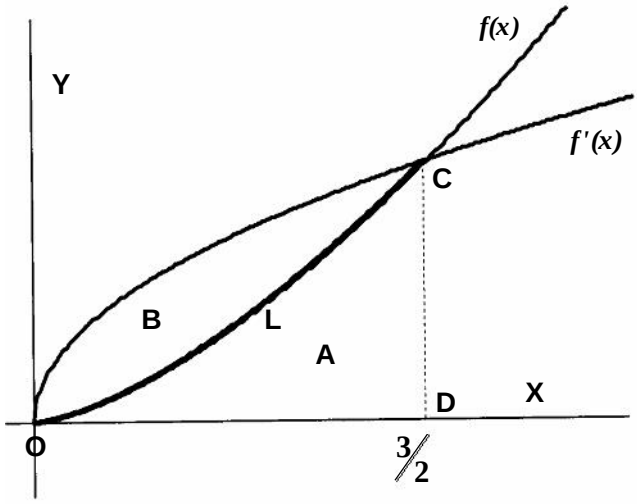
QUESTION 9

Give all answers in this question correct to 3 decimal places.
The length, L, of a curve $f(x)$ between $x = a$ and $x = b$ is given by the formula

$$\mathbf{L} = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

9.1 Use this formula to find the length, **L**, of the curve $f(x) = \frac{2}{3}x^{\frac{3}{2}}$ between $x = 0$ and $x = \frac{3}{2}$. (Give your answer correct to three decimal places.) (12)

9.2 Given below is the sketch of $f(x)$ with the section calculated in Question 9.1 in bold.



9.2.1 Determine the perimeter of area A, that is $L + OD + CD$, if $L = 1{,}969$. (6)

9.2.2 The curve of $f'(x)$ is also shown. Calculate the volume of the solid of revolution generated when the area between the two curves, **B**, is rotated about the x axis. (14)

[32]

TOTAL FOR SECTION A: [200]

Answer any **TWO** of the following **FOUR** sections.

SECTION B
FINANCIAL MATHEMATICS

WHERE APPROPRIATE, GIVE ALL ANSWERS IN THIS SECTION TO THE NEAREST CENT.

QUESTION 10

The total cost function of producing x wooden tables is $C(x) = \dfrac{200x^2 + 150x + 80}{x + 1}$

10.1 Determine:

10.1.1 The marginal cost function (6)

10.1.2 The marginal cost if $x = 10$ (4)

10.2 Determine the average cost of producing the first 10 tables. (8)

[18]

QUESTION 11

At the end of 2005 a company has R20 000 in an account which pays 13,5% interest per annum, compounded monthly.

During 2006, the company carries out the following transactions. (Each transaction is done at the end of the month.)

Month 1 : Deposited R5 000
Month 4 : Withdrew Rx
Month 6 : Deposited R$2x$
Month 8 : Deposited R12 000
Month 9 : Withdrew R$3x$

At the end of 2006 the balance in the company's account will be R31 308,34.

11.1 Represent the above information on a timeline. (4)

11.2 Calculate x. (12)

[16]

QUESTION 12

120 regular payments of R1 000 are made at the beginning of each month into an account earning 12% interest per annum compounded quarterly for a full 10 years. Payments begin immediately.

12.1 Show that the effective monthly interest rate is $\frac{i^{(12)}}{12} = 0,0099$ (6)

12.2 Calculate the total value of the annuity at the end of the 10-year period. (12)

[18]

QUESTION 13

Mr Ngwenya takes out a mortgage bond of R350 000 to help pay for his new house. The bond is repayable in monthly instalments over a period of 20 years at an interest rate of 16% per annum compounded monthly.

13.1 Calculate his monthly instalment if payments begin at the end of the first month. (12)

13.2 After 10 years Mr Ngwenya finds a bank who can offer him a better deal so he decides to calculate his balance outstanding and take out a loan for this amount with the new bank. What is the balance outstanding immediately after the 120^{th} payment if the payments were R4 869,40? (12)

13.3 Assume that he borrows an amount of R290 700,00 from the new bank which charges 15% interest compounded quarterly. He agrees to pay quarterly instalments of R20 000. How many payments will he have to make? He makes the first payment at the end of the first quarter. (14)

13.4 Calculate the final 22^{nd} payment, which will be smaller than R20 000. (10)

[48]

TOTAL FOR SECTION B: [100]

SECTION C
ANALYTICAL GEOMETRY

QUESTION 14

Two circles with equations $(x + 2)^2 + (y - 1)^2 = 16$ and $(x - 2)^2 + (y - 4)^2 = 9$ intersect.

14.1 Calculate the equation of the common chord. (6)

14.2 Calculate the length of the common chord. (18)

[24]

QUESTION 15

15.1 Determine the equation of a parabola with axis $x = -a$ and focus $(a;0)$. (8)

15.2 Determine the equation of a parabola with focus $\left(\frac{1}{2};0\right)$ and axis $2x + 1 = 0$. (2)

15.3 Determine the equations of the tangent and the normal to the parabola $y^2 = 16x$ at the point $(1;4)$. (12)

[22]

QUESTION 16

Given: points $A(6 ; 4 ; 3)$; $B(2 ; 6 ; 7)$; $C(-2 ; 2 ; 5)$ and $D(2 ; 0 ; 1)$

16.1 Prove that ABCD is a square. (14)

16.2 If **P** is the point $(1 ; 9 ; 6)$, determine the equation of line l_1 with direction normal to the plane P_1. (6)

16.3 Determine **Q**, the point of intersection of line l_1 and plane P_1. (6)

[26]

QUESTION 17

17.1 Determine the parametric equations of line l_1 through $A(1 ; 0 ; 3)$ and $B(0 ; -1 ; 4)$. (6)

17.2 Determine if there is a point of intersection between line l_1, and line l_2 with parametric equations $x = 2 + s$, $y = 1 + 2s$ and $z = 3 + s$. (14)

17.3 Determine the distance from point $A(1 ; 0 ; 3)$ to the flat surface P_2 with equation $3x - 2y + z = 7$. (6)

17.4 What does this distance represent in terms of lines l_1 and l_2? (2)

[28]

TOTAL FOR SECTION C: [100]

SECTION D
ALGEBRA

QUESTION 18

18.1 Decompose $\frac{11x^2 + 3x - 1}{2x^3 + x^2}$ into partial fractions. (14)

18.2 $f(x) = x^3 + 12x^2 + 4x - 6$ with zeros **a**, **b** and **c**. Determine the polynomial in $Z[x]$ with zeros $\frac{abc}{6}$, $\frac{a+b+c}{ab+ac+bc}$ and -3. (14)

18.3 Use mathematical induction to prove that

$\log x + 2\log x + 3\log x + \ldots + n\log x = \frac{n}{2}\log x^{n+1}$ for all natural numbers $n \in N$. (20)

[48]

QUESTION 19

19.1 19.1.1 Determine the **HCF** of $x^4 - 5x^3 + 7x^2 - 5x + 6$ and $x^3 + x^2 + x + 1$. (6)

19.1.2 Using the **HCF**, or otherwise, factorise $x^4 - 5x^3 + 7x^2 - 5x + 6$ completely in $\mathbf{Z}[x]$. (8)

19.2 Decompose $f(x) = x^4 - 2x^3 - 8x^2 + 6x - 1$ into linear factors in $\mathbf{R}[x]$, if it is given that $-1 + \sqrt{2}$ is a zero of $f(x)$. (18)

[32]

QUESTION 20

$$f(x) = \frac{5x}{x^2 + 5}$$

The function f is continuous for all $x \in \mathbf{R}$ and the graph intersects the **X** axis and the **Y** axis at the origin. The graph has a horizontal asymptote at $y = 0$.

20.1 Show clearly that $f'(x) = \dfrac{-5(x^2 - 5)}{(x^2 + 5)^2}$ (6)

20.2 Determine the co-ordinates of the stationary points of the graph of $y = f(x)$. (8)

20.3 Sketch the graph of $y = f(x)$. (6)

[20]

TOTAL FOR SECTION D: [100]

SECTION E
STATISTICS

Answers must be given correct to four decimal digits if necessary.

QUESTION 21

The probability that the sun will shine is $\frac{4}{7}$, in which case there is no chance of rain. If it is cloudy, the probability that it will rain is $\frac{4}{11}$. If it does not rain, the soccer practice will definitely take place. If it rains, the probability that the soccer practice will take place is $\frac{11}{28}$.

21.1 Draw a tree diagram to represent the situation. (8)

21.2 Determine the probability that the soccer practice will not take place. (6)

[14]

QUESTION 22

The digits 2;3;4;4;4;5 are given.

22.1 How many 6-digit numbers can be formed from the given digits? (6)

22.2 What is the probability that a random number generated using these digits is larger than 400 000? (8)

[14]

QUESTION 23

23.1 Determine how large a group should be in order that 45 combinations of 2 can be chosen from it. That means ${}_nC_2 = 45$. $({}_nC_r = \frac{n!}{(n-r)!r!})$ (6)

23.2 There are 11 white and 9 black balls in a blue bag. A person draws 9 balls, one after the other. Determine the probability that 5 of these balls will be white if

23.2.1 the balls are replaced after each draw. (8)

23.2.2 the balls are not replaced after each draw. (8)

[22]

QUESTION 24

At a local cinema it is known that 54 percent of the viewers buy popcorn, 67 percent buy a soft drink, 23 percent buy sweets and a soft drink, 34 percent buy popcorn and a soft drink, 28 percent buy popcorn and sweets and 16 percent buy sweets, popcorn and a soft drink. Only 1 percent buy nothing.

24.1 Represent the above situation using a Venn diagram, filling in all the given information. (10)

24.2 What percentage of viewers will buy sweets only? (4)

[14]

QUESTION 25

A survey was done of the traffic passing over a certain bridge in a 12-hour period. The density of vehicles was normally distributed with an average of 123 vehicles per minute and standard deviation of 7 vehicles per minute.

25.1 Calculate the probability that, in any minute, between 110 and 120 vehicles pass over the bridge. (12)

25.2 "Rush hour" is defined as the period(s) during which more than **n** vehicles pass over the bridge per minute. Find n if $P(X > n) = 0{,}0123$. (10)

[22]

QUESTION 26

Police records show that a large percentage of teenage drivers involved in car accidents had been under the influence of alcohol. Fifty teenage drivers who have had accidents are randomly chosen and 44 are found to have been drinking.

26.1 Determine a point estimate, **p**, for the proportion of teenage drivers who have had accidents and have been drinking. (4)

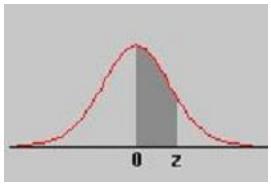
26.2 Determine a 96% confidence interval for the proportion of teenage drivers who have had accidents and have been drinking. (10)

[14]

TOTAL FOR SECTION E: [100]

TOTAL: 400

Normal Distribution/ *Normaalverdeling*



$$P(X \le x) = \frac{1}{\sqrt{2p}} \int_{-\infty}^{x} e^{-\frac{x^2}{2}} dx$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0		0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

FORMULA SHEET/ *FORMULEBLAD*

Differential and Integral Calculus

Differensiaal- en Integraalrekene

$s = r\theta$ $A=\frac{1}{2}r^2\theta$

$\sin^2 x= \frac{1}{2}(1-\cos 2x)$ $\cos^2 x= \frac{1}{2}(1+\cos 2x)$

$\sin A.\cos B= \frac{1}{2}(\sin(A+B)+\sin(A-B))$

$\sin A.\sin B= \frac{1}{2}(\cos(A-B) - \cos(A+B))$

$\cos A.\cos B= \frac{1}{2}(\cos(A-B)+\cos(A+B))$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(2n+1)(n+1)}{6}$$

$$a_{n+1} = a_n - \frac{f(a_n)}{f'(a_n)}$$

$$V = \pi \int_a^b [f(x)]^2 dx$$

$$Riemann\ Sum = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x_i$$

$F(x)$	$F'(x)$
$a.x^n$	$na.x^{n-1}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x.\tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x.\cot x$
arc sin x bg sin x	$\frac{1}{\sqrt{1-x^2}}$
arc cos x bg cos x	$-\frac{1}{\sqrt{1-x^2}}$
arc tan x bg tan x	$\frac{1}{x^2+1}$
$f(x).g(x)$	$f'(x).g(x)+f(x).g'(x)$
$\frac{f(x)}{g(x)}$	$\frac{f'(x).g(x)-f(x).g'(x)}{[g(x)]^2}$
$f(g(x))$	$f'(g(x)).g'(x)$

Finance/ *Finansies*

$F=P(1+i)^n$ $F=P(1-i)^n$

$F=P(1+in)$ $F=P(1-in)$

$$P = x.\frac{1-(1+i)^{-n}}{i} \qquad F = x.\frac{(1+i)^n-1}{i}$$

Analytical Geometry/ *Analitiese Meetkunde*

$y=4ax^2$ $yy_1=2a(x+x_1)$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad \frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$$

Algebra

$$\alpha+\beta=-\frac{b}{a} \qquad \alpha+\beta+\gamma=-\frac{b}{a}$$

$$a.ß=\frac{c}{a} \qquad \alpha\beta+\beta\gamma+\alpha\gamma=\frac{c}{a}$$

$$\alpha.\beta.\gamma=-\frac{d}{a}$$

Statistics / *Statistiek*

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$${}_nP_r = \frac{n!}{(n-r)!} \qquad {}_nC_r = \frac{n!}{(n-r)!r!}$$

$$P(X=x)=\binom{n}{x}p^x(1-p)^{n-x}$$

$$P(X=x)=\frac{\binom{p}{x}\binom{N-p}{n-x}}{\binom{N}{n}}$$

$$z=\frac{X-\mu}{\sigma}$$

$$P\left(X-1.96\frac{\sigma}{\sqrt{n}}<\mu<X+1.96\frac{\sigma}{\sqrt{n}}\right)=0.95$$

$$P\left(p-1.96\sqrt{\frac{p(1-p)}{n}}<\pi<p+1.96\sqrt{\frac{p(1-p)}{n}}\right)=0,95$$

Mathematics For mula Sheet / *Wiskund e Formuleblad*

1. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

2. $T_n = a + (n-1)d$

3. $S_n = \dfrac{n}{2}(a + l)$

4. $S_n = \dfrac{n}{2}[2a + (n-1)d]$

5. $T_n = ar^{n-1}$

6. $S_n = \dfrac{a(1 - r^n)}{1 - r}$

7. $S_n = \dfrac{a(r^n - 1)}{r - 1}$

8. $S_\infty = \dfrac{a}{1 - r}$

9. $A = P(1 + \dfrac{r}{100})^n$

10. $A = P(1 - \dfrac{r}{100})^n$

11. $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

12. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

13. $y = mx + c$

14. $y - y_1 = m(x - x_1)$

15. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

16. $m = \tan\theta$

17. $(\dfrac{x_1 + x_2}{2} ; \dfrac{y_1 + y_2}{2})$

18. $y^2 + x^2 = r^2$

19. $(x - p)^2 + (y - q)^2 = r^2$

20. $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

21. $a^2 = b^2 + c^2 - 2bc.\cos A$

22. $\text{area } \Delta ABC = \frac{1}{2}ab.\sin C$

23. $\cos(A + B) = \cos A.\cos B - \sin A.\sin B$

24. $\sin(A + B) = \sin A.\cos B + \cos A.\sin B$

25. $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

26. $\cos 2A = \cos^2 A - \sin^2 A$

27. $\sin 2A = 2\sin A \cos A$

END / *EINDE*