GAUTENG DEPARTMENT OF EDUCATION

SENIOR CERTIFICATE EXAMINATION

ADDITIONAL MATHEMATICS HG

FEB / MAR 2006

TIME: 3 hours

MARKS: 400

INSTRUCTIONS:

- This examination paper consists of FIVE sections.
- Section A is COMPULSORY.
- A further TWO sections should be answered from Sections B, C, D and E.
- Each section should be answered **in a separate answer book and the relevant section must be clearly indicated on the cover**. Place all answer books inside the answer book for Section A before handing all the answer books in together.
- Non-programmable calculators may be used, unless indicated otherwise.
- This question paper consists of 17 pages. Statistical tables and formula sheets may be found on pages 15 to 17.
- All necessary calculations should be shown.
- All angles are in radians and answers should also be given in radians.
- Handwriting should be legible.

SECTION A
COMPULSORY
CALCULUS

QUESTION 1

1.1 Sketch on the same system of axes the graphs of f and g where

$f : x \to arc\sin x + \frac{\pi}{2}$ and $g : x \to arc\cos x$ (12)

1.2 Answer this question without the use of a calculator:

1.2.1 Calculate $\arccos(\cos\frac{7p}{6})$ (6)

1.2.2 Calculate $\sin(\arctan m + \arctan\frac{1}{m})$ (12)

[30]

QUESTION 2

2.1 $f(x) = \begin{cases} px + 2q - 4 & if \quad x < 2 \\ qx^2 - px + p & if \quad x \geq 2 \end{cases}$ f is differentiable for all x.

Determine the values of p and q. (12)

2.2 $g(x) = \begin{cases} x^2 - 4 & \text{if} \quad x < 3 \\ 4 & \text{if} \quad x = 3 \\ x + 2 & \text{if} \quad x > 3 \end{cases}$

2.2.1 Determine if g is continuous at $x = 3$, and substantiate fully. If not continuous, state the type of discontinuity. (10)

2.2.2 Determine, with substantiation, if g is differentiable at $x = 3$. (4)

[26]

QUESTION 3

3.1 Determine the following limits and show if the limit does not ex ist:

3.1.1 $\lim\limits_{n\to\infty} \dfrac{2+n-4n^3}{5+2n^3}$ (4)

3.1.2 $\lim\limits_{x\to 9^-} \dfrac{x^2-81}{|x-9|}$ (10)

3.2 3.2.1 Determine $f'(x)$ from first principles if $f(x)=\dfrac{1}{\sqrt{2-3x}}$ (10)

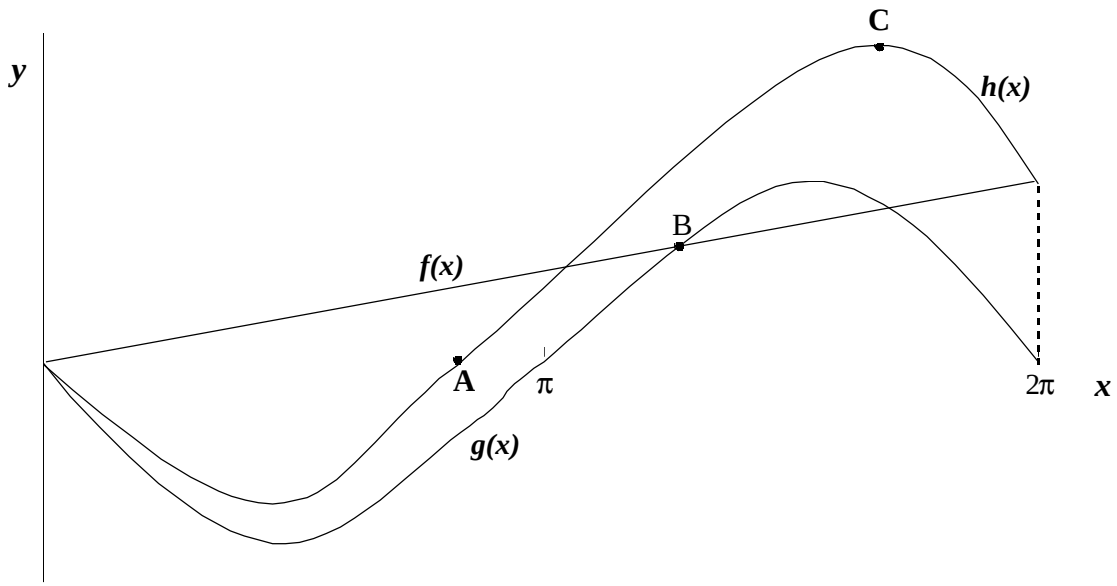
3.2.2 Determine $g'(1)$ if $g(x)=\dfrac{x^2+1}{(x^5-x^3+x)}$ (12)

3.3 Determine a formula for the n-th derivative of $f(x)=(1-5x)^{40}$, $n\le 40$. (10)

[46]

QUESTION 4

Three graphs are drawn in the sketch below for $0\le x\le 2\pi$:
$f(x)=\frac{1}{2}x$ and $g(x)=-3\sin x$ and $h(x)=f(x)+g(x)$.
Three points **A**, **B** and **C** (shown on t he sketch) are to be found.



4.1 Write down Newton's Formula for finding the x co-ordinate of point **A**. Suggest a suitable starting value. Do not calculate the answer. (6)

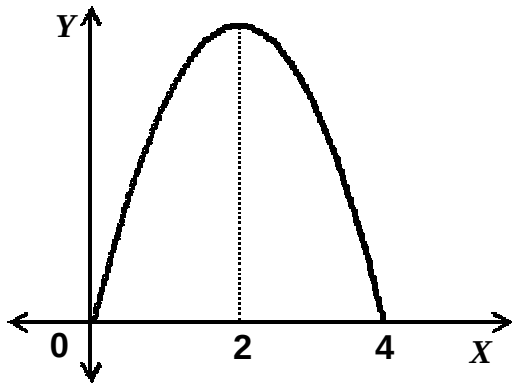
4.2 Write down Newton's Formula for finding the x co-ordinate of **B**. (Do not calculate the answer!) (6)

4.3 Write down Newton's Formula for finding the x co-ordinate of **C**, the maximum turning point of $h(x)$. (Do not calculate the answer!) (6)

[18]

QUESTION 5

The sketch below shows the graph of the curve $y = 4x - x^2$ above the **X**-axis.



5.1 Use the Riemann Sum Formula where $n \to \infty$ to find the area between the graph of the curve $y = 4x - x^2$ and the positive **X**-axis for $x < 4$. (20)

5.2 Allow this curve to be rotated about the **X**-axis and determine the volume of the solid of rotation which is generated. Leave your answer in terms of π. (12)

[32]

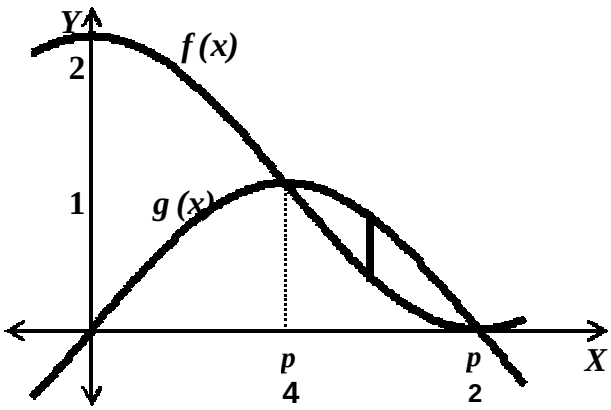
QUESTION 6

6.1 Determine the following integrals:

6.1.1 $\int \frac{\sin\theta}{\sqrt{1-\cos\theta}}\, d\theta$ (8)

6.1.2 $\int \left(\frac{1}{9t^2 + 6t + 2} \right) dt$ (8)

6.2 The curves of $f(x) = 2\cos^2 x$ and $g(x) = \sin 2x$ are given below:



6.2.1 Determine the area of the region between $f(x)$ and $g(x)$ from $x = \frac{\pi}{4}$ to $x = \frac{\pi}{2}$. (16)

6.2.2 AB is a vertical line representing the maximum distance between $f(x)$ and $g(x)$ in the interval $\frac{\pi}{4} \leq x \leq \frac{\pi}{2}$. Determine the value of x at which this maximum occurs, and show that it is indeed a maximum. (16)

[48]

TOTAL FOR SECTION A: [200]

Answer any TWO of the following FOUR sections.

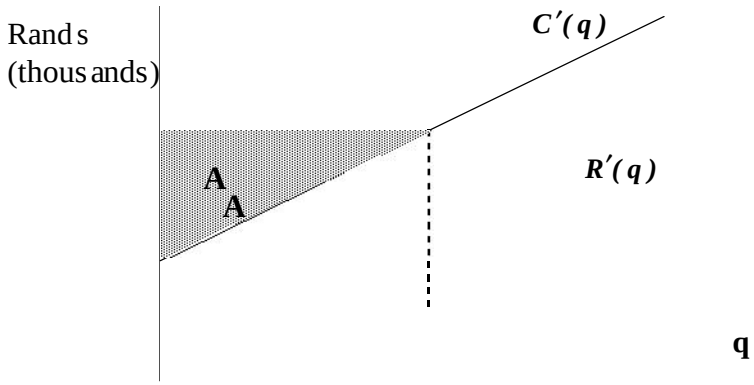
SECTION B
FINANCIAL MATHEMATICS

QUESTION 7

The marginal cost and revenue functions (in thousands of Rands) for a company are

$C'(q) = mq + 10$
$R'(q) = 30$ where q represents the number of items produced.

These functions are illustrated below:



7.1 What does the area **A** represent? (2)

7.2 If a total profit of 100 thousand rands is made when 200 items are being produced, calculate **m**. There is no fixed cost. (12)
[14]

QUESTION 8

8.1 An amount of R1 000 is invested for 5 years at a compound interest rate of 13% per annum. What will the value of the investment be after 5 years? Give your answer to the nearest cent. (4)

8.2 How much (to the nearest cent) should be invested at a simple interest rate of 13% per annum to yield the same amount after 5 years? (6)
[10]

QUESTION 9

Mr Thanksdad wants to invest R30 000 now to cater for a 21st birthday present for each of his 3 daughters when they turn 21. They have presently just turned 3, 5 and 8 and each must receive the same amount. How much will each receive if interest is at 12,5% compounded monthly? **[12]**

QUESTION 10

10.1 A school wants to replace its photocopy machine after five years with a new one. The value of the machine is currently R8 000 and they estimate that a new machine will cost R13 480 in five years' time. Calculate the compound yearly inflation rate that the school used in their calculations. Give the answer correct to a whole number. (4)

10.2 They realise that they can swop the old machine. They estimate that the value of the old machine will decrease with 11% p.a. on the book value. Show that they will need R9 012,75 in five years' time. (6)

10.3 They save monthly with a bank and get 9% p.a. interest, compounded monthly, on the account. They will start saving in one month's time, and their last payment will be two months before the new machine is bought. Calculate how much their monthly deposits should be. (10)

[20]

QUESTION 11

I take out a home loan from a bank to fund the purchase of a house costing R725 000. The interest charged is 12% per annum compounded monthly.

11.1 If I can afford to pay back R6 000 per month, will I ever be able to amortise the loan? Explain your answer. (6)

11.2 I do some creative budgeting and manage to scrape together R8 000 per month. How long will I take to amortise the loan? Payments are made at the end of each month, starting in one month's time. (10)

11.3 After 10 years I find I can't afford to carry on paying back R8 000 a month. How much do I still owe immediately after the 120th payment? (10)

11.4 Fortunately, a rich aunt hears of my predicament described in Question 11.3 and offers to give me R5 000 at the beginning of each year for 5 years to help pay off the balance of R552 471,00 on the house. These yearly payments, together with my lesser monthly amount x, paid at the end of each month, pay off the loan in a further 10 years.

Determine x to the nearest cent. (18)

[44]

TOTAL FOR SECTION B: [100]

SECTION C
ANALYTICAL GEOMETRY

QUESTION 12

Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ be the canonical equation of an ellipse with eccentricity $e=\frac{\sqrt{5}}{3}$ and focus $(\sqrt{5};0)$.

12.1 Show that $a=3$ and $b=2$. (12)

12.2 Find the equation of the normal to the ellipse at the point $P\left(\sqrt{3};\sqrt{\frac{8}{3}}\right)$. (14)

12.3 Find the angle between the normal at P and the diameter of the ellipse which passes through P. Give the answer in degrees. (14)

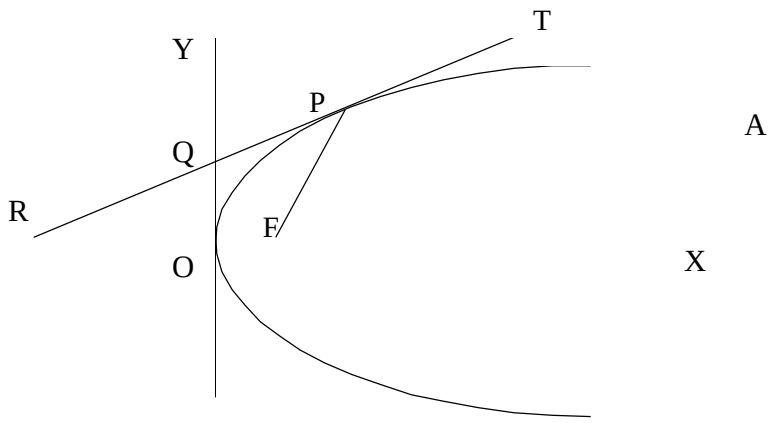
[40]

QUESTION 13

13.1 Show that $x=at^2$ and $y=2at$; $t\in R$ are parametric equations for $y^2=4ax$. (6)

13.2 Prove that the equation of the tangent to $y^2=4ax$ at $(at^2;2at)$ is given by $x-ty+at^2=0$. (10)

13.3 In the figure, P is any point on the parabola $y^2 = 4ax$. RPT is a tangent. PA is parallel to the X-axis. F is the focus of the parabola. The tangent TP through point P cuts the X-axis at R.



Prove that $A\hat{P}T = F\hat{P}R$. (18)

[34]

QUESTION 14

14.1 Given the points $A(0\ ;\ -1\ ;\ -1)$; $B(2\ ;\ -2\ ;\ 2)$; $C(-3\ ;\ 8\ ;\ 7)$ and $D(-1\ ;\ 2\ ;\ 0)$:

14.1.1 Find the equation of the plane through the points **A**, **B** and **C**. (12)

14.1.2 Show that the points **A**, **B**, **C** and **D** are co-planer. (2)

14.1.3 Prove that ABCD is a parallelogram. (6)

14.2 If **P** is the point $(1\ ;\ 2\ ;\ 3)$, give the equation of the line **AP**. (2)

14.3 Calculate the distance from **P** to the plane ABC. (4)

[26]

TOTAL FOR SECTION C: [100]

SECTION D
ALGEBRA

QUESTION 15

15.1 State the conjugate surd theorem. (6)

15.2 If $-1+2\sqrt{3}$ is a zero of $f(x)$, factorize $f(x)=x^5+5x^4-2x^3-25x^2-29x-22$ fully in $\mathbb{Q}[x]$. (20)

15.3 Write down a polynomial with integer coefficients that meets the following conditions: It should be of grade 3, have 4 terms and be irreducible in Q[x] according to Eisenstein (it is not necessary to show this). (4)

[30]

QUESTION 16

16.1 Use mathematical induction to prove 8^n-7n+6 is divisible by 7, for all natural numbers $n\in N$. (16)

16.2 If the zeroes of the polynomial $4x^3+3x^2+2x+1$, are ***a, ß and ?***, find the coefficients of the third degree polynomial of which the zeroes are $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$; $\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\alpha\gamma}$ and $\frac{1}{\alpha\beta\gamma}$. (18)

[34]

QUESTION 17

$$f(x)=x+5+\frac{7}{x}+\frac{3}{x^2}$$

17.1 Write down the asymptotes of the graph of $y=f(x)$. (4)

17.2 Calculate at which x-value the graph cuts its oblique asymptote. (4)

17.3 The graph of $f(x)$ has x-intercepts at $x=-1$ and $x=-3$. It has minimum turning points at (-1; 0) and at (3; 10,7), and a maximum turning point at (-2; ¼). Sketch the graph of $f(x)$, showing all asymptotes and intercepts with the axis. It is not necessary to show the y-values of the stationary points. (12)

[20]

QUESTION 18

Decompose $\dfrac{3x^2+8x-3}{(x+1)^2(x-1)}$ into partial fractions. **[16]**

TOTAL FOR SECTION D: [100]

SECTION E
STATISTICS

Give answers correct to 4 decimal digits where applicable.

QUESTION 19

19.1 Six numbers between 1 and 49 are chosen for a lotto ticket. Order does not matter. A ticket costs R5. How much money must you spend this week to be 50% certain of winning the jackpot? (8)

19.2 Joe's car runs out of petrol between the filling station and his house. On the way back to his car, a stranger approaches him and asks for money. In his pocket Joe has three R10 notes, four R20 notes, two R50-notes and one R100 note. He randomly takes a note out of his pocket.

19.2.1 He wants to lose the smallest possible amount. What is the probability that he will take a R10 note out of his pocket? (2)

19.2.2 The note which he takes out is a R20 note and he gives it to the beggar, but the person is not satisfied and asks for R50. What is the probability that Joe will now take a R50 note out of his pocket? (2)

19.2.3 What is the probability that the total amount would have been R80 if Joe took out three notes simultaneously from his pocket? (8)

19.3 $P(A) = 0,6$; $P(B) = x$ and $P(A \cup B) = 0,88$.
Calculate x if A and B are independent events. (4)
[24]

QUESTION 20

Amanda must write a "true or false" test on ancient Greek literature, but she knows nothing about the topic. She decides to toss a coin before she answers each question. Heads or tails will then determine whether she should answer true or false for each of 30 questions.

20.1 What is the probability (to 4 significant figures) that she will answer 20 questions correctly? (8)

20.2 Determine the probability that she will answer all the questions correctly. Leave your answer in exponential form. (4)

20.3 What is the probability that she will answer at least one question correctly? Do not round off your answer. (6)

[18]

QUESTION 21

Out of a class of 100 students, the following information is obtained over a period of a year after their studies are completed:

(i) 40 are women and 60 are men

(ii) 30 of these students went to work in London after completing their studies

(iii) 50 students got married in this year, of which 35 are men

(iv) 20 got married in London

(v) 10 female students went to work in London

(vi) x is the number of women who got married in London

(vii) 20 are men who are not married and have not worked in London

21.1 Present this information on a Venn diagram.
Use the following sets in the diagram:
W: this is a woman
L: these are students who went to work in London
M: these are the students who got married within the specific year (16)

21.2 Determine how many women got married in London. (4)

[20]

QUESTION 22

The East-Gauteng Centre for Additional Mathematics only wants to enrol learners who are in the top 5% of the population. It is known that IQ is normally distributed, with a mean of 100 and a standard deviation of 15.

22.1 What IQ should be used as the minimum requirement for acceptance? (10)

22.2 If the IQs of 1 500 learners in a big school are also normally distributed, how many learners can be accepted into the centre? (4)

22.3 Learners with an IQ of less than 85 may struggle with Mathematics. Determine the percentage of learners in this category. (8)

[22]

QUESTION 23

A manufacturer of medical products claims that at least 99% of the condoms which they manufacture are effective. This claim is questioned by a group of women.

A sample of 500 condoms is randomly chosen and tested. It is found that 10 of these have defects, and are thus not 100% safe for use.

23.1 Find a point estimate for the proportion of effective condoms. (4)

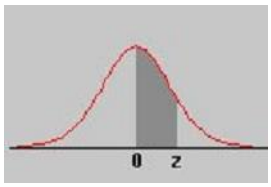
23.2 Determine a 95% confidence interval for the proportion of condoms which are at least 99% effective (accurate to 3 decimals). (12)

[16]

TOTAL FOR SECTION E: [100]

TOTAL: 400

Normal Distribution/ *Normaalverspreiding*



$$P(X \le x) = \frac{1}{\sqrt{2p}} \int_{-\infty}^{x} e^{\frac{-x^2}{2}} dx$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0		0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

FORMULA SHEET/ *FORMULEBLAD*

Differential and Integral Calculus

Differensiaal- en Integraalrekene

$s = r\theta$

$\sin^2 x = \frac{1}{2}(1-\cos 2x) \quad \cos^2 x = \frac{1}{2}(1+\cos 2x)$

$\sin A.\cos B = \frac{1}{2}(\sin(A+B)+\sin(A-B))$

$\sin A.\sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$

$\cos A.\cos B = \frac{1}{2}(\cos(A-B)+\cos(A+B))$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(2n+1)(n+1)}{6}$$

$$a_{n+1} = a_n - \frac{f(a_n)}{f'(a_n)}$$

$$V = \pi \int_a^b [f(x)]^2 dx$$

$$Riemann\ Sum = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x_i$$

$F(x)$	$F'(x)$
$a.x^n$	$na.x^{n-1}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x.\tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x.\cot x$
$\arcsin x$ $\operatorname{bgsin} x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$ $\operatorname{bgcos} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\arctan x$ $\operatorname{bgtan} x$	$\frac{1}{x^2+1}$
$f(x).g(x)$	$f'(x).g(x)+f(x).g'(x)$
$\frac{f(x)}{g(x)}$	$\frac{f'(x).g(x)-f(x).g'(x)}{[g(x)]^2}$
$f(g(x))$	$f'(g(x)).g'(x)$

Finance/ *Finansies*

$F=P(1+i)^n \qquad F=P(1-i)^n$

$F=P(1+in) \qquad F=P(1-in)$

$$P = x.\frac{1-(1+i)^{-n}}{i} \qquad F = x.\frac{(1+i)^n-1}{i}$$

Analytical Geometry/ *Analitiese Meetkunde*

$y=4ax^2 \qquad yy_1=2a(x+x_1)$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad \frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$$

Algebra

$$\alpha+\beta=-\frac{b}{a} \qquad \alpha+\beta+\gamma=-\frac{b}{a}$$

$$\alpha.\beta=\frac{c}{a} \qquad \alpha\beta+\beta\gamma+\alpha\gamma=\frac{c}{a}$$

$$\alpha.\beta.\gamma=-\frac{d}{a}$$

Statistics / *Statistiek*

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$${}_nP_r = \frac{n!}{(n-r)!} \qquad {}_nC_r = \frac{n!}{(n-r)!r!}$$

$$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$P(X=x) = \frac{\binom{p}{x}\binom{N-p}{n-x}}{\binom{N}{n}}$$

$$z = \frac{X-\mu}{\sigma}$$

$$P\left(X - 1.96\frac{\sigma}{\sqrt{n}} < \mu < X + 1.96\frac{\sigma}{\sqrt{n}}\right) = 0.95$$

$$P\left(p - 1.96\sqrt{\frac{p(1-p)}{n}} < \pi < p + 1.96\sqrt{\frac{p(1-p)}{n}}\right) = 0,95$$

Wiskun de For mu leblad / Mathematics For mula Shee t

1. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

2. $T_n = a + (n-1)d$

3. $S_n = \dfrac{n}{2}(a + l)$

4. $S_n = \dfrac{n}{2}[2a + (n-1)d]$

5. $T_n = ar^{n-1}$

6. $S_n = \dfrac{a(1 - r^n)}{1 - r}$

7. $S_n = \dfrac{a(r^n - 1)}{r - 1}$

8. $S_\infty = \dfrac{a}{1 - r}$

9. $A = P(1 + \dfrac{r}{100})^n$

10. $A = P(1 - \dfrac{r}{100})^n$

11. $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

12. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

13. $y = mx + c$

14. $y - y_1 = m(x - x_1)$

15. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

16. $m = \tan\theta$

17. $(\dfrac{x_1 + x_2}{2} ; \dfrac{y_1 + y_2}{2})$

18. $y^2 + x^2 = r^2$

19. $(x - p)^2 + (y - q)^2 = r^2$

20. $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

21. $a^2 = b^2 + c^2 - 2bc.\cos A$

22. $\text{area } \Delta ABC = \frac{1}{2}ab.\sin C$

23. $\cos(A + B) = \cos A.\cos B - \sin A.\sin B$

24. $\sin(A + B) = \sin A.\cos B + \cos A.\sin B$

25. $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

26. $\cos 2A = \cos^2 A - \sin^2 A$

27. $\sin 2A = 2\sin A \cos A$

END / *EINDE*

END / *EINDE*