Additional Mathematics HG

Possible Answers Feb/Mar 2005

302-1/0

Section A

Question 1

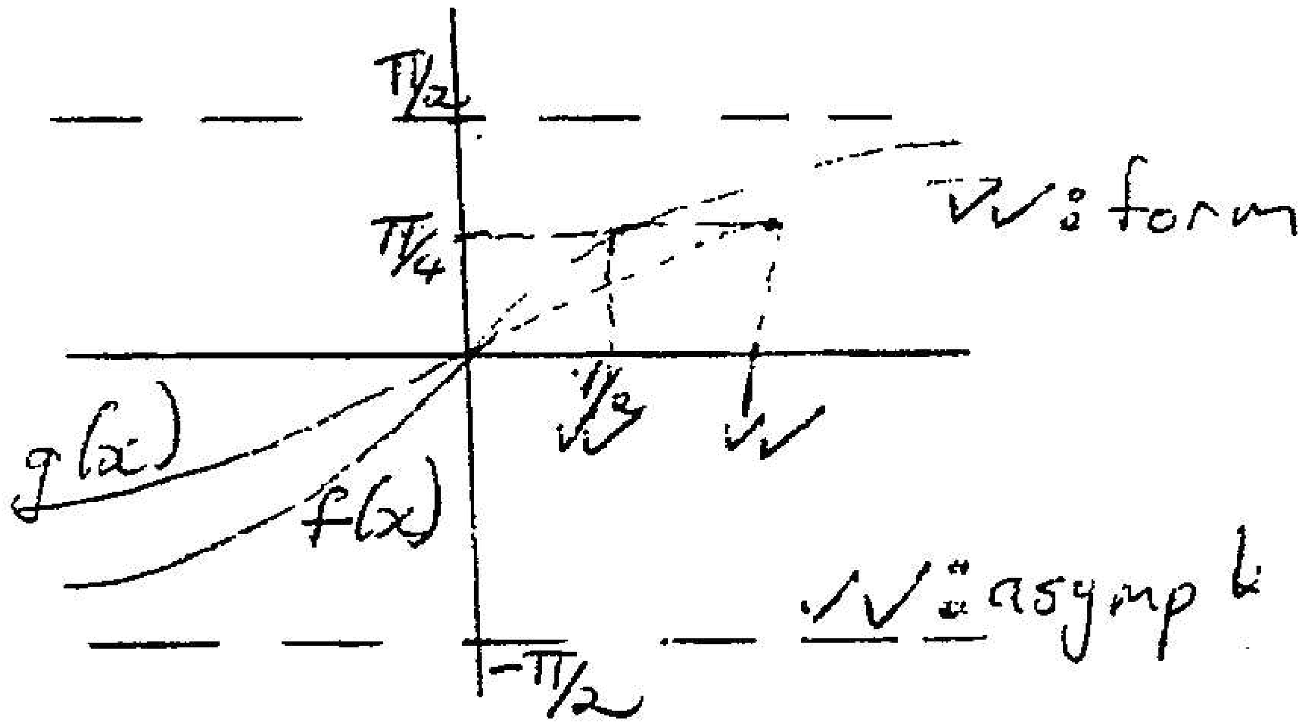
1.1

$y = \arctan 2x$

$2x = \tan y$

$x = \frac{1}{2}\tan y$

or on graph.



(1)

1.2.1 $2\arctan(-\frac{1}{\sqrt{3}}) = 2 \times -\frac{\pi}{6} = -\frac{\pi}{3}$ (4)

1.2.2 $\arcsin(\sin \frac{3\pi}{2}) = \arcsin(-\sin\frac{\pi}{2}) = \arcsin(-1) = -\frac{\pi}{2}$ (6)

or $\arcsin(\sin -\frac{\pi}{2}) = -\frac{\pi}{2}$

1.2.3 $\tan(\arccos(-\frac{\sqrt{2}}{2})) = \tan(\pi - \frac{\pi}{4}) = \tan\frac{3\pi}{4} = -1$ (6)

Question 2

2.1.1 $\lim_{x\to -3^-} f(x) = -1$ and $\lim_{x \to -3^+} f(x) = 1$

∴ Not continuous; Jump (8)

2.1.2 $\lim_{x\to -1^-} f(x) = -1$ and $\lim_{x\to -1^+} f(x) = -1$

and $f(-1) = -1$

∴ Continuous at $x=-1$ (8)

2.1.3 $f(2)$ ∄ ∴ Not continuous: Removable (6)

2.2 $\lim_{x\to -1^-} f'(x) = -2$ and $\lim_{x\to -1^+} f'(x) = \lim_{x\to -1^+} 2x = -2$

∴ Differentiable at $x = -1$. (8)

Question 3

3.1 $\frac{x^2-4}{|x-2|} = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x \geq 2 \\ \frac{x^2-4}{-x+2} & \text{if } x < 2 \end{cases}$

$\lim_{x\to 2^+} \frac{x^2-4}{x-2} = \lim_{x\to 2^+} \frac{(x-2)(x+2)}{x-2} = 4$

$\lim_{x\to 2^-} \frac{x^2-4}{-x+2} = -4$

∴ Limit does not exist. (10)

3.2.1 $(g\circ f)(x) = \text{bg}\cos(\sqrt{2x+1})$ (4)

3.2.2 $D_x(g\circ f(x)) = \frac{-1}{\sqrt{1-(2x+1)}} \cdot \frac{1}{2}(2x+1)^{-1/2} \cdot 2$

$= \frac{-1}{\sqrt{-2x}} \cdot \frac{1}{\sqrt{2x+1}}$

$D_x(g\circ f(-\frac{1}{4})) = \frac{-1}{\sqrt{1/2}} \cdot \frac{1}{\sqrt{1/2}} = -2$ (10)

3.3 $h'(x) = \frac{2\sin 3x . 3 . \sin x - (1-\cos 3x) 2\sin x}{4\sin^2 x}$ (6)

3.4 $y = (1+2x)^{-1}$

$\frac{dy}{dx} = -(1+2x)^{-2} . 2$

$\frac{d^2y}{dx^2} = 2(1+2x)^{-3} . 2 . 2$

$\frac{d^3y}{dx^3} = -3 . 2(1+2x)^{-4} . 2 . 2 . 2$

$\frac{d^ny}{dx^n} = (-1)^n n! (1+2x)^{-n-1} . 2^n$ (12)

Question 4.

4.1. 0,8 is very near the stationary point, 1. If you draw a tangent here, it cuts the x-axis on the left of −1, which gives the next approximation. If he carried on, he would obtain the x-intercept B. (8)

4.2. 1 is a stationary point, which means that a tangent drawn here will be ~~paralle~~ parallel to the x-axis. OR Here $f'(x) = 0$, this results in dividing by zero. (6)

4.3. $$a_2 = a_1 - \frac{f(a_1)}{f'(a_1)}$$
$$= a_1 - \frac{3a_1^4 - 4a_1^3 - 6a_1^2 + 12a_1 - 2}{12a_1^3 - 12a_1^2 - 12a_1 + 12}$$ (6)
$$= 0{,}1689$$

Question 5

5.1. $$\frac{2\cos^3 3x}{3 \cdot 3} + k$$ (8)

5.2. $$\left.\frac{\arcsin 2x}{2}\right|_0^{\sqrt{3}/4} = \frac{\arcsin 2 \cdot \sqrt{3}/4}{2} - \frac{\arcsin 0}{2}$$ (10)
$$= \frac{\arcsin \sqrt{3}/2}{2} = \frac{\pi}{6} \quad \pi/3/2$$

Question 6.

6.1. $$\int_0^1 x\sqrt{1-x^2}\,dx = \left.\frac{(1-x^2)^{3/2}}{3/2 \cdot (-2)}\right|_0^1 = 0 - \frac{-(1-0)^{3/2}}{3} = 1/3$$ (12)

6.2. $$V = \pi\int_0^1 x^2(1-x^2)\,dx$$ (10)
$$= \pi\int_0^1 (x^2 - x^4)\,dx$$
$$= \pi\left(\frac{x^3}{3} - \frac{x^5}{5}\right)\Big|_0^1 = \pi\left(\frac{1}{3} - \frac{1}{5}\right) - 0 = \frac{2}{15}\pi$$

Question 7.

$\Delta x_i = \frac{2}{n}$ $\quad x_0 = 0$, $x_1 = \frac{2}{n}$, $x_i = \frac{2i}{n}$ $\quad f(x_i) = 2\left(\frac{2i}{n}\right) - \left(\frac{2i}{n}\right)^2 = \frac{4i}{n} - \frac{4i^2}{n^2}$

$$f(x_i) \cdot \Delta x_i = \frac{8i}{n^2} - \frac{8i^2}{n^3}$$
$$\sum_{i=1}^{n} f(x_i) \cdot \Delta x_i = \sum \frac{8i}{n^2} - \sum \frac{8i^2}{n^3}$$
$$= \frac{8}{n^2} \cdot \frac{n}{2}(n+1) - \frac{8}{n^3} \cdot \frac{n}{6}(2n+1)(n+1)$$ (18)
$$= 4 + \frac{4}{n} - \frac{8}{3} - \frac{12}{3n} - \frac{4}{3n^2}$$
$$\therefore RS: \lim_{n\to\infty} \sum f(x_i) \cdot \Delta x_i = \frac{4}{3}$$

Question 8

8.1. $V = \frac{1}{4} \cdot \pi(2x)^2 \cdot y \quad \therefore 2 = \frac{1}{4}\pi \cdot 4x^2 y$ (4)
$$y = \frac{2}{\pi x^2}$$

8.2. $$A = \frac{1}{4} \cdot \pi(2x)^2 + y \cdot \frac{1}{4} \cdot 2\pi(2x) + 2 \cdot 2x \cdot y$$ (6)
$$= \pi x^2 + xy(4+\pi)$$

8.3. $$A = \pi x^2 + x \cdot \frac{2}{\pi x^2}(4+\pi)$$
$$= \pi x^2 + x^{-1}\frac{(4+\pi)}{\pi} \cdot 2$$
$$A' = 2\pi x - \frac{2}{\pi}(4+\pi)x^{-2} = 0$$ (14)
$$2\pi x^3 = \frac{2}{\pi}(4+\pi)$$
$$x^3 = \frac{1}{\pi^2}(4+\pi)$$
$$x = 0{,}898$$

Section B

Question 9.

9.1 T_0: x — T_2: $2x$ — T_3: $3x$ — T_6: 60900.35

$i = 0{,}125$ $\quad$ $i = 0{,}15$

$60900.35 = 3x(1{,}15)^3 + 2x(1{,}125)(1{,}15)^3 + x(1{,}125)^3(1{,}15)^3$ (12)

$x(10{,}1500\ldots) = 60900.35$

$x = R6000$

9.2. $3000(1{,}07)^{42} = 51432.77$ No. (8)

Question 10

10.1. $289 \times 36 = 10404$

$10404 = 7500(1+3i)$

$i = 0{,}129 \quad \therefore r = 12{,}9\%$ (6)

10.2. $7500 = x\left(\dfrac{1-\left(\frac{.155}{12}+1\right)^{-36}}{\frac{.155}{12}}\right)$ form (10)

$x = R261.83.$

10.3. $BO = 7500(1+i)^{12} - x\left(\dfrac{(1+i)^{12}-1}{i}\right)$ (8)

$= R5373.68$

10.4. $5373.68(1+i)^3 = 261.83\left(\dfrac{1-(1+i)^{-n}}{i}\right)$ (12)

$(1+i)^{-n} = 0{,}7244\ldots$

$-n = \dfrac{\log 0{,}7244\ldots}{\log(1+i)}$

$n = 25$ months.

Question 11

11.1. $350000 - 270000(1-0{,}095)^5$ {163910.46} (6)

$= 186089.54$

11.2. $187000 + 5000\left[\dfrac{(1+i_*)^4-1}{i_*}\right](1+i_*) =$

$x\left(\dfrac{1{,}03^{20}-1}{{,}03}\right)$ form (20)

and $(1+i_*)^1 = (1{,}03)^4 \quad \therefore i_* = 0{,}1255\ldots$

$\therefore x(26{,}87\ldots) = 187000 + 27113.73$

$x = R7968.39$

Question 12.

12.1. $W = -45(5)^2 + 130(5) - (10.5^3 - 30.5^2 + 40.5 + 1000)$

$= -2175.$

12.2. Maks W as $C' = R'$

$\therefore 30x^2 - 60x + 40 = -90x + 130$

$\cancel{1200} \quad 30x^2 + 30x - 90 = 0$

$x^2 + x - 3 = 0$

$x^2 + x + 1/4 = 3 + 1/4$

$(x + \tfrac{1}{2}) = \pm\sqrt{13/4}$

$x = -1/2 \pm \sqrt{13/4}$

$x = 1{,}30$

ANALYTICAL GEOMETRY 1

Question 13

13.1 $x = \sec\theta$ $\quad$ $y = 2\tan\theta$

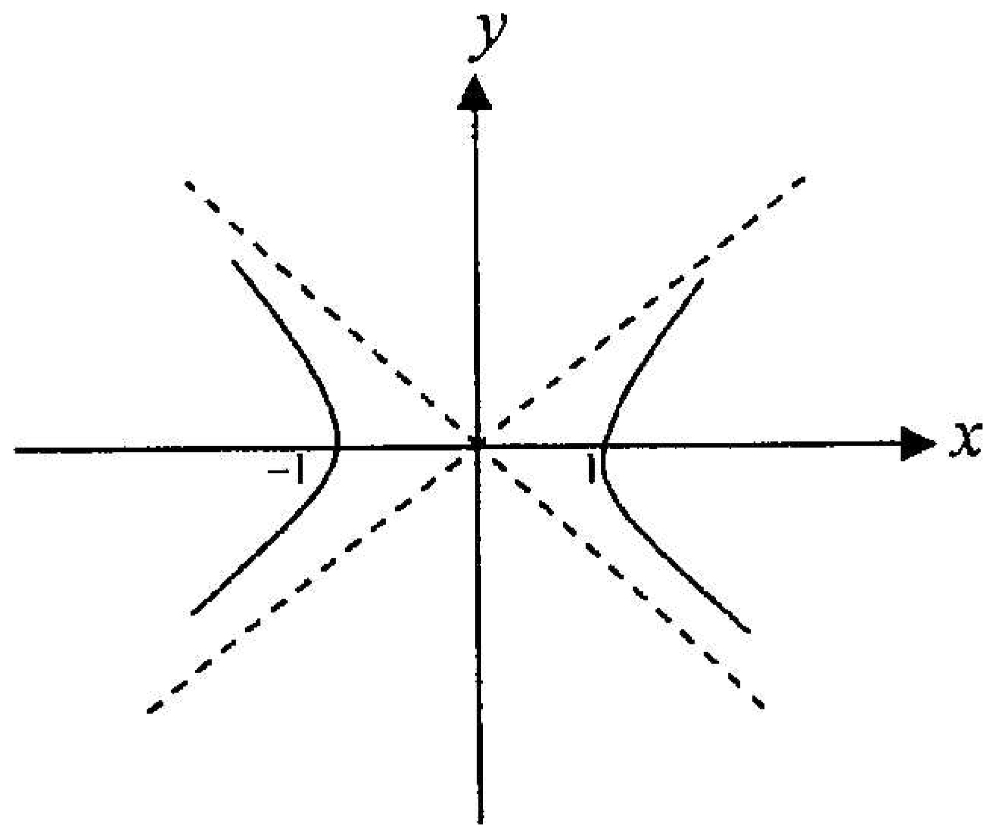
$a = 1 \qquad b = 2$ $\quad$ a hyperbola

$$\frac{x^2}{1} - \frac{y^2}{4} = 1 \qquad (4x^2 - y^2 = 4)$$

13.2

Asymptotes : $y = \pm \frac{b}{a}x$

$y = \pm 2x$



Question 14

14.1 $\frac{(x-1)^2}{25} + \frac{(y+2)^2}{9} = 1$

$a^2 = 25 \quad b^2 = 9$

$a = 5 \qquad b = 3$

$e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{9}{25} = \frac{16}{25}$

$e = \frac{4}{5}$

14.2

$\frac{x^2}{25} + \frac{y^3}{9} = 1$

$(\pm ae\ ;\ 0)$

$(\pm 4\ ;\ 0)$

$(5\ ;\ -2) \qquad (-3\ ;\ -2)$

Question 15

15.1 $P(2t^2\ ;\ 4t)$ $\qquad y' = \frac{2a}{y_1} = \frac{4}{4t} = \frac{1}{t}$

$y - 4t = \frac{1}{t}(x - 2t^2)$

$y = \frac{x}{t} - 2t + 4t$

$y = \frac{1}{t}x + 2t$

directrix: $x = -a \qquad \therefore\ x = -2$

$y = \frac{1}{t}(-2) + 2t$

$$y = \frac{-2}{t} + 2t$$

$$\underrightarrow{\left(-2 \; ; \; \frac{-2}{t} + 2t\right)}$$

15.2 $y^2 = 8x$ $\quad\quad$ $(2 \; ; \; 4)$

15.2.1 $\quad a = 2$

tgt: $yy_1 = 2a(x + x_1)$

$$m_{tgt} = \frac{2a}{y_1} = \frac{4}{4} = 1$$

$m_\perp = -1$

$y - 4 = -1(x - 2)$

$y = x + 2 + 4$

$y = \underrightarrow{-x + 6}$

15.2.2 $(-x + 6)^2 = 8x$

$x^2 - 12x + 36 = 8x$

$x^2 - 20x + 36 = 0$

$(x - 18)(x - 2) = 0$

$x = 18$ or $x = 2$

$\therefore x = 18$

ie $\underrightarrow{(18 \; ; \; -12)}$

15.3

15.3.1 $\quad A(2 \; ; \; 0)$

$m_\perp = -t$

$y - 0 = -t(x - 2)$

$y = -tx + 2t$ $\quad\quad\quad\quad$ $y = \frac{1}{t}x + 2t$

$$-tx + 2t = \frac{1}{t}x + 2t$$

$-t^2x + 2t^2 = x + 2t^2$

$x + t^2x = 0$

$x(1 + t^2) = 0$

$x = 0$

$\underrightarrow{Q(0 \; ; \; 2t)}$

15.3..2 $P(2t^2 \; ; \; 4t)$ $\quad\quad\quad\quad$ $Q(0 \; ; \; 2t)$

$$R\left(\frac{2t^2}{2} \; ; \; \frac{6t}{2}\right)$$

$R(t^2 \; ; \; 3t)$

$x = t^2$ $\quad\quad$ $y = 3t$

$$\left(\frac{y}{3}\right)^2 = x$$

$$9x = y^2$$

QUESTION 16

16.1 $A(1\ ;\ 4\ ;\ 6) \qquad B(2\ ;\ 7\ ;\ 5) \qquad C(-3\ ;\ 8\ ;\ 7)$

$AB : 1\ ;\ 3\ ;\ -1$

$AC : -4\ ;\ 4\ ;\ 1$

$a + 3b - c = 0$

$-4a + 4b + c = 0$

$-3a + 7b = 0$

$7b = 3a$

$b = \frac{3a}{7}$

$7\ ;\ 3\ ;\ 16$ normal

$7(x - 1) + 3(y - 4) + 16(z - 6) = 0$

$7x - 7 + 3y - 12 + 16z + 96 = 0$

$7x + 3y + 16z - 115 = 0$

16.2 $3x - y - 4z = 7 \qquad 3\ ;\ -1\ ;\ -4$

$2x + 3y - z = 11 \qquad 2\ ;\ 3\ ;\ -1$

$$\cos\phi = \frac{(3)(2) + (-1)(3) + (-4)(-1)}{\sqrt{9 + 1 + 16}\sqrt{4 + 9 + 1}}$$

$$= \frac{6 - 3 + 4}{\sqrt{26}\sqrt{14}}$$

$$\frac{7}{\sqrt{364}} = 0{,}3668\ldots$$

$$\therefore \phi = 64{,}48^0$$

Question 17

$$\begin{cases} x = 1 + 2s \\ y = 1 + s \\ z = s \end{cases} \quad \text{and} \quad \begin{cases} x = 3 + t \\ y = 2 - t \\ z = 4 + 2t \end{cases}$$

Solve: $1 + 2s = 3 + 2t \leftarrow; 1 + s = 2 - t \uparrow; s = 4 + 2t \rightarrow$

$\rightarrow$ in $\uparrow$: $1 + (4 + 2t) = 2 - t$

$5 + 2t = 2 - t$

$3t = -3$

$t = -1$

$\therefore s = 2$

into $\leftarrow$ $\quad LHS = 1 + 4 \qquad RHS = 3 + (-1)$

$= 5 \qquad = 2$

not a solution

$\therefore$ no intersection

Section D

Question 18

18.1. As $a+b\sqrt{c}$, met a, b en $c \in Q$, 'n irrasionale nulpunt is van $f(x) \in Z(x)$, dan sal $a-b\sqrt{c}$ ook 'n nulpunt wees (6)

18.2. $\therefore x+3-\sqrt{2}$ is a factor and $-3-\sqrt{2}$ a zero.

$\therefore (x+3-\sqrt{2})(x+3+\sqrt{2})$ a factor

$= x^2+6x+9-2 = x^2+6x+7$

$$\begin{array}{ccc|ccccc} & & & 1 & 0 & -4 & & \\ 1 & 6 & 7 & 1 & 6 & 3 & -24 & -28 \\ & & & 1 & 6 & 7 & & \\ & & & & & -4 & -24 & -28 \\ & & & & & -4 & -24 & -28 \end{array}$$

$\therefore (x^2+6x+7)(x^2-4)$

$= (x^2+6x+7)(x-2)(x+2)$ (14)

Question 19

19.1.

$$\begin{array}{ccc|cccc} & & & 1 & -2 & & \\ 1 & 2 & 1 & 1 & 0 & 0 & -5 \\ & & & 1 & 2 & 1 & \\ & & & & -2 & -1 & -5 \\ & & & & -2 & -4 & -2 \\ & & & & & 3 & -3 \end{array}
\qquad
\begin{array}{cc|ccc} & & 1/3 & 1 & \\ 3 & -3 & 1 & 2 & 1 \\ & & 1 & -1 & \\ & & & 3 & 1 \\ & & & 3 & -3 \\ & & & & 4 \end{array}$$

$\therefore$ 4 is HCF. (10)

19.2. $g = f.(x-2) + 3x - 3$ ... ①

$f = (3x-3)(\tfrac{1}{3}x+1) + 4$ ... ②

① in ②: $f = [g - f.(x-2)](\tfrac{1}{3}x+1) + 4$

$4 = f[1 + \tfrac{1}{3}x^2 + \tfrac{1}{3}x - 2] + g(\tfrac{1}{3}x+1)$ 12

$4 \equiv f(x).(\tfrac{1}{3}x^2 + \tfrac{1}{3}x - 1) + g(x).(\tfrac{1}{3}x+1)$

19.3. Let $x = \alpha = \sqrt[3]{5}$: $4 = (\alpha^2+2\alpha+1)(\tfrac{1}{3}\alpha^2+\tfrac{1}{3}\alpha-1)$ (12)

$\therefore \dfrac{12}{\alpha^2+2\alpha+1} = \alpha^2+\alpha-3$

19.4 $x = -3$ (2)

Question 20

Let $p=1$: $7^2-2^2 = 49-4 = 45$.

$\therefore$ True for $p=1$

Assume true for $p=k$:

$7^{2k}-2^{2k}$ is divis by 5.

Let $p=k+1$: $7^{2k+2}-2^{2k+2} = 49.7^{2k} - 4.2^{2k}$ (14)

$= 49(7^{2k}-2^{2k}) + 49.2^{2k} - 4.2^{2k}$

$= 49(7^{2k}-2^{2k}) + 45.2^{2k}$ and this is div. by 5.

$\therefore$ If statement true for $p=k$, it is also true for $p=k+1$. $\therefore$ True for all $p \in N$.

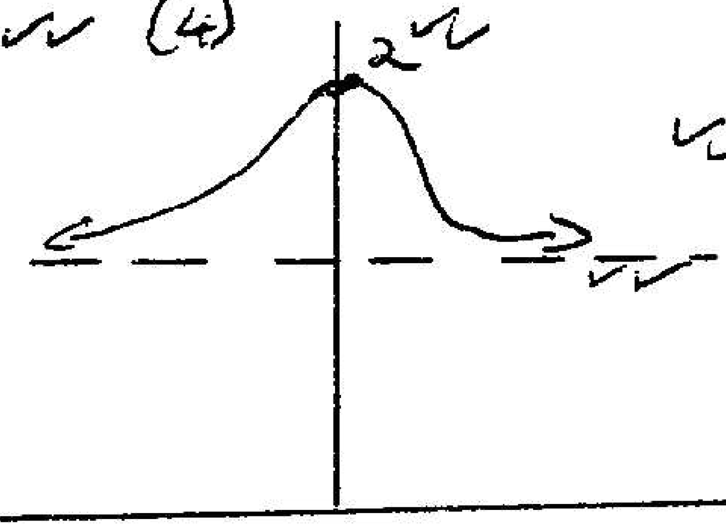
Question 21.

21.1: $y=2$ (2)

21.2. $4x^2+2=0$ has no real roots. (4)

21.3. $x=0$ (4)

21.4.



form (6)

Question 22

$\dfrac{2x^3+3x^2+x+2}{x^2(x^2+1)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{x^2+1}$

$2x^3+3x^2+x+2 = Ax(x^2+1) + B(x^2+1) + (Cx+D)x^2$

$x=0$: $2=B$

x^3: $2 = A+C$ (16)

x^2: $3 = B+D$ $\therefore D=1$

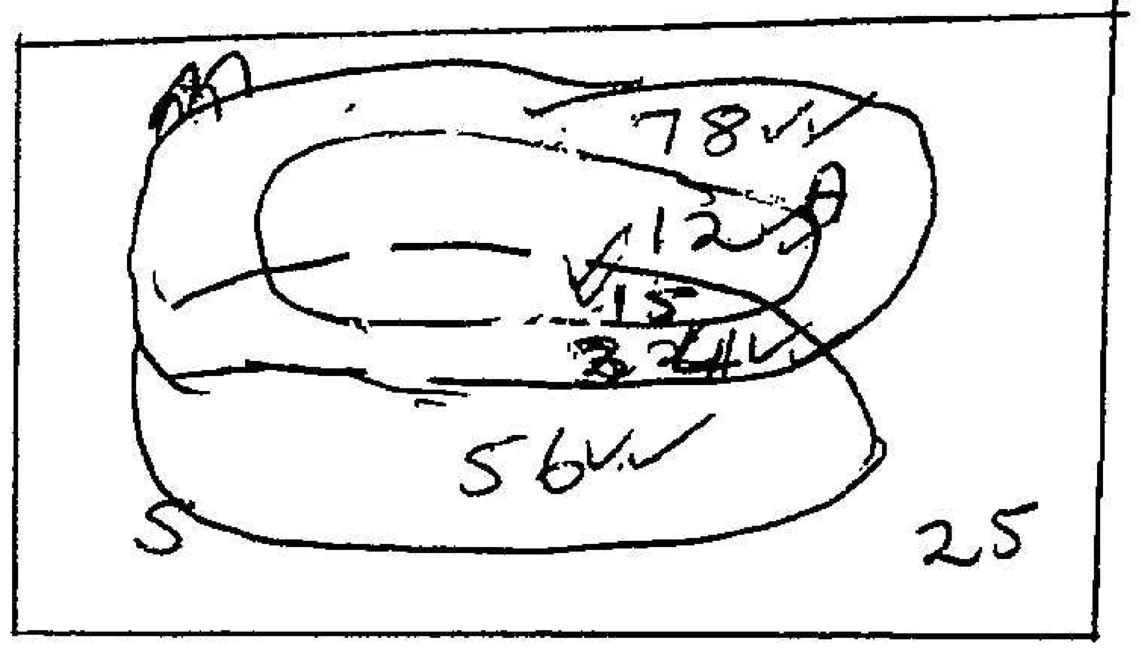
x: $1=A$

$\therefore C=1$ $\therefore \dfrac{1}{x} + \dfrac{2}{x^2} + \dfrac{x+1}{x^2+1}$

Section E

Question 23.

23.1.



(10)

23.2. 78

23.3. $\frac{12+34}{220} = 0,2091$ (6)

23.4 $24+78=102$ (4)

Question 24

24.1.1. $\frac{\binom{11}{4}\binom{6}{1}}{\binom{17}{5}} = 0,3200$

24.1.2. $P(X \le 4) = 1 - P(X=5)$

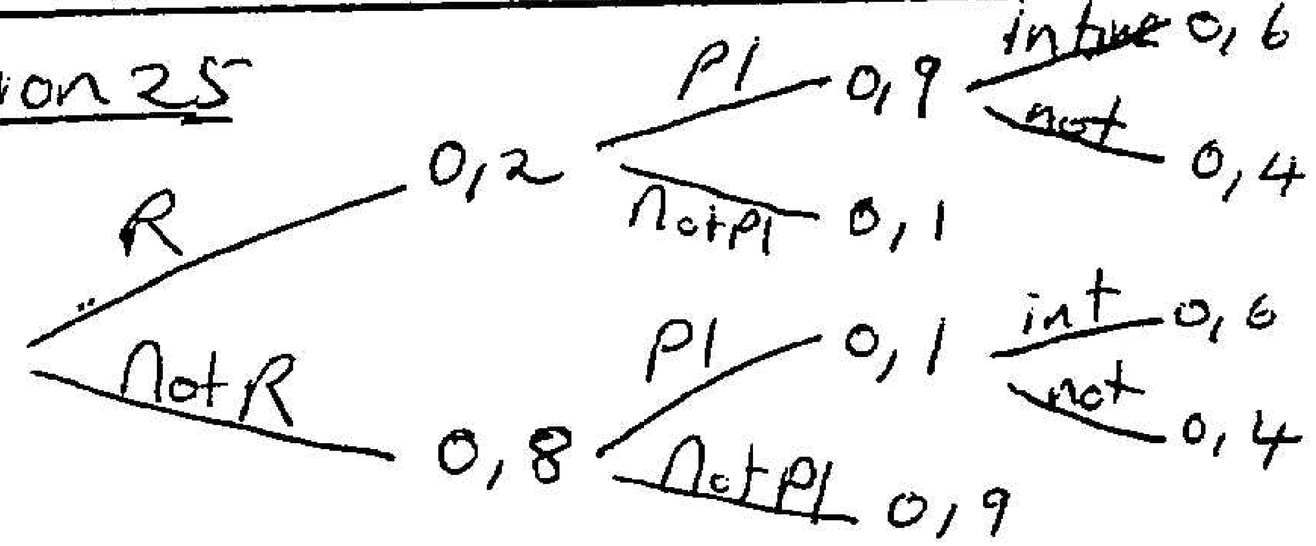
$= 1 - \frac{\binom{11}{5}\binom{6}{0}}{\binom{17}{5}} = 0,9253.$

24.2.1 $P(X=x) = \begin{cases} \binom{5}{x}\left(\frac{11}{17}\right)^x\left(\frac{6}{17}\right)^{5-x} & x = 0,1,\dots 5 \\ 0 & \text{otherwise} \end{cases}$ (8)

24.2.2. $\binom{5}{4}\frac{11}{17}^4\frac{6}{17}^1 = 0,3093$ (6)

24.3 If there were a great number of balls. (2)

Question 25



P (late) =
$0,2 . 0,9 . 0,4 +$
$0,8 . 0,1 . 0,4$
$= 0,104$

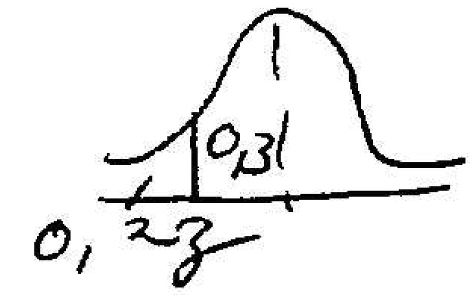
Question 26

26.1. $P(X>30) = P\left(Z > \frac{30-25}{3,4}\right)$

$= P(Z > 1,47)$

$= 0,5 - 0,4292 = 0,0708$ (10)

26.2. $z = -0,84$

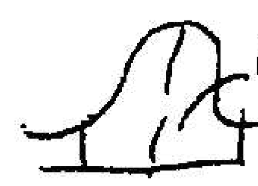


$\therefore \frac{X-25}{3,4} = -0,84$

$X = 22,144 = 22\text{cm}.$

Question 27

27.1.



$z = 2,57$ (or 2,58)

$25 \pm 2,58 . \frac{2,7}{\sqrt{60}}$ (8)

$(24,1042 ; 25,8958)$

27.2. No. (2)